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Sean Yuxiang Wu & Lü Wu

ABSTRACT

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3 General Concerns and 12 Problems in Einstein's Paper and Book on Special Relativity

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ABSTRACT

The theory of relativity has been at the pinnacle of human scientific thought for 120 years. However, whether it should continue to lead is a question for us. Voices questioning the theory of relativity have been continuing, but since they are mostly from a mathematical or experimental point of view, they have not yielded convincing results. We have pioneered a new way of analyzing mathematical models from the perspective of reviewing the rationality of physical models, thus we have seen many problems with relativistic models from different perspectives, which we have grouped into three general concerns and 12 obvious or easy to prove problems. Through these intuitive discussions, we believe that the theory of relativity should no longer lead the scientific and technological thinking of mankind. This article focuses on special relativity. We will continue to discuss general relativity in future articles.

Keywords: relativity, einstein, special theory of relativity.

I. INTRODUCTION

The theory of relativity put Einstein on the altar of science. For 120 years, people also have been questioning the theory of relativity, but to no avail. The reason is that people always question it by mathematics or experiments, which is of no much effect.

Pioneering from the perspective of an application engineer, we saw that Einstein did not correctly and strictly define those physical models like design a precise engineering project, and thus could not use those models correctly according to his own intentions. Therefore, when he used those models, there were already full of loopholes in his applications.

This made us believe that the theory of relativity should no longer lead the scientific thinking of mankind.

This article is a critique of Einstein's special theory of relativity. Therefore, it first describes three key points that need to be concerned in the systems of special theory of relativity. Then, 12 problems of the special theory of relativity were analyzed.

1.1 Three General Concerns

In general, in Einstein's theory of relativity, he did not clearly point out the application restrictions between two reference bodies. This led to many general application problems. Three major concerns are listed below, because of their absence the relative system makes the relativistic application lack a scientific basis, especially when the three are used in combination.

First concern

Theory of relativity does not specify the distance between the two reference bodies. If a clock on the airplane can be relative to the ground clock, then the American flag pole on the moon should be able to be relative to a light beam on the Earth. If the distance between the two reference bodies is more than one light year, the length change or clock slow down would still happen?

Second concern

The theory of relativity does not specify the quality of a reference body. If a moving alloy rod has a diameter of 1000 meters, a ray flies back and forth over the moving rod, will the rod become shorter?

Third concern

The model of the light ray and the rigid rod does not explain how the two reference bodies are

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bound to each other? How do they form a relative system?

Assuming there are 100 rigid rods, how could Einstein make his ray form a relative system with the No.4 rigid rod he wanted to be relative to? Is there any way to bind his ray and No.4 rod to each other so that they can be relative without disturbed by other rods?

For another example, in the experiment of a clock on the airplane and a ground navy clock, how does the airplane clock know it should be relative to the navy clock? Does the airplane clock also be relative to the clocks on space shuttles, on trains, or on cars……?

This relative situation in which the objects join the relative system automatically without knowing by the experimenter's mind is called Passive Relative. Due to the unscientific system design of Einstein's physical model, the Passive Relative is unavoidable.

B. 12 Problems

We listed 12 problems about the special theory of relativity from Einstein's paper [1] and book [3] below. It will follow the following format: In each problem, Einstein's quotation corresponding to the discussing problem is extracted from Einstein's paper [1] or book [3] into the problem. Then, "what is wrong" is discussed and analyzed through this problem, and the key concern in the problem is pointed out.

Problem 1

There are several problems associated with Einstein's Quotation-1 from Section VII of [3]: "*Let us suppose our old friend the railway carriage to be travelling along the rails with a constant velocity v, and that a man traverses the length of the carriage in the direction of travel with a velocity w. How quickly or, in other words, with what velocity W does the man advance relative to the embankment during the process?*"

https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch6

Using the "man-carriage-embankment" system in above Quotation-1, together with following Quotation-2, we prove that the synchronous transmission rule given by Einstein is an obvious error, and is not a "universally valid" rule.

From "§1. Definition of simultaneity" of [1], Einstein's Quotation-2 is such:

"*Suppose a ray of light leaves from A toward B at "A time" t_A, is reflected from B toward A at "B-time" t_B, and arrives back at A at "A-time" t'_A, The two clocks are synchronous by definition if*

$$t_B - t_A = t'_A - t_B.$$

We assume that it is possible for this definition of synchronism to be free from contradictions, and to be so for arbitrarily many points, and that the following relations are therefore generally valid:

1. *If the clock is B is synchronous with the clock in A, then the clock in A is synchronous.*
2. *If the clock in A is synchronous with the clock in B as well as with the clock in C, then the clocks in B and C are also synchronous relative to each other.*

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To easily refer to, we label Einstein's above formula as (1) shown below, since formula (1) will be used from beginning to the end in this paper.

$$t_B - t_A = t'_A - t_B. \tag{1}$$

Now, we take the carriage as A, the man as B, and the embankment as C. Then, according to (1):
A and B are synchronous, that is, the carriage and the man are synchronous.

A and C are also synchronous, that is, the carriage and the embankment are synchronous.

But B and C are not synchronous. That is, the man and the embankment are not synchronous. Because the speed of the man relative to the embankment is (the speed of the man + the speed of the carriage), the back-and-forth speed is not equal. The back and forth times are different, which violates (1).

This is to say item 2 in Quotation-2 is wrong.

Problem 2

The following is an exactly same error as Problem 1. We take Footnote of Section VIII of [3] as Einstein's Quotation-3:

"*We suppose further that, when three events A, B and C take place in different places in such a manner that, if A is simultaneous with B, and B is simultaneous with C (simultaneous in the sense of the above definition), then the criterion for the simultaneity of the pair of events A, C is also satisfied. This assumption is a physical hypothesis about the law of propagation of light; it must certainly be fulfilled if we are to maintain the law of the constancy of the velocity of light in vacuo.*"https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch8

In Quotation-3, Einstein defined a *simultaneity* transmission rule. The meaning of two events simultaneously is that they satisfy (1). Using the same method and steps as the proof in *Problem 1*, we can prove that the simultaneity transmission rule defined by Einstein is also wrong.

In simple terms, let the moving carriage in Einstein's Quotation-1 be B, the moving man in the carriage be A, and the embankment be C. Then, A and B meet (1) so they are simultaneous, and B is simultaneous with C; but A is not simultaneous with C.

Problem 3

The last sentence of Quotation-3 also leads to a serious problem. Einstein said that if this simultaneity transfer assumption is not fulfilled, then the law of the constancy of the velocity of light in vacuo is not held. Now according to our proof in above Problem-2, the simultaneity transfer assumption is really not fulfilled. How should we handle Einstein's law of the constancy of the velocity of light in vacuo?

Problem 4

One of the problems in Quotation-1 is: Since the composite of the man-and-carriage moves relative to the embankment, then, let's assume that there are two points A and B on the embankment that are far apart. According to (1), we need to calculate the time required for the composite speed of the man-and- carriage to move back and forth between A and B. Assume that the train is heading from A to B. Then, when the man moves from B to A in the opposite direction of the carriage, W = w - v. Since the man's speed w is much smaller than the running speed v of the carriage, so w - v < 0, means the man will only move further and further away from point A and will never reach point A. Therefore, the result of the calculation using (1) is $t'_A - t_B = -$ infinity. Or we can say that the man-and-carriage complex only moves in the direction of the train's movement, and has no movement against the direction of the train's movement. This is a situation not handled within Einstein's theory. It can be seen from this that the physical model of man-and-carriage relative to the embankment given by Einstein in Quotation-1 is not a model of relativity, and should not be used to discuss relativity at all.

Problem 5

Another problem in Quotation-1 is that the physical model in this passage is that a man is walking in a moving carriage, but the man is required to be a reference body relative to the embankment, while the embankment is another reference body. The problem is that the speed W of the man moving relative to the embankment is the combination of the man's speed w and the carriage's speed v. The man and the carriage are together formed a reference body in the relative system, and the embankment is another reference body. Then, W is the speed of the reference body composed of the man-and-carriage. The back-and-forth speeds of W are different (coming: W = w + v; going: W = w - v), which violates the regulations of the qualified reference body that must have uniform speed stipulated by Einstein. Therefore, the composite of the man-and- carriage cannot be used as a reference body in Einstein's theory of relativity. In other words, Einstein widely used this unqualified composite object as a reference body in his book [3] to discuss his theory of relativity.

Someone may ask: Einstein uses such a pattern in many places. For example, in "§2. On the relativity of lengths and times" of [1], Quotation-4 says:

Let a ray of light depart from A at the time t_A, let it be reflected at B at the time t_B, and reach A again at the time t'_A. Taking into consideration the principle of the constancy of the velocity of light we find that

$$t_B - t_A = \frac{r_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c + v}$$

where r_{AB} denotes the length of the moving rod—measured in the stationary system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous.

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The time calculation formula used is labeled as (2) as following for later reference:

$$t_B - t_A = \frac{r_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c + v} \qquad (2)$$

Then, doesn't formula (2) also mean that the speed of light does not meet the requirement that the reference body must move at a uniform speed? Because the speeds go back and forth in (2) look like (c – v) and (c + v), they are different, just like (w - v) and (w + v). Many of such patterns discussed by Einstein, like lightning striking from both ends of the carriage, or the raven flying over the carriage, are all the same pattern. Are they all wrong?

The answer is: None of them are wrong. In all of the works of Einstein that we know of, only this system model of man-and-carriage, and embankment that Einstein used extensively in [3] is wrong!

Regarding this question, please see the detailed discussion about the speed of light below. In Problem 7 we will give an answer to it. This is a difficult question. Try first to see if you can answer it.

Problem 6

Einstein emphasized the constancy speed of light in [4]. But in his work, there are contradictory statements.

In the second paragraph at the beginning of [1], Einstein emphasized the principle of the constancy of the speed of light. In the following §1, he said four times that the speed of light in a vacuum is constant and has nothing to do with the motion state of the observer or the light source.

However, in [1] and [3], his writing repeatedly violated this principle set by himself.

Quotation-5 is from "§4. The physical meaning of the equations obtained concerning moving rigid bodies and moving clocks" of [1]:

An analogous consideration—applied to the axes of Y and Z—it being borne in mind that light is always propagated along these axes, when viewed from the stationary system, with the velocity $\sqrt{(c^2 - v^2)}$ *gives us*

$$\partial\tau/\partial y = 0, \partial\tau/\partial z = 0.$$

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Here Einstein calculated that "*light is always propagated along these axes, when viewed from the stationary system, with the velocity* $\sqrt{(c^2 - v^2)}$ ......"

Problem 7

Quotation-6 is From Section VII of [3]:

"w is the required velocity of light with respect to the carriage, and we have w = c – v.
The velocity of propagation of a ray of light relative to the carriage thus comes out smaller than c."

The above description obviously violates "the constancy of the velocity of light in vacuo." At the end of §2 in [1], a similar problem with the speed of light also appears.

This problem is not a big deal and can be corrected by the following calculation. Since the speed of light is constant, we cannot say that w is the speed of light relative to the carriage w = c – v. The speed of light is constant and has nothing to do with the motion state of the observer or the light source. The correct statement should be follows:

The speed of light is completely independent and will not be affected by anything. Suppose the time required for light to travel the length L of the carriage in the stationary system is T. When light at the speed of light travels from point B through the length of the moving carriage L, and arrives the original position of point A, but point A has already moved forward a certain distance at the speed v. The light needs more time to catch up to A. So, the total used time is bigger than T. Similarly, the total time required for light to reach B from A is slightly less than T.

Thus, the back-and-forth times the light used to travel moving distance L in different directions are not the same, and do not satisfy (1).

In this way, we have proved that (1) does not hold in this model, as Einstein wanted to prove; and we have also correctly explained the problem that the speed of light in (2) has not changed. In our proof, the speed of light is always constant, and it is the movement of another reference body - the carriage, that makes (1) not hold. Moreover, the speed v of the reference body (here is carriage) is always uniform, and the carriage can be used as another reference body in the relative system. Therefore, this is a qualified relative system.

This also answers the question that the reader was asked to think about at the end of Problem-5 before reading this section.

In the relative systems composed of light, lightning, flying raven, etc., which Einstein often used, each of them is completely independent to another reference body. In Einstein's theory of relativity, each of them is not affected by another reference body in the system and exists independently with a uniform speed.

Problem 8

Einstein's physical models often fail to take into account the application conditions, leading to various errors. Here is an example.

In Section V of [3] Einstein gave us a new protagonist raven in Quotation-7 below:

Let us imagine a raven flying through the air in such a manner that its motion, as observed from the embankment, is uniform and in a straight line. If we were to observe the flying raven from the moving railway carriage, we would find that the motion of the raven would be one of different velocity and direction, but that it would still be uniform and in a straight line.
https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch6

But there are obvious problems with the physical mode. The raven is different from the light beam in the relative system because their speeds are very different.

When two moving reference bodies are independent of each other, the relative system composed of them cannot maintain synchronization. In addition, it has certain requirements for the reference bodies. The raven flying over the carriage is independent of the carriage. Because the raven's speed is smaller than the speed of the carriage, the raven can never catch up to the other end of the carriage, and there is no way of using it as a reference body to form a relative system. The mathematical model abstracted from this physical model is completely invalid since it cannot use formula (1).

Problem 9

Using Einstein's theory to wipe out any enemy.

First, Einstein said in following Quotation-8 from section XVIII of [3]:

If we formulate the general laws of nature as they are obtained from experience, by making use of

(a) the embankment as reference-body,
(b) the railway carriage as reference-body,

then these general laws of nature (e.g. the laws of mechanics or the law of the propagation of light in vacuo) have exactly the same form in both cases.......

As long as it is moving uniformly, the occupant of the carriage is not sensible of its motion, and it is for this reason that he can unreluctantly interpret the facts of the case as indicating that the carriage is at rest, but the embankment in motion. Moreover, according to the special *principle of relativity, this interpretation is quite justified also from a physical point of view.* https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch18

Quotation-8 describes the relative meaning between two reference bodies in a relative system. Everything in Quotation-8 seems perfect. But if we replace the protagonists with "a light beam" and "an enemy," who is staying in a position or moving at a uniform speed, and replace the carriage with a light beam, and replace the embankment with the enemy. Now let the light and the enemy form a relative system.

What will happen after the replacement?

Using the model of above Quotation-8, the light is not moving, instead the enemy is moving with the speed of light.

In the case of the matter (enemy) as m moving at the speed of the light, according to Einstein's theory $E = mC^2$, the enemy is converted into energy E. He is wiped out, and exists as energy.

Einstein said in *Quotation-9*: *"It is clear that the same results hold good for bodies at rest in the "stationary" system, viewed from a system in uniform motion.*

So, using a light beam to be one reference-body, the enemy be another reference-body; by applying Quotation-8, we can easily and remotely wipe out any enemy.

If we can't wipe out the enemy, it means somehow the theory is wrong.

Problem 10

Quotation-9 is from § 4. Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks of [1]: *the X dimension appears shortened in the ratio 1: $\sqrt{(1 - v^2/c^2)}$, i.e. the greater the value of v, the greater the shortening. For v = c all moving objects—viewed from the "stationary" system—shrivel up into plane figures...*

It is clear that the same results hold good for bodies at rest in the "stationary" system, viewed from a system in uniform motion.

...the travel clock on its arrival at A will be 1/2 $t(v/c)^2$ second slow," and "*a balance-clock at the equator must go more slowly...*"
https://einsteinpapers.press.princeton.edu/vol2-trans/166

In Quotation-9 Einstein discussed the physical meaning of the equations obtained for a moving rigid body and a moving clock.

In whole §4 of [1], there is no other physical or matter content except coordinate motion and transformation. That is a pure mathematics section. But playing the pure mathematic, using the motion of the reference bodies, Einstein concluded that the moving length be shortened *in the ratio 1: $\sqrt{(1 - v^2/c^2)}$*, and *the travelled clock will be 1/2 t $(v/c)^2$ second slow,"* and "*a balance-clock at the equator must go more slowly.*"

If we take two beams of light with the same conditions but moving in completely opposite directions, and make them be relative to the same one rigid rod at the same time, what will be the result?

Can a moving diamond rod become shorter by relative motion?

We cannot prevent moving objects in the world passively relative to Einstein's moving rod. We also cannot prevent moving clocks in the airplanes, in the running trains...., be passively relative to Einstein's clock at any location.

We want to ask: Does the §4 of [1] mean that as long as the mathematics is beautiful, the application can be arbitrary? Will the material world be changed according to pure mathematical inference or calculation?

Lorentz transformation is a theory about electromagnetic fields. Can it be extended to rigid rods, carriages, and other matters at will?

Problem 11

The Quotation-10 in Section VII of [3] Einstein says

"since *the ray of light plays the part of the man walking along relatively to the carriage. The* *velocity W of the man relative to the embankment is here replaced by the velocity of light relative to the embankment.*" This sentence is incorrect. The "light relative to the *embankment* " and "the man relative to the *embankment* " are two completely different modes.

The light and the carriage are independent, so their speeds cannot be superimposed! The light relative to the embankment is also good to form a static relative system.

But for the man walking in a moving carriage, his speed and the carriage's speed must be superimposed. (1) is hold in the relative system they composed. But the motion of the man can't form a relative system with the embankment, which we discussed in Problem-4 and Problem-5. More importantly, it damaged Einstein's conclusion that "absolute simultaneity does not exist." We continue discussing this below.

Problem 12

Einstein emphasized the *relativity of simultaneity* and rejected absolute simultaneity in the theory of relativity. It seems that if there is absolute simultaneity, the whole relative system will crash. Quotation-11 below are the stories about definition of simultaneity.

a) From [5]: *That is why the theory of relativity rejects the concept of absolute simultaneity, absolute speed, absolute acceleration, etc., they can have no unequivocal link with experiences.*
b) From [1], after eighteen years, Two key words [*"simultaneous," or*] were added into this paragraph: *Thus with the help of certain imaginary physical experiments we have settled what is to be understood by synchronous stationary clocks located at different places, and have evidently obtained a definition of "simultaneous,"* or "synchronous" and *of "time."*
https://www.physics.umd.edu/courses/Phys606/spring_2011/einstein_electrodynamics_of_moving_bodies.pdf

c) From [2]: (This paragraph missing two key words *"simultaneous," or*) *With the help of some physical (thought) experiments, we have thus laid down what is to be understood by synchronous clocks at rest that are situated at different places, and have obviously obtained thereby a definition of "synchronous" and of "time."*
https://einsteinpapers.press.princeton.edu/vol2-trans/157

Then, what is simultaneity? How can a system be judged as a relativistic system with absolute simultaneity?

The title of §1 in [2] is "§1 Definition of simultaneity". However, it is strange that in this about 1,000-word paragraph of §1, readers cannot find the definition of simultaneity. There is only one sentence related to the definition of "simultaneity" in §1 of the paper (Quotation-2), but still readers have no way to figure out the precise meaning of simultaneity.

Instead, there is a formula defined as synchronous. In §1, "simultaneous" appears 5 times and "synchronous" appears 7 times. The precise definition of synchronous was given by the formula which we referred to as (2) in Problem 1. But people still don't have a clear definition of "simultaneous."

If we don't have a precise definition of anything, how can we comment on this thing?

So, Einstein set a trap for readers in §1: the title is "§1 Definition of Simultaneity", but he did not give a clear definition of it, instead he gave us a precise mathematical definition for "synchronous". Generally speaking, "simultaneity" is not equal to "synchronous".

This has trapped many people, and it is certain that many so-called "masters of theory of relativity" do not truly understand the theory of relativity.

This trap also protected Einstein's theory of relativity. Because people could not accurately understand the key concepts of relativity, they had to follow the so-called masters who "understood" relativity to support relativity. 18 years later, relativity theory had established its unshakable position, few people would still be interested in what Einstein's simultaneity was.

After eighteen years, the two key words were quietly added into his paper "On the Electrodynamics of Moving Bodies" by Einstein. This modified paper was included in the book "The Principle of Relativity" [1] which Einstein personally arranged to reprint in 1923. He quietly inserted two key words into this English version of the paper 18 years later. Thus, the definition of simultaneity became Quotation-11 b), made it clear that "simultaneity" and "synchronous" in Einstein's paper are the same!

However, in other languages besides English [6-8], the articles still do not contain these two keywords. Einstein's secret revision of the key points of his paper 18 years later is not a decent behavior.

Now we know simultaneity = synchronous, and synchronous has a precise definition by (1). Then we can judge if a relative system is an absolute simultaneity system or not. The judging rule should be:

If a relativistic system always meets (1), then it is an absolute simultaneity relativistic system.

We believe that relative systems satisfying absolute simultaneity exist according to equation (1). An obvious example is the system composed of railway embankment, train carriage, and a man, an old friend in Quotation-1 of [3] discussed above. Continue Problem 11, taking out the man and the train carriage to build up a relativistic system; taking out the man and the embankment to build up another relativistic system. Both relativistic systems maintain absolute simultaneity. Because even if observed from the moon, the systems composed of the man and the train carriage always satisfies the equation (1) – the back-and-forth time will always be the same.

According to Einstein's definitions on absolute and relative synchronous or simultaneity of a relative system, we can find that there are a large number of relative systems that maintain absolute simultaneity in reality, such as the raven walking back and forth on a running train, stewardess walking on flying plane..., their movement always satisfies equation (1), and they are all relative systems that maintain absolute simultaneity. And that's why a sprinter doesn't need to consider running in the same or different direction of the Earth's rotation.

The existence of the absolute simultaneity relative system damaged and negated Einstein's theory of relativity.

III. RESULTS

Since Einstein did not attach importance to the physical model used to abstract out the mathematical model, from the perspective of the rationality of the physical model, the special theory of relativity has various theoretical defects; due to the lack of rigor in Einstein's theory and writing, there are many self-contradictory and unjustifiable statements in his paper and monograph as we listed.

IV. DISCUSSION

Over the past century, under the influence of Einstein, the academic world has been filled with an atmosphere of mathematical supremacy, and Einstein's theory of relativity seems to be.

Starting from analyzing the rationality of the physical model and then discussing the mathematical model derived from it can ensure the rationality of the mathematical model, especially in analyzing the relativistic model that is closely integrated with the application. This also applies to analyzing the mathematical model of general relativity.

V. CONCLUSIONS

Summarizing all problems above, we would like to ask: is the theory of special relativity worth to be the top scientific holy object to continue leading the scientific thinking for another 120 years and more? The answer is negative.

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While aeroponic systems have shown promise for seed potato production, light LED lighting, especially lights, remains largely unexplored. This study, therefore, aimed to uncover potential by determining the light compensation point (Ic) required for potato growth under LED lighting. Consequently, it could contribute to advancing the field of aeroponic farming, offering a promising future for sustainable agriculture.

The light compensation point is the irradiance where photosynthesis equals respiration, and it varies by species and environment. Selecting an appropriate light intensity is crucial for crop productivity and cost-efficiency. Limited research on potato cultivation under LED lighting is available, especially concerning plant photosynthetic rates. This study evaluated the effects of LEDs on potato plants, focusing on CO_2 exchange and biomass production.

Our results revealed valuable insights for aeroponic farming. For instance, we discovered that blue light was the most efficient for CO_2 absorption, with an Ic of ~50 μmol m^{-2} s^{-1}, while red light required 67 μmol m^{-2} s^{-1}. White and mixed lights exhibited higher Ic values (84.6 and 108.5 μmol m^{-2} s^{-1}, respectively). Plant exposure to blue light develops robust structures. Still, they had lower biomass than red treatments, underscoring the need for further optimization of LED spectra in aeroponic farming systems, thereby enhancing crop productivity and cost-efficiency.

Keywords: aeroponic, potatoes, LED, light compensation point, net photosynthesis.

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I. INTRODUCTION

Artificial light source is a critical component in indoor farming since light quality and intensity are one of the most important environmental factors affecting plant growth and morphology (Rabara et al., 2017) but also to prevail economically within the limits of plant growth and cost reduction (Domurath et al., 2012). The light compensation point is the irradiance at which the photosynthetic activity of a plant is equal to its respiratory activity by CO_2 exchange (Nobel, 2009), and it depends on species and growing environments (Taiz & Zeiger, 2002). So, it provides a basis for selecting light intensity suitable for developing a particular crop. Moreover, when we graph the Net Photosynthesis vs irradiance, the slope of the line at the light compensation point indicates quantum yield (φ), mol of CO_2 fixed by mol of photons irradiated (Long & Hällgren, 1993). It is also necessary to consider that 9 mol photons are required to fix a mol of CO_2 (Osborne & Geider, 1987). So, these measurements help better understand the effect of irradiance on crops.

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As an artificial illumination source, Light-emitting Diodes (LEDs) are the most popular for vertical farming or urban agriculture since they can emit a specific wavelength or a combination of them, they have a variety of designs, low heat emission, energy efficiency as well as their long lifetime (Janda et al., 2015; Rabara et al., 2017).

Potatoes are a highly productive crop, and tubers constitute an excellent source of carbohydrates and proteins. Field farming is associated with several risks and uncertainties in vital and environmental stresses, such as high winds, floods, droughts, and pest attacks (Tunio et al., 2020), and also the poor efficiency in the use of natural resources such as soil producing degradation on it and water waste (Lakhiar et al., 2018). Aeroponic culture is a soiless method that offers an innovative solution to ensure the environmental and economic sustainability of food supplies with high nutritional quality, nontoxic food, and labor (Lakhiar et al., 2018).

The potato (*Solanum tuberosum L*) is, due to its nutritional and energy value, an essential and necessary food in the diet of Mexicans, and its cultivation has excellent economic and social importance for many Mexican families (*NOM-041-FITO-2002*, 2003). One of the main challenges for the producers of this tuber is the quality of the plant propagation material, since the seed degenerates due to pathogens and diseases in the planting material continued in the cycles of vegetative propagation, affecting yields and the quality of the crop. The *in vitro* production of potato seed is very convenient since the seedlings are manipulated more easily to eradicate any pathogen present in the tissue (Tapia y Figueroa et al., n.d..). The pre-basic seed is obtained from *in vitro* plants that originate from the culture of meristems, and these can be used directly for the production of mini tubers (*ex vitro*) or the formation of microtubers (*in vitro*) (*NOM-041-FITO-2002*, 2003; Tapia y Figueroa et al., n.d..).

Potatoes are traditionally grown under open-field conditions, and fewer studies have been conducted on cultivation in a controlled environment illuminated with LEDs. The related lack of information in the literature excludes the benefits of LED or dichromatic light *on* potato plants or, particularly, the photosynthetic influence on tuberization.

Our research aimed to experimentally determine the minimum light necessary for growth in this culture. We tested photosynthetic rates in aeroponic potato culture under industry-relevant light intensities and various light qualities, measuring CO_2 leaf gas exchange. These experiments allowed us to determine the light compensation points (LCP) under different conditions, thereby proposing a suitable light intensity for growing *Solanum Tuberosum L.* aeroponic plantlets.

II. MATERIALS AND METHODS

2.1 Plant material, growth chamber, environmental conditions, and experimental design

Potato (*Solanum tuberosum L.)* variety Fianna plants *in vitro* were donated by the Plant Biotechnology Laboratory of the Instituto Tecnológico de Tlajomulco and transplanted *in vivo* in an aeroponic chamber divided into four zones. Each zone was isolated from any external light source and was illuminated for 10 days with white LED light by Philips HUE multicolor smart bulbs. After transplantation, 10 plants were allocated to each of the four light treatments and kept for the 8-week experimental period, Figure 1. The environmental conditions inside the growth chamber were set to a 12/12 h photoperiod. The energy provided by the LEDs in the chamber was fixed at 10.65 J m^{-2} s^{-1}, which means an irradiance of 61, 42, 51 and 48 µmol m^{-2} s^{-1} for the colors red, blue, 50% blue:50% red (mix) and white (33% blue, 32% green. 8% yellow, 15% orange and 12% red) respectively, measured with the limited by the technology of the HUE lamps for the blue light. Planck-Einstein's equation (6) was used to convert from PPFF to energy.

The Planck-Einstein equation relates the energy of a photon to its wavelength. It is expressed as:

$$E = h(\frac{c}{\ }) \quad (6)$$

Where:

- E is the energy of the photon (in joules).
- h is Planck's constant, approximately 6.626×10^{-34} J m² s⁻¹
- c is the speed of light in a vacuum, approximately 3.00×10^{8} m s^{-1}
- λ is the wavelength of the photon (in meters).

Relative humidity 70 ± 10%, and temperature of 24 ± 1°C. Water spray irrigation was one minute every two hours. The plant nutrient solution was made with (meq/L) 10.5 of N, 11.6 of P, 9 of K, 6.1 of Ca, 1.1 of Mg, and 0.5 of S.

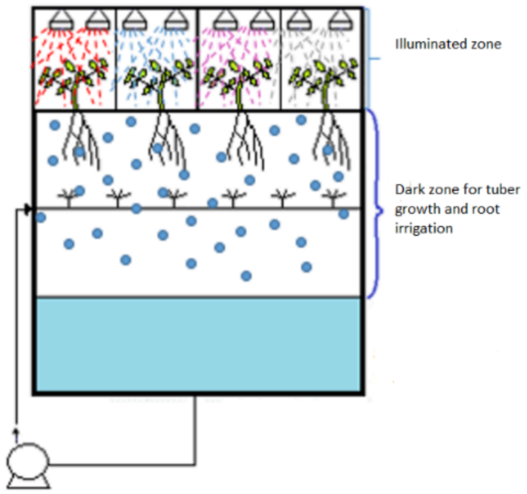


Figure 1: Aeroponic chamber scheme for potato growth

The experiment was performed twice for a randomized design of growth conditions.

2.2 Net photosynthetic rate determination

To determine the net photosynthesis, Pn, as the CO_2 measurements were made at 100 minutes of illumination, the Pn_{100}, not the total Pn for the plant. The equation proposed by Long & Hallgreen (1993) for closed systems was used:

$$Pn = \frac{C_1 - C_2}{(t_1 - t_2)} * \frac{V}{22.4} * \frac{P}{101.325} * \frac{273.15}{T * s} \quad (7)$$

Where:

Pn = Net photosynthetic rate (μmol m^{-2} s^{-1})
C = molar fraction CO_2 inside the cabin (μmol mol^{-1})
V = cabin's volume (L)
P = atmospheric pressure
S = leaf surface (m^2)
t = time (h)

The Light Compensation Point, LCP, is obtained from the graph Pn vs irradiance for each treatment; more specifically, it is the point cross with the x axis. The slope at this point is called an apparent quantum yield, φ, and it indicates the amount of CO_2 fixed by mol of photons incident and not absorbed (Long & Hällgren, 1993).

2.3 Phenological monitoring

At the end of the treatment, each plant was measured to determine shoot length, shoot diameter, number of leaves, and root length. Plant tissue samples were dried in an oven for 48 h at 90°C before being weighed.

2.4 Statistical Analyses

Statistical analyses were carried out using the InfoStat Professional v.1.1 program. All the parameters were subjected to variance (analysis ANOVA) and a t-test. Differences were accepted as statistically significant when $P < 0.05$. Tuckey's test was carried out to identify significance among the samples.

III. RESULTS AND DISCUSSION

3.1 CO_2 Absorption at Different Light Intensities

Figure 5 presents the CO_2 absorption by the plants in the cabinet. The first 100 minutes correspond to measurements in the dark. The dotted lines indicate the boundary between dark and illuminated conditions. Overall, the results show that the higher the illumination, the higher the CO_2 absorption.

Under 10 PPFD, all treatments maintained a slope similar to mitochondrial respiration. We can see that even when photosynthesis has started, this light intensity is insufficient to disrupt the balance between the CO_2 absorbed via photosynthesis and the CO_2 produced by mitochondrial respiration. When exposed to 50 PPFD, only plants under blue light reached equilibrium (CO_2 absorbed = CO_2 produced). Plants treated with white and mixed lights reached this equilibrium at approximately 100 PPFD. Meanwhile, plants under red light required 100 PPFD to generate a positive CO_2 balance. Blue light proved the most efficient in facilitating CO_2 absorption, while red light was the least efficient.

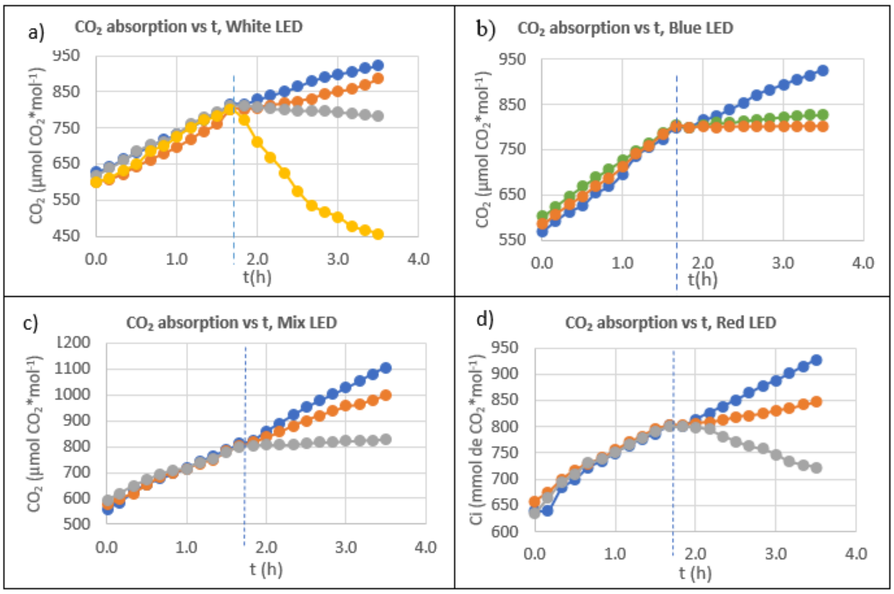
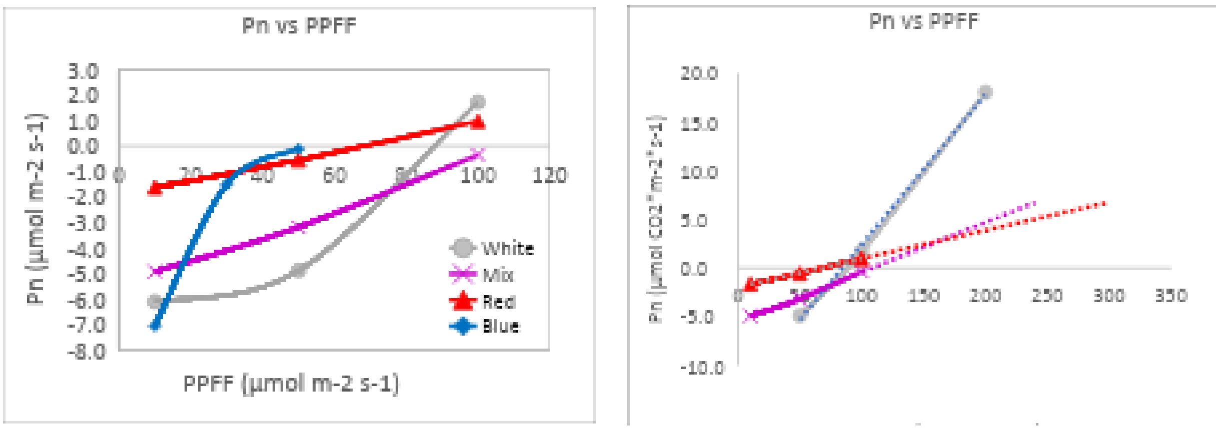


Figure 5: CO_2 absorption at different light intensity. CO_2 absorption vs time measured under a) white LED, b) blue LED, c) mix LED, and d) red LED. The blue line shows the CO_2 absorption by the plant to

10 PPFF, the green line to 30 PPFF to range line is the answer of the plant to illumination of 50 PPFF, gray line is the answer to 100 PPFF and yellow line to 200 PPFF. The vertical dotted lines indicate the beginning of the illumination.

3.2 Light Compensation Point and Quantum Yield

Figure 6 illustrates the relationship between net photosynthesis (Pn100) and light intensity for each treatment. Table 4 summarizes the light compensation point (LCP) and quantum yield (ϕ) calculated from these data. Blue light treatments achieved the lowest LCP (~50 µmol m^2 s^{-1}), significantly lower than white (84.6 µmol m^2 s^{-1}), mixed (108.5 µmol m^2 s^{-1}), and red light (67 µmol m^2 s^{-1}) treatments. Therefore, this suggests that blue light promotes higher photosynthetic efficiency under low light conditions, consistent with findings that highlight the role of blue spectra in enhancing CO_2 uptake and stomatal conductance (Li et al., 2017; Paradiso et al., 2019). The quantum yield was highest for white light (ϕ = 0.1543), followed by mixed, red, and blue lights. In contrast, plants under blue light operated near their LCP, which limited the calculation of a reliable quantum yield equation.



a) b)

Figure 6: a) Pn_{100} vs irradiance until 100 µmol m^{-2} s^{-1}, b) Pn_{100} vs irradiance until 200 µmol m^{-2} s^{-1}, showing the extrapolation lines and their equations at the LCP.

The light compensation point is mainly related to respiration and the light source used for the treatments (Azcón & Osmond, 1983). Due to the characteristics of this experiment, where dark respiration was established at 800 ppm, there are no other differences between the experiments, and calculated LCP was affected only by light source. Plants were grown in a light level close to the blue and red light compensation point. In this way, these plants grow and pass through the vegetative stage. Increasing the PPFD of light above the compensation point with these treatments is necessary to understand the light effect on biomass and the tuberisation process.

The light compensation point (LCP) **(**Table 4, Figure 5) was achieved at the lowest photon flux density with blue light (50 µmol m^{-2} s^{-1}), making it the most efficient for photosynthesis. Red, mix, and white light required higher fluxes (67, 84.6, and 108.5 µmol m^{-2} s^{-1}, respectively). The high efficiency of blue light correlates with its ability to increase stomatal conductance and net photosynthesis (Li et al., 2017). However, white light's quantum yield (Φ) was the highest among treatments, indicating its potential for efficient energy use once plants grow above the LCP. These results support further testing of white light under optimized conditions.

Table 4: Equations of the graphs Pn vs PPFF on light compensation

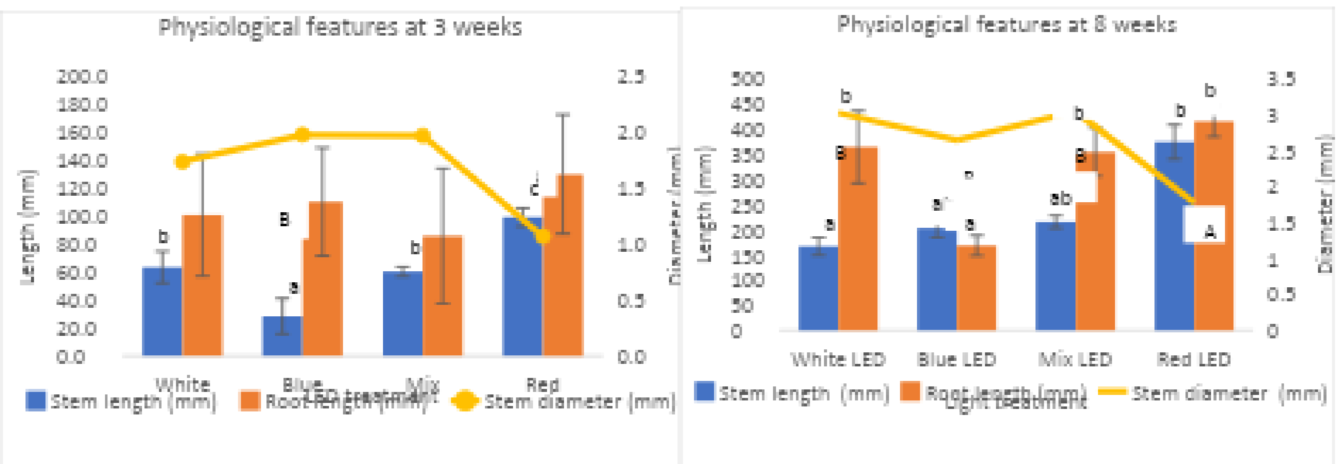
LED light treatment	Eq. Pn vs PPFF	LCP (µmol m^{-2} s^{-1})	Φ (µmol CO_2/ µmol photon)
White	y= 0.1543x - 13.045 R^2= 0.997	84.6	0.1543
Blue	----	~50	---
Mix	y= 0.051x - 5.54 R^2 = 0.9948	108.5	0.051
Red	y= 0.029x - 1.94 R^2=0.9986	67	0.029

LCP, Light Compensation Point; φ, Quantum yield

3.3 Dry Biomass

The results in Figure 7 show that at three weeks, all plants developed longer roots than stems. At this stage, plants grown under red light displayed the longest roots, whereas those under mixed light had the shortest roots. In some studies made with *in vitro* plantlets, monochromatic red and far-red findings were stem elongation and leaf thinning in those seedlings producing fragile plants (Chen et al., 2020; Miyashita et al., 1995). This researcher found that blue light caused leaf expansion and increased leaf thickness (Chen et al., 2020) and hurt the growth of potato seedlings, but the plants were more robust and had broader leaves (Miyashita et al., 1995).. In our experiments, the same growth tendencies were observed in plants at three weeks of transplanting, However, at the end of eight weeks, these differences disappeared as observed in Figure 7b, mainly tainning only the thickness in stem and fragility for plants under red light treatment, which also showed the thinniest leaves. However, to compensate, plants increased the amount of leaf biomass, translating to a more significant number of leaves per plant.

Blue light produced the shortest roots at both time points, significantly smaller than other treatments.



a) b)

Figure 7: Physiological growth at week 4 (a) and eight weeks (b) after transplanting. The left axis refers to the length in mm for stems and roots. The right axis is reffered steam diameter. Data are presented as means ± standard error (n = 4). Different letters indicate significant differences between values ($p < 0.05$).

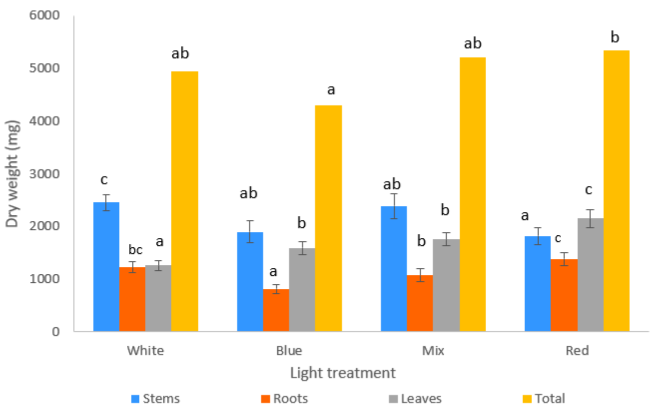


Figure 8: Dry weight of three plants after 8 weeks of light treatment at 10.65 J m^{-2} s^{-1}. Data are presented as means ± standard error (n = 4). Different letters indicate significant differences between values ($p < 0.05$).

IV. CONCLUSIONS

Solanum tuberosum L. plants derived from *in vitro* culture successfully grew in our aeroponic chamber illuminated with LED lights in blue, red, mixed (50:50 red and blue), and white spectrums over eight weeks. The findings highlight significant differences in the physiological responses of plants to varying light qualities and intensities. Blue light most efficiently achieved CO_2 equilibrium at the lowest light compensation point (~50 µmol m^{-2} s^{-1}), promoting robust plant structures. Red light, although less efficient, produced the highest foliar biomass. The mix and white light treatments required higher irradiance levels to reach the compensation point, yet they balanced structural development and biomass production.

These results indicate white and mixed light spectrums as promising candidates for further studies, particularly under conditions above their respective light compensation points. However, this study did not progress to the tuberization stage, a critical phase to validate the suitability of these light treatments for mini-tuber production. Future experiments should evaluate these light spectrums under various intensities above the compensation point to determine the optimum light conditions for maximizing tuber yield and quality. Based on these findings, we recommend using white light for future experiments due to its high Φ and mixed light for balancing energy use and biomass output. Further research should explore the impact of light intensities above the LCP to optimize growth and tuberization.

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A New Method of Solving Chess Problems based on a Purely Mathematical Solution

Mustapha Bakani

ABSTRACT

The practice of the game of chess leads to the development of skills related to memory, logic, concentration, rigor, strategy and the capacity for abstraction. In addition to the benefits observed on learning citizenship, by respecting the rules and others. Solving chess problems is an interesting variation for realizing intellectual development. The common way is to present problems on the chessboard or through diagrams. Here, we present a new method of solving chess problems based on a purely mathematical solution. Concretely, it is a question of solving a chess problem thanks to the solution of equations and the mathematical analysis. Thus with a basic knowledge of mathematics, generally of the secondary level, we can proceed to the resolution with a minimum of knowledge of chess, given that the resolution is done from the algebraic notation of the said problem. Here we advance definitions, properties and theorems. Also we present here an example of a chess problem solved by the method.

Keywords: mathematical solving of chess problems, fundamental mathematics of games, chess composition.

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I. INTRODUCTION

The resolution method that we have developed essentially concerns the didactics of mathematics. This is a new multidisciplinary practical field for teachers and students to test their abilities in mathematical problem solving, manipulation of data displayed on tables programmed by these same students in a simplistic way (Excel for example) and of course the patience and rigor specific to both disciplines. However, we believe that it would be wise to expose - briefly – some other aspects related to algorithms within the framework of game theory. The problem be comes difficult in the case of an nxn board.

For advocates of Chess in Education (CIE), chess offers a powerful tool to build a conceptual understanding of math in children. While the Internet is awash in clever programs that gamify the teaching of early math, chess provides an immediate, direct, and tactile offline tool for teachers. [1]

The game of chess belongs to a set of games sharing common properties, called combinatorial games, of which here are the two main properties: “There is no random intervention. No dice are rolled and no cards are drawn. "These are full-information games. To choose his move, each player has all the information about the game to make his decision. This excludes for example the game of naval battle, where the board of the adversary is hidden". These two properties make it possible to eliminate any luck factor, so that in the presence of two players playing perfectly, the result of the game is known in advance. In education, the use of traditional games and especially chess is recommended. Indeed, on the cognitive level, the game chess "promotes the learning of logic and the development of the spirit of analysis and synthesis, or memory", essential skills for the student to solve a problem. [2]

Problems and modeling: A chess problem, also called a chess composition, is a puzzle set by the composer using chess pieces on a chessboard, which presents the solver with a particular task. For instance, a position may be given with the instruction that White is to move first, and checkmate Black in n moves against any possible defence. A chess problem fundamentally differs from chess play in that the latter involves a struggle between Black and White, whereas the former involves a competition between the composer and the solver. Most positions which occur in a chess problem are 'unrealistic' in the sense that they are very unlikely to occur in chess play. There is a good deal of specialized jargon used in connection with chess problems. There are various different types of chess problems: Direct mates, Help mates, Self mates, Reflex mates, Series mates, Studies, Retrograde analysis problems, Shortest proof games, Construction tasks. There are other types of chess problem which do not fall into any of the above categories. Some of these are really coded mathematical problems, expressed using the geometry and pieces of the chessboard. Famous such problems are the Knight's tour and The Queen puzzle.

Knight's tour problem: The object of the puzzle is to find a sequence of moves that allow the knight to visit every square on the board exactly once. It is a direct mathematical problem, related to the Hamiltonian path problem in graph theory. It appeared for the first time in arabic manuscripts in the 9th century and was very popular among mathematicians from the 18th century due to all possible different solutions. Euler presented a very famous solution in the Berlin Academy of Science in 1759 based on the premise “divide and conquer.” [3]

8 queens problem: The problem initially posed by K.F. Gauss in 1842, proposed by Max Bezzel in Germany in 1848 is as follows: is it possible to place 8 queens[i] on a chessboard without any queen threatening another? Gauss himself found 72 solutions to this problem, but there are 92, if you ignore the natural symmetries of the chessboard. The correct solution was found in 1972 with the help of computers and backtracking; 92 solutions were found in total where 12 of them are linear independent (Ramirez, 2004). It is rather the generalization to an nxn chessboard that poses a problem when n is very large. [4]

These problems are usually solved by a classic technique called backtracking in computer science. Backtracking is a resolution process that is particularly suitable for this type of problem. We can describe the course of a game by a graph whose vertices are the different states of the game and the arcs represent the transitions from one state to another. In the special case where it is impossible to return to a past state in the game timeline, the graph has no cycles and is therefore a tree. The maximum number of alternatives to pass from one state to another of the game fixes the arity of the tree and the beginning of the game is logically the root of the tree. Such a tree is called a decision tree. Backtracking designates a way of traversing such a tree and this traversal is often implicit. [5]

Chess solving software: Many programs have also sprung up to verify the correctness of a chess problem. This type of program is very specific, because contrary to a game program, it must analyze all the possible moves, since a problem which would have other solutions than those wanted by the author would be demolished.

What is meant by "solving the game of chess"? Every chess player has one day faced with a problem of the type "White plays and checkmates in n moves". For such a problem, regardless of Black's responses, White manages to checkmate in n moves or less. The problem is correctly analyzed once all Black answers are taken into account. As the human player progresses, he can study positions of more and more complex, but the length of analyzes required means that the problems rarely exceed 5 or 6 moves. The computer then comes to support the human player in its analysis, and for endgame positions, it can then be determined whether, assuming a perfect game, one of the two players is in a position to win, or if the game will end in a draw.[6]

Chess problems take many different forms. The most common form is given by the specification of a position on the chessboard, the specification of the state of play and a statement of the solution condition. Note that this is an extremely general form and can cause many different kinds of problems.

In this article we wish to present a new method of solving chess problems different from current methods at the theoretical level and complementary to them on the level of general interest.

At the theoretical level, it seems that the subject "solving chess problems in a purely mathematical way: using the solution of equations and mathematical analysis" is not well present in the literature proper to these kinds of topics. In contrast, solving chess problems in a computer context is based on tree algorithms, which are good for the machine. Our method is based on new mathematical functions defining the movements of the pieces and their properties. Thus, we have avoided the theory of graphs present in the other methods, which makes it possible to solve compound chess problems by hand, taking into account the theoretical development carried out, thanks to the resolution of equations and the analysis through a dashboard that can be programmed using simple tools.

This way makes it possible to exploit the properties of chess in the learning of mathematics. A path that might be interesting to explore.

Perspective: Theoretical development reveals hidden links with physics. Indeed, within the complexity of chess lies a network of algorithms, models and structures that can be mathematically explored.

The interaction between moves, positions and strategies in chess can be interpreted as a dynamical system, similar to the behavior of particles in physical systems. This connection has opened new avenues for understanding chess problems and designing new methods for solving them. [7]

Here we present the theoretical development specific to our method of resolution.

II. A MATHEMATICAL CHESS PROBLEM

Note: we denote: K=King, Q=Queen, R=Rook, N=Knight, B=Bishop, P=Pawn.

Without a chess board or diagram available, solve the following chess problem: “Find mate in two moves from the following position: (White: Ke1; Rh1; Ng3; Nf5; e2; h3) and (Black: Kg2; f3).

This problem is simple, and established composers or solvers may find it easy to solve (blindly). But for a beginner who finds it difficult to solve a problem laid out on a chess board or in the form of a diagram, mentally analyzing the initial position, it is not easy to think of all the eventualities from the initial position, provided in algebraic notation. However, a mathematical formalization of some rules of the chess game allows the transfer from the chessboard (3D) or diagram (2D) to a digital scoreboard, supporting mathematical analysis. So, with a modest knowledge of chess, we can try to solve chess problems using basic math (usually secondary school).

To do this, we introduce functions for moving chess pieces and controlling chessboard squares. Programming in Excel (or other) allows us to perform repetitive calculations and draw up control tables. Thanks to a mathematical analysis including the resolution of algebraic equations, one manages to solve certain chess problems composed for this purpose, using the usual mathematical tools; such as classical logic or solving equations and systems of equations, for example.

III. DEFINITIONS AND NOTATIONS

3.1 Geometric representation of the position

We project the chessboard on a suitable finite plane (℘) provided with an orthonormal coordinate system $(O,\vec{i},\vec{j})$ of unit one and of origin O(0; 0). We thus define any square M of the chessboard by its strictly positive integer Cartesian coordinates $(x_M; y_M)$ in the Plane $(\wp)(O,\vec{i},\vec{j})$

The pieces (or figures) are represented by capital letters. These same letters can also represent the squares on which the corresponding pieces are located, if there is no cause for any confusion. Each letter actually represents a type of piece (unique to Orthodox chess) that acts as test functions when it comes to checking the value of a given move, for example.

We will denote ***A*** a white piece and ***A'*** a black piece of the same nature. We denote by ***A*** a square occupied by a piece ***A***.

a) Algebraic writing of the position of the diagram

The following example illustrates a position given in algebraic notation and how to relate it to a mathematical plane.

Whites: Kf8; R1e3; Bd5; Ne6; R2ç7; g6. Blacks: Kd8; Ne4; Bf5; Rg7; ç6.

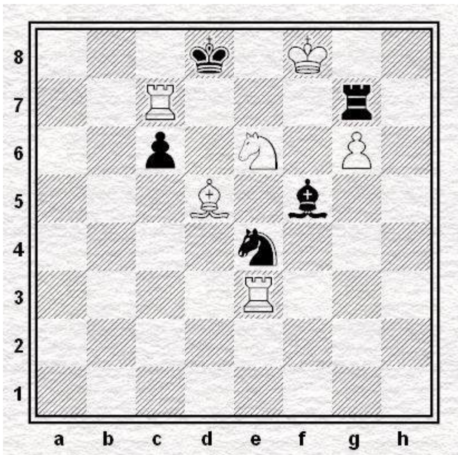

Fig. 1: Position diagram

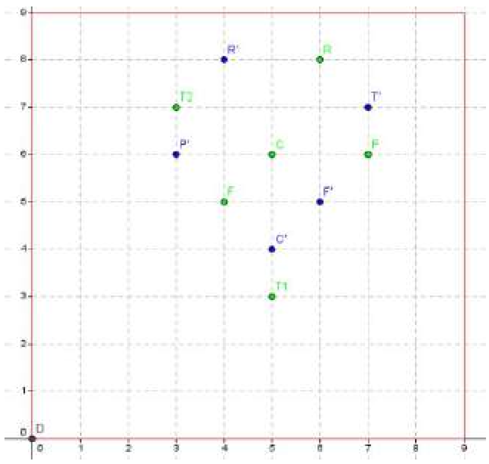

Fig. 2: Representation in Plan (℘)

b) Writing in Cartesian coordinates

Consider that the letters from a to h correspond respectively to integers from 1 to 8.

a	b	c	d	e	f	g	h
1	2	3	4	5	6	7	8

Fig. 3: From letters to numbers

We replace the letters corresponding to the columns by the numbers to have the x-axis. The y-axis are the line numbers.

We thus obtain, for each piece and square, a representation in Cartesian coordinates.

WHITE : $K(^{6}{}_{8})$;$R_1(^{5}{}_{3})$;$B(^{4}{}_{5})$;$N(^{5}{}_{6})$;$R_2(^{3}{}_{7})$; $P(^{7}{}_{6})$
BLACK : $K'(^{4}{}_{8})$;$N'(^{5}{}_{4})$;$B'(^{6}{}_{5})$;$N'(^{7}{}_{7})$;$P'(^{3}{}_{6})$.

Note: The transfer of the pieces to our plane is done from the algebraic notation of the position. The diagram is therefore not necessary to deal with the problem!

3.2 Lines and circles

For some, the movement of the pieces was posed in an arbitrary way by the "inventor" of Chess. Perhaps this proposal is not entirely sound!
In our benchmark, the pieces seem to follow a certain geometric logic.

Explanations:

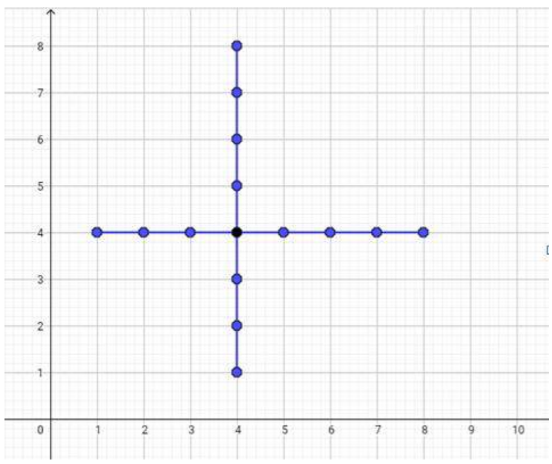

Fig. 4: The Rook moves on the horizontal and vertical squares

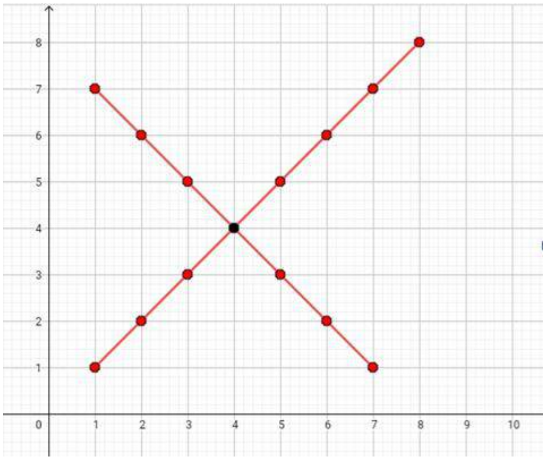

Fig. 5: The Bishop moves on the diagonal squares.

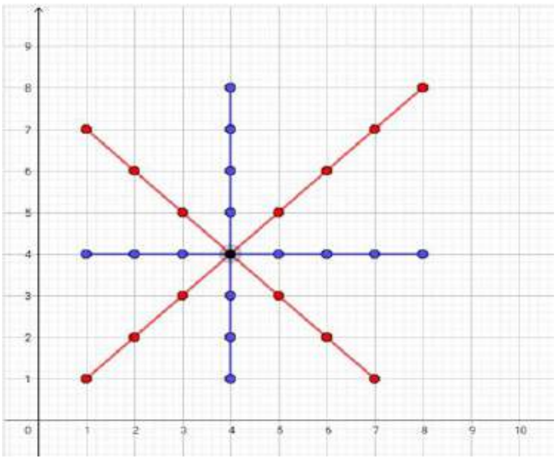

Fig. 6: The Queen moves on the horizontal, vertical and diagonal squares. It possesses both the qualities of the Rook and those of the Bishop.

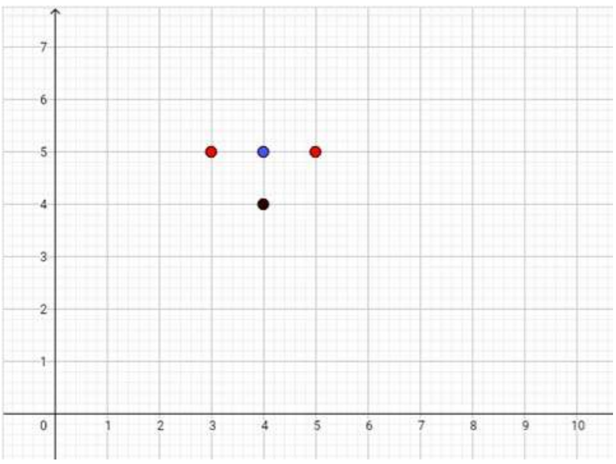

Fig. 7: The pawn plays on the adjacent vertical square and takes on the 2 adjacent diagonal squares above for a white pawn and below for a black pawn.

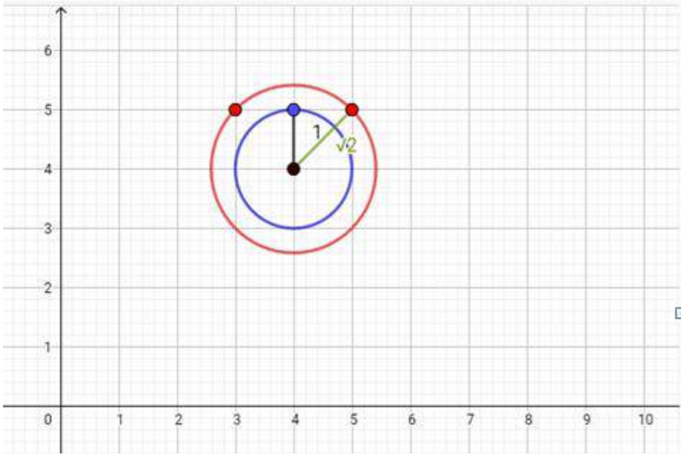

Fig. 8: The arrival points of the pawn belong respectively to the circles of origin the starting point and of radii 1 and √2.

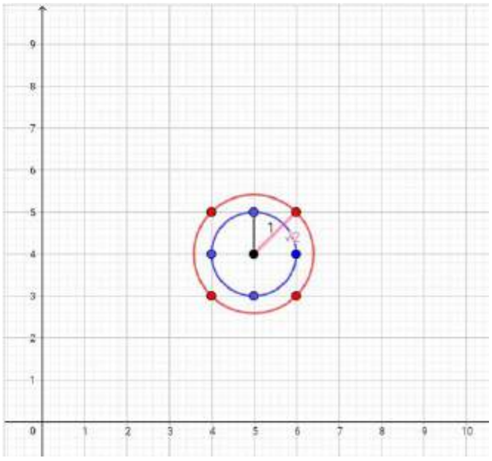

Fig. 9: The King moves on adjacent horizontal, vertical and diagonal squares. The arrival points of the King belong respectively to the circles of origin the starting point and of radii 1 and √2.

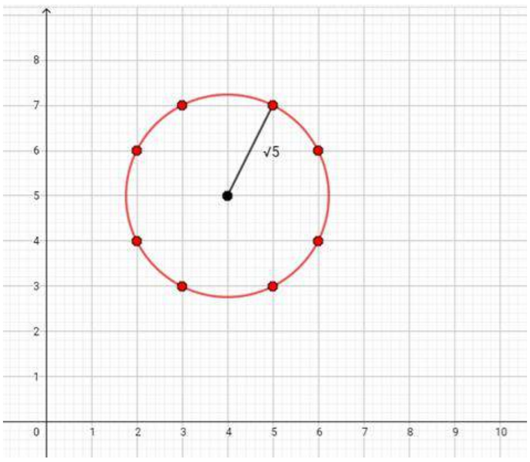

Fig. 10: The end points of the Knight belong to the circle of origin the starting point and of radius √5.

Here we distinguish two families: that of straight lines (Queen, Bishop and Rook) and that of circles (King, Knight and Pawn). In this order the chess pieces seem to represent a good part of the reality of the old wars.

3.3 Squares and Neighborhoods

Let us denote by ξ the set of usual pieces of orthodox chess[ii]

$$\xi = \{K; P; N; B; R; Q\} \cup \{K'; P'; N'; B'; R'; Q'\}$$

Let Ƥ={K; P; N; B; R;Q} and Ƥ'={K′; P′;N′;B′; R′;Q′}

With:

K = King; P = Pawn; K = Knight; B = Bishop;R = Rook and Q = Queen, white pieces and K′,N′, B′, R′ and Q′, black pieces.

Denote by Ω the set of squares on a chessboard. L and M ∈ Ω,

We denote by V(M) the set of adjacent squares called here neighboring to square M. V(M) = {N;NW; W;SW;S;SE; E;NE} ⊂ Ω.

V(M) is said Immediate neighborhood of M.

NW	N	NE
W	M	E
SW	S	SE

Fig. 11: Neighborhood of M

3.4 Pieces and Control Functions

a. Movement Control

Definition: a piece A controls an M square if A can reach M after the next move.

For each type of piece of ξ and any square M of Ω, we define P,N, B, R,Q and K as applications of Ω in ℤ, indicative of the possibility or not of the existence of a given piece on a given square after play of the next move.

These notations can also represent two-variable functions of $\mathbb{Z}^2$in ℤ defined by :

- Knight : $N(M) = f_C(x; y) = x^2 + y^2 - 5$
- Bishop : $B(M) = f_F(x; y) = x^2 - y^2$
- Rook : $R(M) = f_T(x; y) = xy$
- Queen : $Q(M) = f_D(x; y) = xy\,(x^2 - y^2)$
- King : $K(M) = f_R(x; y) = \prod (x^2 + y^2 - m)\ {}^{2}m=1$
- Pawn : $P(M) = f_P(x; y) = (y - 4) \prod (x^2 + y^2 - m)\ {}^{2}m=1$

Such as m = 1 (Vertical / horizontal movement) or m = 2 (diagonal movement), And y > 0 (if white pawn) or y < 0 (if black pawn) ;

With, for each square M of Ω and each piece A of ξ :

$$x = x_M - x_A \text{ and } y = y_M - y_A \text{ Where } M(x_M; y_M) \text{ and } A(x_A; y_A).$$

b. Neighborhood control

The restrictions of the control applications on ⩒(M) allow to test the controllability of the M square and its immediate neighborhood (its adjacent L squares) by a piece of ξ. They are

9 defined by:

- Knight : $N(L) = (X + \alpha_L)^2 + (Y + \beta_L)^2 - 5$
- Bishop : $B(L) = (X + \alpha_L)^2 - (Y + \beta_L)^2$
- Rook : $R(L) = (X + \alpha_L)(Y + \beta_L)$
- Queen : $Q(L) = (X + \alpha_L)(Y + \beta_L)\,[(X + \alpha_L)^2 - (Y + \beta_L)^2]$
- King : $K(L) = \prod ((X + \alpha_L)^2 + (Y + \beta_L)^2 - m)$ ^{2}m=1
- Pawn : $P(L) = (X + \alpha_L)^2 + (Y + \beta_L)^2 - 2$, such as :

$Y + \beta_L > 0$ (if white pawn) or $Y + \beta_L < 0$ (*if* black pawn).

With, for each square M of Ω and each piece A of ξ :

- $X = x_M - x_A$ et $Y = y_M - y_A$ where $M(x_M; y_M)$ and $A(x_A; y_A)$;

$\alpha_L$et $\beta_L$parameters of the set {0;±1} linked to the neighboring cells of M and are defined in the following table :

L	M	N	NW	W	SW	S	SE	E	NE
α_L	0	0	-1	-1	-1	0	1	1	1
β_L	0	1	1	0	-1	-1	-1	0	1

Fig. 12: Neighboring squares of M parameters

IV. PROPERTIES

Suppose the trait is white.

Either A ∈ Ƥ and generally $A_i(i = 1, 2, 3, \ldots)$ white pieces of ξ.

A′ ∈ Ƥ′, A'_i(i∊ℕ) black pieces of ξ.

In what follows we will deal with the concepts within the strict framework of orthodox chess and we will assume that: $A(x_A; y_A) \in \xi : (x_A; y_A) \in \mathbb{N}^2 / 1 \le x_A \le 8$ et $1 \le y_A \le 8$.

1. Check the King

In a given position of a chess problem, the Black King K′ is put on failure by one (or more) piece A (A_i, i ∈ ℕ) if and only if A(K′) = 0 (A_i(K′) = 0).

2. Control of the royal neighborhood

In a given position of a chess problem, a square L adjacent to that of the Black King is controlled by one (or more) piece A (A_i, i ∈ ℕ) if and only if A(L) = 0 (A_i(L) = 0).

3. Self-locking

a. Definition

Let A′ ∈ Ƥ’ and L ∈ ⩒(K′). Let's ask : $\mathcal{B}_{A'}(L) = (x_L - x_{A'})^2 + (y_L - y_{A'})^2$ Where $A'(x_{A'}; y_{A'})$ and $L(x_L; y_L)$.

In a given position of a chess problem, Black King K’ is blocked by A’ if and only if it exists L ∈ ⩒(K′) such that $\mathcal{B}_{A'}(L) = 0$.

b. Lemma

In a given position of a chess problem, Black King K' is blocked by A' if and only if K'(A') = 0.

4. Small castling

For castling to be possible, it is necessary and sufficient that all of the following conditions be verified:

a) The King and the Rook must be on their original primitive (game) spaces. We must therefore necessarily have K(5; 1) and R(8; 1) for whites and K′(5; 8) and R′(8; 8) when it comes to castling blacks.
b) The spaces B(6; 1) and N(7; 1) for white castling and B′(6; 8) and N′(7;8) in the case of black castling must not be controlled or blocked.
c) The King is not in check and must not have moved before.
d) The execution of castling produces the following condition: K(7; 1) and R(6; 1) for whites and K′(7;8) and R′(6;8) for blacks.

5. Large castling

For Large castling to be possible, it is necessary and sufficient that all of the following conditions be verified:

a. The King and the Rook must be on their original primitive (game) squares. We must therefore necessarily have K(5; 1) and R(1; 1) for whites and K′(5; 8) and R′(1; 8) when it comes to castling blacks.
b. The spaces Q(4; 1), B(3; 1) and N(2; 1) for the white castling and
Q′(4; 8),B′(3; 8),N′(2; 8) in the case of the black castling, must not be controlled or blocked.
c. The King is not in a state of check and must not have moved before. d. Large castling produces the following condition: K(3; 1) and R(4;1) for whites and K′(3; 8) and R′(4; 8) for blacks.

III. INITIALIZATION AND CONTROL TABLES

The results needed to deal with a problem are gathered in tables (non-exhaustive list) that help us determine the squares checked and the pieces involved, carry out tests and build a checkmate plan.

These tables expose the following data:

Table (A): Initialization / pieces coordinates

Table (B): The movement indicators applied to the cells of the opposing Kings Table (C): Control of the Black King's neighborhood by white pieces
Table (D): Possible blockages of the Black King

Table (E): Coordinates of neighboring cells and satisfaction of the rules Note: We use the symbol Ø to denote a correct result at the level of the application but which contradicts the rules of the game. For example, a pawn cannot retreat; a Rook cannot ensure its own defense against the opposing King on the ground one of its neighboring squares... etc.

VI. APPLICATION (PROBLEM)

Solve the following chess problem, using its algebraic notation only:

Position A: White: Ke1 ; Rh1 ; Ng3 ; Nf5 ; e2 ; h3 and Blacks: Kg2 ; f3. White mate in 2 moves.

4.1 Reformulation

Initialization

a) Redefine the pieces as moving objects, located by their Cartesian coordinates, and the squares as points in a suitable discrete plane;

b) Draw up the control tables using a calculation program (Excel for example); c) Check whether one or the other of the Kings is in a state of failure; d) Define a resolution plan;

To analyze

e) Study the possible movements of blacks;

f) Deduce a first move (the most satisfactory) for White;

g) Play* the White move (the key) and mathematically define the new position;

h) After analyzing the possibilities of Black, play the black move and define the new position;

i) Find the checkmate from this position.

* play = in the control table, replace the old coordinates of the square on which the piece concerned is located with the new ones.

4. 2 Solution

Note: To avoid repetitive manual work in the sense of formulas, you can use a simplistic program such as Excel to perform the calculations, knowing that the tables are reusable for each new position (new move). This should lighten the work, but also serve as a guide for mathematical analysis.

a. Initial position

In what follows we consider the following notations:

$$R = king\ (Roi), D = queen\ (Dame), T = rook\ (Tour), C = knight\ (Cavalier),$$
$$P = pawn\ (Pion)$$

Whites: R(5; 1); T(8; 1); C_1(6; 5); C_2(7; 3); P_1(5; 2); P_2(8;3),

Blacks: R′(7; 2); P′(6; 3).

b. Satisfaction with preliminary rules

According to (B), $\forall B \in \mathcal{P} : B(R') \neq 0$, so the black king is not in a state of failure and like $P'(R) \neq 0$, the white king is not either, in the state of the initial position of the problem.

c. Resolution strategy

To solve a chess problem such as a straight checkmate in 2 moves, one starts by studying the possibilities of Black; the King's escape spaces and the possible movements of the pieces. However, the escape squares are the squares adjacent to the Black King. We therefore calculate the movement and control indicators for each white piece applied to the squares adjacent to the Black King. If the indicative specific to a piece is zero for a square then the latter is controlled by said piece.

BLANCS						
B	R	T	C1	C2	P1	P2
x	5	8	6	7	5	8
y	1	1	5	3	2	3
x	2	-1	1	0	2	-1
Y	1	1	-3	-1	0	-1
B(R')	12	-1	5	-4	2	-2

NOIRS		
R'	P'	B'
7	6	x
2	3	y
-2	-1	x'
-1	-2	Y'
	-1	B'(R)

Fig. 13: Tables A and B

L	R(L)	T(L)	C1(L)	C2(L)	P1(L)	P2(L)	α_L	β_L	NC	PC
N	42	-2	0	-5	3	-1	0	1	1	C1
NW	12	-4	-1	-4	0	2	-1	1	1	P1
W	0	-2	4	-3	-1	3	-1	0	1	R
SW	0	0	11	0	Ø	6	-1	-1	3	R, T, C2
S	6	0	12	-1	3	3	0	-1	1	T
SE	56	Ø	15	0	8	2	1	-1	1	C2
E	72	0	8	-3	7	-1	1	0	1	T
NE	132	0	3	-4	8	-2	1	1	1	T

Fig. 14: Table C

L	x_L	y_L	Y+β_L_P1
N	7	3	1
NW	6	3	1
W	6	2	0
SW	6	1	-1
S	7	1	-1
SE	8	1	-1
E	8	2	0
NE	8	3	1

Fig. 15: Table E

Notes :

- NC = Number of Controls on square L and PC = Pieces which control L
- $P_1(SW) = Ø$ Because $Y + \beta_{SW} < 0$ and $T(SE) = Ø$ since $T \equiv SE$, according to the Tables (A) and (E).

To analyze:

In our example, all of the Black King's escape squares are controlled according to the Table (C) ; since $\forall L \in V(R'), \exists A \in \mathcal{P}$ tel que $A(L) = 0$. The King therefore cannot move from the initial position.

Let's study the movement of the black Pawn. Let M be an end square that we want to determine its position. So we have $P'(M) = 0$.

Which implies $P'(M) = x^2 + y^2 - m = 0$, with $m = 1$ *ou* $m = 2$ s depending on the nature of the movement.

If $m = 1$, we operate with the vertical displacement indicator; so $x^2 + y^2 = 1$. As x *et* y are relative integers, this equation has for solutions $(0; \pm 1)$ *and* $(\pm 1; 0)$ Now, $y < 0$ since this is a black pawn. There is only one solution which therefore holds: (0; –1).

As a result, $x = x_M - x_{P'} = 0$ *et* $y = y_M - y_{P'} = -1$. Thus, $x_M = x_{P'}$ and $y_M = y_{P'} - 1$. The square that will be occupied by the black pawn after the next move is therefore $M(6; \mathbf{2})$.

Let's take a look at the Black Pawn's control indicator on the White King's throne, once it has been moved. For this, we calculate $\boldsymbol{P'(R) = x^2 + y^2 - 2}$ since it is a control test.

We have $\boldsymbol{x = x_R - x_{P'} = 5 - 6 = -1}$ ***et*** $\boldsymbol{y = y_R - y_{P'} = 1 - 2 = -1}$,
So $\boldsymbol{P'(R) = (-1)^2 + (-1)^2 - 2 = 0}$.

The indicator being zero, the King will therefore be put in a state of failure by the black pawn. This fatal blow would jeopardize any white plan to do Mate in 2 moves. We must therefore remember to avoid it. Let us continue our analysis with which implies a diagonal displacement. Determine the arrival square M and check if it is indeed occupied by an opponent's piece so that the movement is possible.

$\boldsymbol{P'(M) = x^2 + y^2 - 2 = O \implies x^2 + y^2 = 2}$. The solutions are $(\pm \mathbf{1}; \pm \mathbf{1})$. Like $\boldsymbol{y < 0}$, we hold back $(\pm \mathbf{1}; -\mathbf{1})$. As a result, $\boldsymbol{x = x_M - x_{P'} = \pm 1}$ ***and*** $\boldsymbol{y = y_M - y_{P'} = -1}$.
Thus, $\boldsymbol{x_M = x_{P'} \pm 1}$ and $\boldsymbol{y_M = y_{P'} - 1}$.

The square that can be occupied by the black pawn after the next move, if possible, is $\boldsymbol{M(7; 2)}$ ***or*** $\boldsymbol{M(5; 2)}$.

According to Table (A), the squares (7; 2) and (5; 2) are respectively occupied by the black king and a white pawn, so it is possible to move to (5 ; 2). However, these are 2 opposing pawns; which means that the black pawn can also be taken by the white pawn. It will therefore be interesting to verify this white move.

In Table (A), we replace the coordinates of the white Pawn P1 by those of the black Pawn P' which we eliminate by granting it the origin $\boldsymbol{O(0; 0)}$ which is off the board.

	BLANCS						NOIRS		
B	***R***	***T***	***C1***	***C2***	***P1***	***P2***	***R'***	***P'***	***B'***
x	5	8	6	7	**6**	8	7	**0**	***x***
y	1	1	5	3	**3**	3	2	**0**	***y***

x	*2*	*-1*	*1*	*0*	*1*	*-1*	*-2*	*5*	***x'***
Y	*1*	*1*	*-3*	***-1***	*-1*	*-1*	*-1*	*1*	***Y'***

B(R')	*12*	*-1*	*5*	*-4*	***Ø***	*-2*		*####*	***B'(R)***

FIG.16: Tables A1 and B1

L	R(L)	T(L)	C1(L)	C2(L)	P1(L)	P2(L)	β_L	NC	PC
N	42	-2	**0**	-5	-1	-1	1	1	C1
NW	**12**	**-4**	**-1**	**-4**	**-2**	**2**	1	**0**	**inc**
W	**0**	-2	4	-3	-1	3	0	1	R
SW	**0**	**0**	11	**0**	**Ø**	6	**-1**	3	R,T,C2
S	6	**0**	12	-1	3	3	-1	1	T
SE	56	**Ø**	15	**0**	6	2	-1	1	C2
E	72	**0**	8	-3	3	-1	0	1	T
NE	132	**0**	3	-4	2	-2	1	1	T

Fig.17: Table C1 inc = square not checked

According to Table (C1), the NW space is not controlled by any white piece and is occupied by the white Pawn P1 (Tables (E) and (A1)). The Black King therefore has only one escape space on which there is an opponent's piece. King R' is forced to take the P1 pawn since it is the only black piece on the board.

In the new position the black king gains access to the square (6 ; 3) while the white pawn disappears from the board. In Table (A1), we therefore replace the coordinates of King R' by (6 ; 3) and we assign to P1 those of the origin.

The following results are obtained:

L	R(L)	T(L)	C1(L)	C2(L)	P2(L)	α_L	β_L	NC	PC
N	**72**	**-6**	**-4**	**-3**	**3**	0	1	**0**	**inc**
NW	56	-9	-3	**0**	8	-1	1	1	C2
W	6	-6	**0**	-1	7	-1	0	1	C1
SW	**0**	-3	5	**0**	8	-1	-1	2	R, C2
S	**0**	-2	4	-3	3	0	-1	1	R
SE	**12**	**-1**	**5**	**-4**	**Ø**	1	**-1**	**0**	**inc**
E	42	-2	**0**	-5	-1	1	0	1	C1
NE	132	-3	-3	-4	**0**	1	1	1	P2

Fig. 18: Table C2

In this new position, the Black King has in front of him 2 escape spaces: N and SE. Knowing that it is now about to checkmate, the white move must cover the 2 spaces N and SE in addition to the one where R' sits. This is too much for a single piece like the Knight or the Pawn, but not for the Rook by the nature of its movement.

So let's play this move of the Rook.

To do this, let's first determine which squares our Rook must land on in order to defeat the Black King.

Let M be one of these squares. So we have on the one hand $\boldsymbol{T(M) = 0}$ and on the other hand $\boldsymbol{T(R') = 0}$ when $\boldsymbol{T \equiv M}$ In other words, we have to solve the following integer system:

$$\begin{cases} (x_M - x_T)(y_M - y_T) = 0 \\ (x_{R'} - x_M)(y_{R'} - y_M) = 0 \end{cases}$$

We therefore have the following 4 possibilities:

$$(x_M = x_T \text{ or } y_M = y_T) \text{ and } (x_M = x_{R'} \text{ or } y_M = y_{R'}).$$

\# If $x_M = x_T$ and $x_M = x_{R'}$, then $x_T = x_{R'}$, that is 8=6. Which is absurd.

\# If $x_M = x_T$ and $y_M = y_{R'}$, then $M\binom{x_T}{y_{R'}}$, from where $M\binom{8}{3}$. However, according to Table (A) : $P_2(8;3)$. Which means that $M \equiv P_2$. This possibility is excluded since the white Rook cannot access a square occupied by a white pawn.

\# If $y_M = y_T$ and $y_M = y_{R'}$, then $y_T = y_{R'}$ and $1 = 3$. Which is absurd.

✓ The last case gives us $y_M = y_T$ and $x_M = x_{R'}$. So $M(6;1)$.

We replace in (A2) the coordinates of the Rook by (6 ; 1) and we obtain the following tables:

		BLANCS					NOIRS	
B	***R***	***T***	***C1***	***C2***	***P2***	***R'***	***P'***	***B'***
x	5	**6**	6	7	8	**6**	0	***x***
y	1	**1**	5	3	3	**3**	0	***y***
x	1	0	0	-1	-2	-1	5	***x'***
Y	2	2	-2	0	0	-2	1	***Y'***
B(R')	12	**0**	-1	-4	####		#####	***B'(R)***

Fig. 19: Tables A3 and B3

L	***R(L)***	***T(L)***	***C1(L)***	***C2(L)***	***P2(L)***	**β_L**	**NC**	**PC**
N	72	**0**	-4	-3	3	1	1	T
NW	56	-3	-3	**0**	8	1	1	C2
W	6	-2	**0**	-1	7	0	1	C1
SW	**0**	-1	5	**0**	8	-1	2	R, C2
S	**0**	**0**	4	-3	3	-1	2	R, T
SE	**12**	**1**	**5**	**-4**	**Ø**	**-1**	**0**	**inc**
E	42	2	**0**	-5	-1	0	1	C1
NE	132	3	-3	-4	**0**	1	1	P2

Fig. 20: Table C3

Note: Like $\beta_{SE} < 0, we\ have\ P_2(SE) = \emptyset$ From (B3), $T(R') = 0$. The Black King is therefore in a state of failure by the Rook, but he still has an escape space (the SE space according to (C3). The Rook move is therefore not the right one. Note, however, that $R(5;1)$ ans $T(8;1)$ one of the conditions for white castling is fulfilled and the Rook after having played his last move ($T(6;1)$) does indeed occupy the finish square after performing a small white castling. What motivates us to play the small castling rather than the Rook.

In Table (A3), we change the coordinates of the White King to have $R(7;1)$.

B	***R***	***T***	***C1***	***C2***	***P1***	***P2***	***R'***	***P'***	***B'***
x	**7**	**6**	6	7	**0**	8	**6**	**0**	***x***
y	**1**	**1**	5	3	**0**	3	**3**	**0**	***y***
x	-1	0	0	-1	6	-2	1	7	***x'***
Y	2	2	-2	0	3	0	-2	1	***Y'***
B(R')	12	**0**	-1	-4	**Ø**	###		###	***B'(R)***

Fig. 21: Tables A4 and B4

L	R(L)	T(L)	C1(L)	C2(L)	P2(L)	α_L	β_L	NC	PC
N	72	0	-4	-3	3	0	1	1	T
NW	132	-3	-3	0	8	-1	1	1	C2
W	42	-2	0	-1	7	-1	0	1	C1
SW	12	-1	5	0	8	-1	-1	1	C2
S	0	0	4	-3	3	0	-1	2	R, T
SE	0	1	5	-4	Ø	1	-1	1	R
E	6	2	0	-5	-1	1	0	1	C1
NE	56	3	-3	-4	0	1	1	1	P2

Fig. 22: Table C4

It can be seen from (C4) that all of the black king's escape cells are under white control, since:
$\forall L \in V(R') : NC(L) \neq 0.$

Since the Black King is in a check state according to (B4), it is concluded that he is checkmate.

By following the development of the position of the pieces involved in the checkmate, we manage to present the result in its digital form and then the chess form:

$0. P_1(5;2) \quad 1. P_1(6;3)! \quad R'(6;3) \quad 2. R(7;1), T(6;1)$

$$\textbf{Solution} : \mathbf{1.e \times f3!} \quad \mathbf{R' \times f3} \quad \mathbf{2.0-0\,\#}$$

#2 Math Solution (6+2)

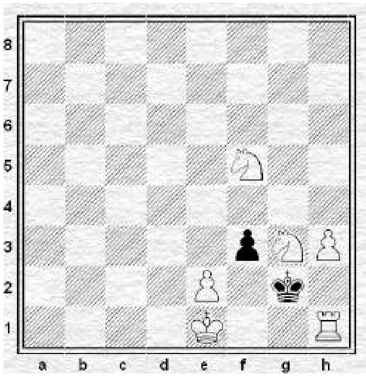

Fig. 23: mate in 2 moves to solve with math tools

The application example shows us that it is quite possible to solve chess problems with the help of equations and analysis using properties and theorems. This brings closer the two disciplines Chess and Mathematics and opens the way to new ideas around a composition of problems oriented towards education.

VII. CONCLUSION

So there are other ways to value chess problems. The fact of exploring the other possibilities should increase the presence, in various forms, of Chess in particular the resolution of the problems. Certainly some fun and useful combinations come out of it.

What can we think about the diversification of published problem genres?

A mathematics teacher who seeks, in special sessions, to offer a playful challenge to his students may find no better than to give them chess problems to solve using mathematical tools. But first he will have to teach them a method, which should be done quickly and without hassle, then compose himself problems or seek out them from the composers who indicate under their compositions the terms: "math solution". These will have to respect certain rules (which will have to be defined) and certainly the level of the target audience; A one-move problem being easier than a two-moves one. For the student it is a fun session solving puzzles and proving his ability to use mathematical analysis with the appropriate tools and for the teacher it is a question of testing at fair value certain technical and moral skills in its students.

The link between the two disciplines gives rise to connections whose result can be in favor of the promotion of the chess problem and the human cognitive values it must convey.

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Heat stress is one of most domineering abiotic stress influences that border barley production. Herein, three different field screening locations were carried out at Sakha, Mallawi and New-valley research stations, to identify the response of ten barley genotypes to different temperatures degrees using phenotypic diversity and, multivariable analysis during two consecutive seasons 2020/2021 and 2021/2022 under different temperatures degrees. Heat stress index (HI) activated a reduction in all traits ranged from lowest average reduction in plant height PH by (5.43 and 20.37%) to highest average reduction in no. of tillers /m²TM by (14.49 and 40.83 %) under Malawi (T2) and New Valley (T3) locations respectively as camper by Sakha, also high temperature enhancement all the genotypes to quicken flowering and days to maturity by average (7.24 and 8.35 %) under New Valley. Days to heading HD and to maturity MD exhibited a strong and significant negative relationship with all studied traits Loading principal component analysis PCA accounted 86.1% of the total variability, which PCA2 clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative). Scatter plot of PCA categorizing all the barley genotypes in four groups indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line5, line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA1 analysis cluster with a significant distance which could be considered as a heat tolerance genotypes.

Keywords: hordeum vulgar, phenotypic diversity - heat stress index hi, pca, heatmap analysis.

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ABSTRACT

Heat stress is one of most domineering abiotic stress influences that border barley production. Herein, three different field screening locations were carried out at Sakha, Mallawi and New-valley research stations, to identify the response of ten barley genotypes to different temperatures degrees using phenotypic diversity and, multivariable analysis during two consecutive seasons 2020/2021 and 2021/2022 under different temperatures degrees. Heat stress index (HI) activated a reduction in all traits ranged from lowest average reduction in plant height PH by (5.43 and 20.37%) to highest average reduction in no. of tillers /m²TM by (14.49 and 40.83 %) under Malawi (T2) and New Valley (T3) locations respectively as camper by Sakha, also high temperature enhancement all the genotypes to quicken flowering and days to maturity by average (7.24 and 8.35 %) under New Valley. Days to heading HD and to maturity MD exhibited a strong and significant negative relationship with all studied traits Loading principal component analysis PCA accounted 86.1% of the total variability, which PCA2 clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative). Scatter plot of PCA categorizing all the barley genotypes in four groups indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line5, line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA1 analysis cluster with a significant distance which could be considered as a heat tolerance genotypes. A cluster heatmap according their resulted form all studied traits showing that the ten barley genotypes were clustered into two main clusters, Giza 137, Giza 138, line5, line 1 and line 3 were the most closed genotypes together due to their tolerance to heat stress while line 2 and line 8 were the most closed genotypes together due to their sensitive to heat stress. Thus, we could use them as a source for future barley breeding programs for heat stress such as important step to get a new genotype with high heat tolerance and high yield.

Keywords: hordeum vulgar, phenotypic diversity - heat stress index hi, pca, heatmap analysis.

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I. INTRODUCTION

Heat stress is one of most vital climate change influences, which there was a universal will increase the average of temperature by 1.8–4 ◦C in the 21st century, the increasing will due to a significant yield losses with great dangers for the future global food safety around the worldwide *(Mariey et al., 2023a, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025).* In Egypt temperature changes from low and worm in coastal zone to hot in the Upper area that the winter season is from December to February and the summer season from June to August, so these weather change had negative influence on Egypt agricultural strategy. *(Elbasiouny et al., 2017 and Mahmoud et al., 2017 and Mariey et al., 2023a).*

Barley (*Hordeum vulgare L.*) is a chief cereal crop that has well improved to numerous abiotic stresses in dry areas, it was found to be moderately tolerant to drought stress, due to it is the restricted amount

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of water that is available for irrigation *(Habib et al., 2021 and Mariey et al., 2022)*. In Egypt, barley is a major winter crop cultivated in old and newly reclaimed lands that hurt from a dearth of irrigation, low soil fecundity, and salinity of both soil and water. However, there is a lack of consciousness of the nutritional role of barley for both humans and animals *(Mariey et al., 2023b and Horváth et al., 2024)* The important responsibilities for plant breeders is to increase yield per unit area by evolving high tolerant genotypes to be suitable for sowing in bad area which surfing from abiotic stresses, These tasks could be realized over using effective methods that help the breeders to screening and documentation the response of genotypes for stress. Agro-morphological parameters and yielding period were the most useful selection criteria to evaluation barley response to heat salinity stress under real field conditions. *(Mariey et al., 2021, 2023 a , Horváth et al ., 2024 ,Kim et al., 2024 and Habouh and Abo-Sapra, 2025).*

Multi-traits include several related traits that could be included in a multivariate analysis. Bi-plot, principal component and cluster analysis were the most influence mathematical methods that could provide a simultaneous analysis of multiple variables to improve the ranking accuracy of the genotypes for abiotic stress in barley *(Mansour et al., 2021, Mariey et al., 2022 &2023b, Habouh , Abo-Sapra, 2025 and Kumar et al ., 2025)*

Consequently, understanding the phenotypic diversity among genotypes will help to ensure that the breeding program has the genetic diversity to improve biotic and abiotic stresses tolerance by crossing genetically-diverse parents having desirable characters, estimate of genetic diversity using phenotypic diversity is one of the primary and important steps in breeding programs for abiotic stresses tolerance *(Mariey et al., 2021 & 2023, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025).*

The present study aimed to investigate the phenotypic diversity of ten Egyptian barley genotypes using some relative importance of some agronomical traits and classify them using multivariable analysis in order to offer genetic evidence for the future breeding programs for heat to intensification the production of barley under heat stress

II. MATERIAL AND METHODS

2.1. Barley plant materials

Ten barley genotypes were kindly provided by Barley Dep., , Field Crops Research Institute, ARC, Egypt, were used in this study their names and pedigree shown in (Table 1).

Table 1: Name, and pedigree of ten barley genotypes used in this study

No.	Name	Pedigree
1	Line 1	Giza 124/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
2	Line 2	BLLU/PETUNIA1//CABUYA/3/Alanda// Lignee527 / Arar
3	Line 3	Giza 118/3/Alanda/Hamra//Alanda,01
4	Line 4	Rihane03/7/Bda/5/Cr.115/Pro/Bc/3/Api/CM67/4/Giza120/6/Dd/4/Rihane,03
5	Line 5	Giza 2000/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
6	Line 6	Giza 119/3/Alanda/Hamra//Alanda,01
7	Line 7	Giza 117/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
8	Line 8	Giza 123/5/Furat 1/4/M,Att,73,337,1/3/Mari/Aths*2//Attiki
9	Giza 137	Giza 118 /4/Rhn-03/3/Mr25-//Att//Mari/Aths*3-02
10	Giza 138	Acsad1164/3/Mari/Aths*2//M-Att-73-337-1/5/Aths/ lignee686 /3/Deir Alla 106//Sv.Asa/ Attiki /4/Cen/Bglo."S")

2.2. Field investigational description

2.2.1. Field experimental Locations

Three field experimentations were achieved in three dissimilar heat stress sites were growing during two winter sowing seasons of 2020/ 2021 and 2021. /2022 to study the effect of heat stress on ten barley genotypes yield production as shown in Fig 1:

1. Sakha station, locating in the center of the Delta -Kafer EL-Sheik governorate, has an elevation of 8.30 above sea level, with Latitude: 31° 6' 22.75" N" and Longitude: 30° 56' 31.11" E" .
2. Malawi station, locating in Minya governorate with Latitude: 27° 43' 53.04" N Longitude: 30° 50' 29.94" E
3. EL-Dakhla, Oasis station, locating in New valley research governorate with Latitude: 25° 30' 59.99" N and Longitude: 29° 09' 60.00" E,.

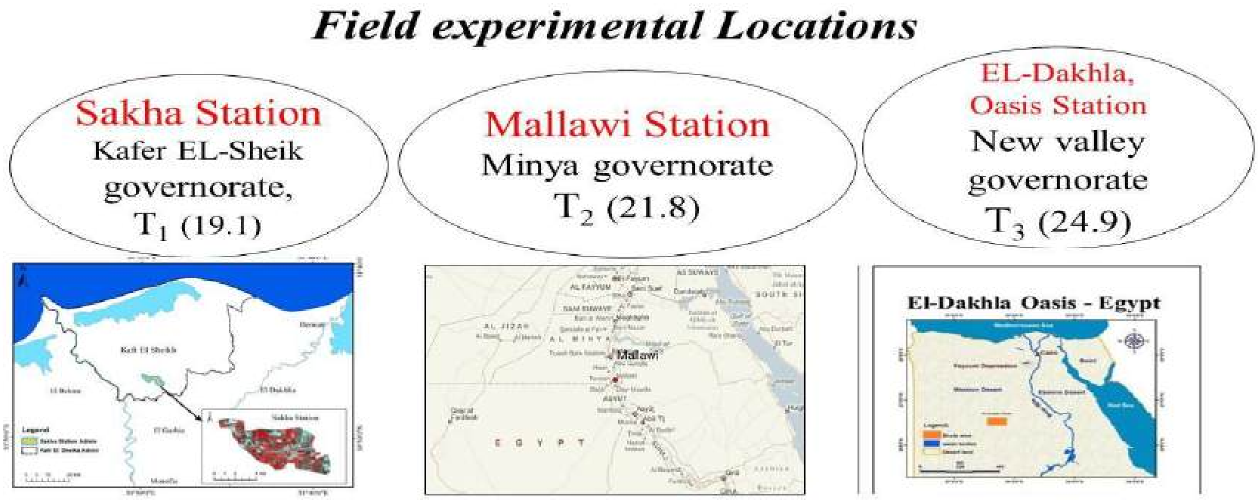

Fig. 1: The maps of Field experimental Locations

2.2.2. Field experimental design

The ten genotypes were growing in a randomized complete block design (RCBD) with three replicates using (plot area =3.6 m²) for each plot, to evaluate the related traits to grain yield and heat stress index

2.2.3. Field experimental Soil samples

Soil samples were taken before land preparation in two depths from the soil surface; i.e. 0-15 cm and 15-30 cm. The physical and chemical analysis of different experimental sites were presented (Table 2)

Table 2: The average of physical and chemical properties for soil samples from the field experiments sites during two growing seasons 2020/2021 and 2021/2022

Soil analysis	Sakha Station	Mallawi Station	New valley Station
A: Physical analysis			
Sand (%)	18.94	14.1	67.1
Silt (%)	28.15	43.1	9.0

Clay (%)	51.35	40.2	23.9
Texture	Clayey	Silty caly	Sandy clay loam
B: Chemical analysis			
EC(dSm^{-1})	2.76	1.62	5.78
PH	7.6	7.86	7.85
K^+ meq100^{1} g soil	0.1	0.57	0.58
$CaCO_3^{--}$ meq100^{1} g soil	0	2.21	4.52
So_4^{--} meq100^{1} g soil	4.95	0.55	-

2.2.4. The Agro- meteorological information

The data of average month maximum and minimum temperatures (C°) and relative humidity (RH., %), were documented for weather station belonging to the Sakha (T_1), Mallawi (T_2) and New Valley (T_3) Station, Egypt during two growing winter seasons 2020/2021 and 2021/2022 were shown in (Table 3).

Table 3: The Meteorological of the experimental area during the two-growing seasons of barely 2020/2021 and 2021/2022 under three different locations Sakha, Mallawi and New valley sites

Season	Month	Temperature, C°									Relative humidity, RH %		
		Sakha (T_1) Normal temperature			Mallawi (T_2) Medium temperature			New valley (T_3) High temperature					
		Max.	Min	Mean	Max.	Min	Mean	Max	Min	Mean	Sakha	Malawi	New valley
Season 2020/2021	Dec.	21.4	13.4	17.4	20.7	9.15	14.93	21.9	9.9	15.9	86.9	64.83	54.2
	Jan.	18.4	11.8	15.1	18.7	6.13	12.42	28.0	7.6	17.8	86.7	64.83	54.5
	Feb.	20.4	12.7	16.6	22.7	9.82	16.26	25.7	9.8	17.75	84.6	61.81	41.9
	Marc.	22.6	15.6	19.1	28.7	14.2	21.45	30.0	13.8	22.9	81.1	61.19	32.9
	Apr.	26.0	18.9	22.5	32.53	17.1	24.82	35.5	18.2	26.85	80.0	53.46	25.2
	seasonal	21.71	14.4	18.4	24.6	11.8	17.9	28.2	11.8	20.4	83.6	61.2	39.9
Season 2021/2022	Dec.	22.9	13.7	18.3	25.0	14.0	19.50	25.3	11.2	18.25	87.7	54.56	51.7
	Jan.	21.0	13.5	17.25	24.5	12.5	18.50	23.1	6.6	14.85	86.7	54.54	44.9
	Feb.	21.5	12.5	17.0	23.5	9.71	16.61	25.2	8.4	16.8	87.5	54.20	43.6
	Marc.	23.8	15.2	19.5	29.3	13.9	21.60	31.6	14.6	23.1	83.8	52.35	35.2
	Apr.	27.6	19.4	23.5	31.0	14.6	22.80	31.9	15.5	23.7	74.6	45.51	27.3
	seasonal	23.35	14.8	19.11	26.6	12.9	19.8	27.4	11.26	19.34	84.06	52.23	40.5

2.2.5. Measured Characteristics:

At the heading stage days to heading were recorded, at maturity stage days to maturity were recorded and at the harvest stage ten guarded plants were randomly taken from each plot to measure plant height cm, number of tillers m^{-2}, number of grains spike^{-1}, and grain yield was determined using the full plot area (3.6 m^{-2}).

2.2.6 Multivariable studied analysis

1. *Heat stress index*: The relative change due to heat stress was computed for each trait according to (Bouslama and Schapaugh, 1984)

2. *Correlation coefficient:* person and matrix were used to study the relationship between each two studied traits were done using Minitab 18.1 statistical software (Minitab Inc., Coventry, UK) and
3. *Principal Component Analysis (PCA):* Loading and scatted plot were performed to study the differences and interrelations between genotypes with respect to measured phenotypic traits using Minitab 18.1 statistical software (Minitab Inc., Coventry, UK) and
4. *Heatmaps cluster:* ClustVis: is a web tool for visualizing clustering of multivariate data, was used to constructed heatmaps (https://biit.cs.ut.ee/clustvis/) *(Metsalu, et al., 2015)*

2.7. Data analysis

All the data of the examined traits from the two seasons were homogeneity and statistically analyzed were exposed to ANOVA in a randomized complete block design (RCBD) to conclude the effects of genotypes, salinity levels and their interaction on the studied traits was performed using SAS software ver. 9.1 *(SAS 2011).* Duncan's test was used to compare mean values at 95% levels of probability *(Duncan, 1955).*

III. RESULTS

3.1 Effect of different temperatures degrees on studied traits for barley genotypes

The ANOVA analysis of all phenotypic studied traits including days to heading (HD days), days to maturity (DM, days), plant height (PH, cm), number of tillers m^2 (TM , tillers /m^2), number of grain spike^{-1} (NGS^{-1} grain /spike), thoued kernel weight (TKW, g) and grain yield (GY ard/fad) indicated a significant statistical effect ($P < 0.01$) by different temperatures degrees under three locations Sakha (T_1) , Malawi (T_2) and New Valley (T_3), cultivars (C), and years (Y) as shown in (Table 4).

A significant two-way interaction between temperatures degrees and barley genotypes (G X T) were observed for all studied traits. While, the two-way interaction between years x temperatures degrees (Y X T) and years x genotypes (Y X G) were significant. across all traits expect the, TM were non-significant. Similarly, the combined ANOVA indicated significant effect for three-ways interaction (G X T X Y) across all traits, expect for TM, which were non-significant

The results indicate that high temperatures at Malawi and New Valley (T_2 and T_3) caused a significant decrease in all studied characters, while caused a significant increase in HD and MD as compared with normal temperature at Sakha station (T_1). which induced all genotypes to flowering and maturity early in Malawi and New Valley (T_2 and T_3) more than Sakha station (T_1). Correspondingly, the results showed diverse significantly which were found among all the Egyptian barley genotypes according their average of mean performances of all studied traits due to the differ temperature (Table 4), which the results showed that Giza 138, Line 1 line 3, line 5 and Giza 137 high average values for all studied under the high temperatures degrees than other genotypes recognized as greater heat tolerance barley cultivars traits, while Line 2 and Line 8 had low average values which they were more affected by heat stress .For grain yield GY, the results indicated that the heat stress significant reduced GY at Malawi and New Valley (T_2 and T_3) more than Sakha station (T_1) as showed in Fig 2. Giza138 gave the maximum values was (21.11, 19.03 and 17.47 ard/fad) at Sakha, Malawi and New Valley sites respectively with followed by Giza 137, Line 5, Line 3 and line 1, which all get high grain yield at Sakha, Malawi and New Valley sites respectively. However, line 2 had minimum GY values (15.11,13.33 and 9.47 ard/fad) at Sakha, Malawi and New Valley sites respectively followed by Line 8 get lower GY were (15.89, 13.47 and 10.14 ard/fad) at Sakha, Malawi and New Valley locations respectively as showed in Fig 2. These results were in gooh harmony with (Kaseva et al., 2023, Mariey et al 2023a Kim et al., 2024, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025). They reported that heat stress significantly reduced most of the grain-yield related traits, which they confirm that interaction G × E in grain yield was strong when applied across the years for each barley genotypes under heat stress, and they conveyed that high temperatures also have indirect negative significances on yield by margarine the plant cycle or unsettling optimal development forms.

Table 4: ANOVA analysis of years, different temperature degrees (three location) and barley genotypes on agronomic, and their interactions during two growing seasons 2020/2021and 2021/2022.

Studied traits	Days to heading days	Days to maturity days	Plant height cm	No. of tillers /m2	No. of grain spike-1	Thoued kernel weight (g)	Grain Yield ard/fad
Years							
2019/20	85.18	113.37	98.07	379.48	56.72	44.91	17.81
2020/21	84.77	113.26	99.33	378.36	55.73	44.36	17.50
Temperature Degrees							
(Sakha) T1	85.67	118.02	109.05	488.83	66.77	53.24	19.78
(Malawi) T1	82.15	115.81	103.13	418.01	62.08	49.85	17.59
(New valley) T2	79.47	108.17	86.83	289.23	44.52	34.19	14.78
Barley cultivars							
line 1	80.67	114.00	101.11	396.25	56.78	48.79	19.08
line 2	84.44	113.72	83.67	256.19	52.11	43.21	13.53
line 3	80.17	115.22	104.17	427.67	56.00	48.99	19.89
line 4	83.83	113.67	96.89	359.89	61.00	44.92	18.61
line 5	81.11	112.67	105.61	407.89	57.33	45.66	19.70
line 6	84.46	113.82	92.70	373.01	55.43	45.56	17.42
line 7	83.06	112.39	95.89	373.44	53.39	44.00	18.09
line 8	85.78	113.44	93.56	259.17	49.44	42.78	14.86
Giza 137	80.72	112.06	102.61	438.78	55.00	41.98	18.96
Giza 138	80.22	112.72	105.22	453.44	62.33	43.18	19.54
ANOVA analysis							
Years (Y)	*	**	*	NS	**	**	**
Cultivars (C)	**	**	**	**	**	**	**
Temperature (T)	**	**	**	**	**	**	**
LSD (0.05)							
Years (Y)	0.381	0.876	0.891	4.54	NS	3.45	1.166
Cultivars (C)	0.808	1.123	1.79	11.15	3.34	3.54	2.18
Temperature (T)	0.442	0.876	0.969	5.56	1.831	1.56	1.906
Interaction							
C X Y	**	**	**	NS	NS	NS	**
TX Y	**	**	**	NS	NS	NS	NS
C X T	**	**	**	**	**	**	**
CX T X Y	**	**	**	NS	NS	**	**

*Which Ns, * and ** non-significant and significant at the 0.05 and 0.01 levels of probability, respectively*

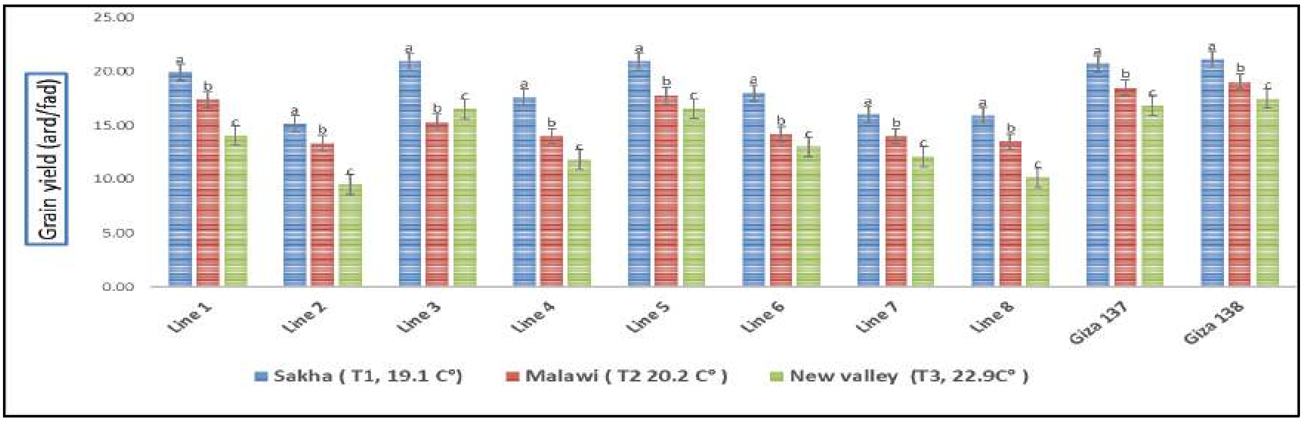

Figure 2: Effect of different temperatures degrees on grain yield among ten barley genotypes at Sakha, Malawi and New Valley locations

3.2 Multivariable analysis

3.2.1. Heat stress index (HI)

The virtual changes reduction due heat stress (HI) on morphological studied traits, were presented in (Figure 3), the results showed that heat stress activated a reduction in all traits ranged from lowest average reduction in PH by (5.43 and 20.37%) to highest average reduction in TM by (14.49 and 40.83 %) under Malawi (T2) and New Valley (T3) locations respectively as camper by Sakha T_1. About the heat index due heat stress on grain yield the results showed that there a reduction was happened due heat stress by average values. Nevertheless, heat stress induced all genotypes to flowered and maturity earlier by an average (4.11 and 1.87 %) respectively under Malawi (T2) and by an average (7.24 and 8.35 %) under New Valley (T3) location respectively as camper by Sakha T_1as shown in (Figure 3). On behalf of the relative changes due heat stress on grain yield the results showed that there a reduction was happened due heat stress by average values (8.06 and 29.0 %) under Malawi and New Valley location respectively. However, heat stress induced all cultivars to flowered earlier by an average (3.93 and 11.39 %) respectively as shown in (Figure 3). This results were agree with (Devi et al 2021, Bhagat et al., 2023, Mariey et al 2023a, Kimet al 2024, Habouh and Abo-Sapra, 2025 and Kumar et al ., 2025) whom, found that barley heat tolerant genotypes were significantly less affected by stress factors than heat sensitive genotypes which heat stress index is an inductor for detect the barley heat tolerant genotypes depends on it grain yielded values.

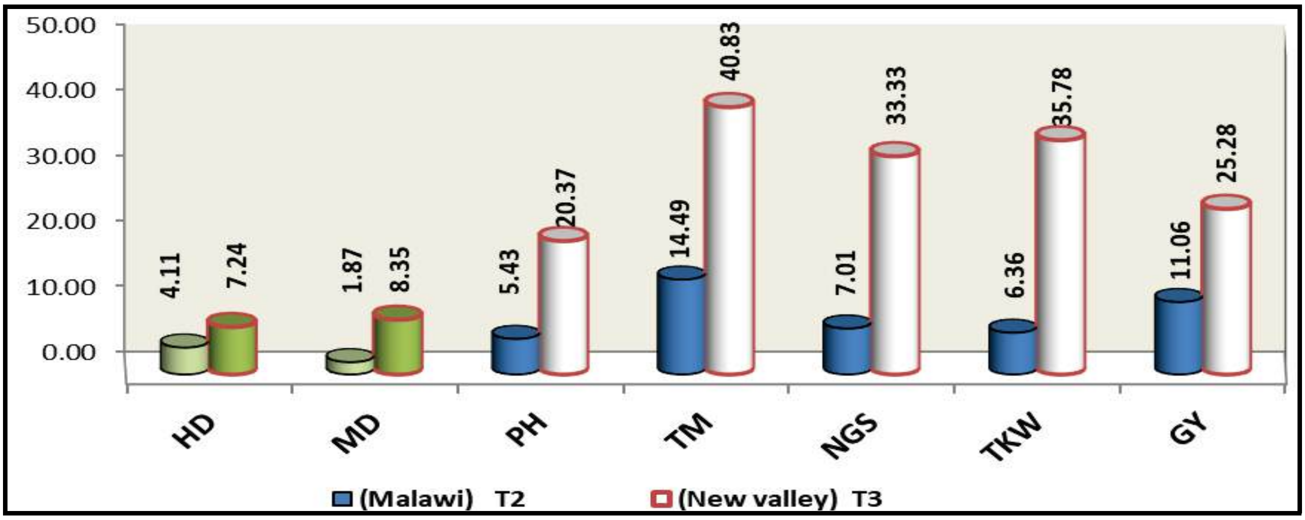

Figure 3: Heat stress index of studied traits under Malawi T2 and New valley T3 as compere by T1 at Sakha station which days to heading (HD), days to maturity (MD), plant height (PH), number of tillers m2 (TM), number of grain spike-1 (NGS-1), thousands kernel weight (TKW and grain yield (GY), which the green refer to inducing flowering and maturity days.

3.3.2 Correlation coefficient

Both Pearson and matrix correlation coefficient was done to recognize the relationships among all studied characters across the three different temperature degrees (three locations) (Figure 4 & 5). Results designated clearly that the correlation coefficients among grain yields GY and PH, TM, TKW and NGS traits were highly positive and significantly correlated. Days to heading HD exhibited a strong and negative relationship with all studied traits grain yield, and days to maturity MD showed negative relationship with all studied traits expect NGS. Theses results were in agreements with (**Mariey et al 2023a, Kim et al., 2024 and Horváth et al., 2024)** whom reported that there was a significant correlation between the heat stress-induced changes in grain-yield related traits.

	HD	MD	PH	TM	TKW	NGS
MD	0.055					
PH	-0.842	-0.148				
TM	-0.878	-0.174	0.872			
TKW	-0.564	-0.039	0.586	0.692		
NGS	-0.395	0.734	0.325	0.301	0.223	
GY	-0.821	-0.068	0.919	0.942	0.712	0.461

Figure 4: Pearson correlation coefficient heatmap among grain yield (GY) and days to heading (HD), days to maturity (MD), plant height (PH), number of tillers m2 (TM), number of grain spike-1 (NGS-1), thousands kernel weight(TKW) across the three heat stress locations. Correlation key and the scale reads, red box indicted strong negative correlation, green box indicted strong positive correlation, white yellow box mean medium positive correlation, orang box mean medium negative correlation

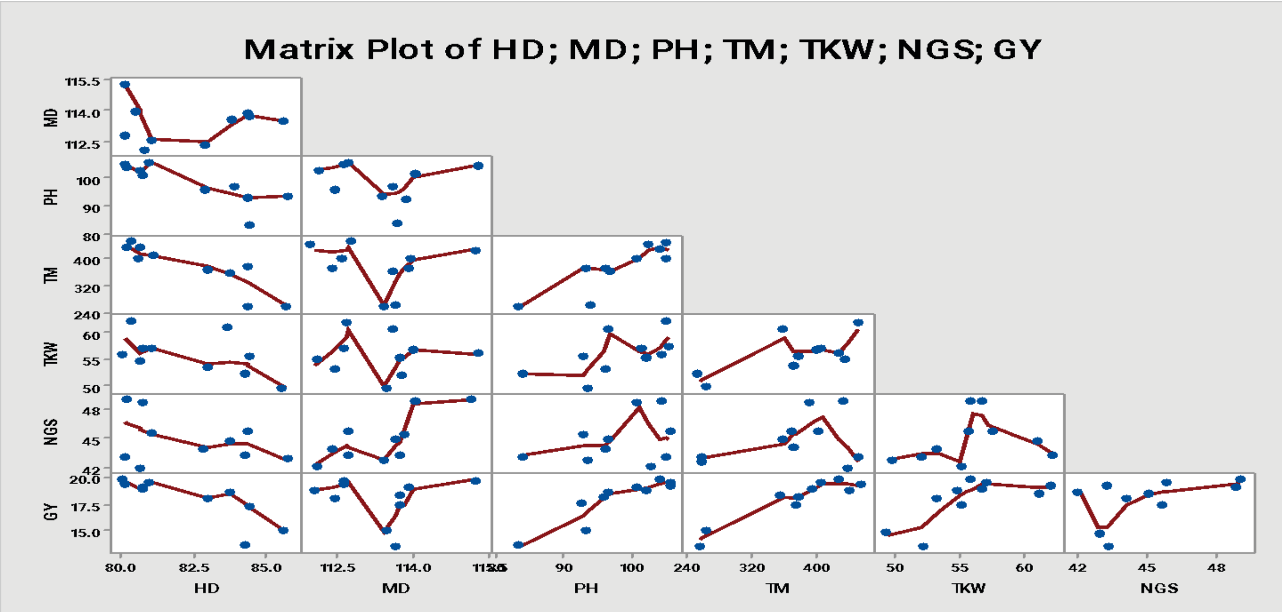

Figure 5: matrix plot correlation coefficient heatmap among grain yield (GY) and days to heading (HD), days to maturity (MD), plant height (PH), number of tillers m2 (TM), number of grain spike-1 (NGS-1), thousands kernel weight(TKW) across the three heat stress locations.

3.2.3. Principal component analysis (PCA)

3.2.3.1. Loading plot PCA

Loading plot was realized using distance matrix available in the horizontal axis chosen the direction of association among all studied characters was showing in (Figure 6). The results presented that principal component analysis PCA accounted 86.1% of the total variability. PCA1 lit 61.9 % of total variation partial by PH, MT, NGS, TKW and GY characters were positioned in positive direction (right side) of the horizontal axis according to their positive significant correlations with other characters under study. The second PCA2 clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative) of the horizontal axis conferring to its negative significant correlations with other characters under this study.

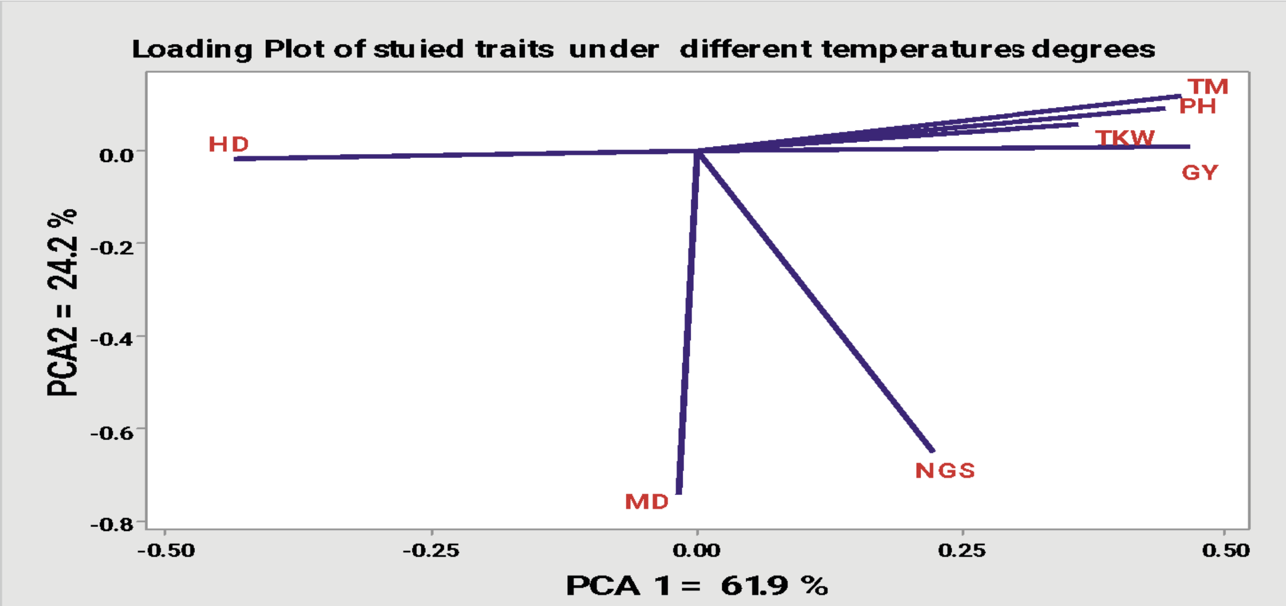

Figure 5: Loading plot graph, showing the first two principal components (PCA) of the correlation matrix among the studied characters which leaf area index (LAI), chlorophyll fluorescence (Fv/Fm), Total chlorophyll content SPAD, days of heading (HD), plant height (PH) numbers of tiller m2 (TM), number of grains spike, 1 (NGS,1), thousand kernel weight (TKW) and grain yield (GY)

3.2.3.2. PCA scatter plot

The scatter plot of PCA analysis based on all studied traits categorizing all the barley genotypes in four groups as shown in (Figure 6), which PCA analysis indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line no5) and (line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA1 analysis cluster with a significant distance, which they had achievement high average of all studied traits under study that could be documented them as heat tolerance genotypes. line 4, line 6 and line 7 which were distributed distance from one other in the scatter plot of PCA analysis cluster based in their medium average value of studied traits could be documented them as moderated heat tolerance genotypes. Whereas both of line 2 and 8 genotypes were scattered distance far from the two other groups which located on left side, as selected by cluster analysis of PCA2 affording to their lowest values of all studied traits with high reduction that could recognized as heat sensitive genotypes. The results agree with (Mariey et al., 2023 and Kumar et al., 2025) they confirmed that principal component analysis PCA was the most impact mathematical devices that could provide a concurrent analysis of multiple variables to improve the position correctness of the genotypes for abiotic stress in barley.

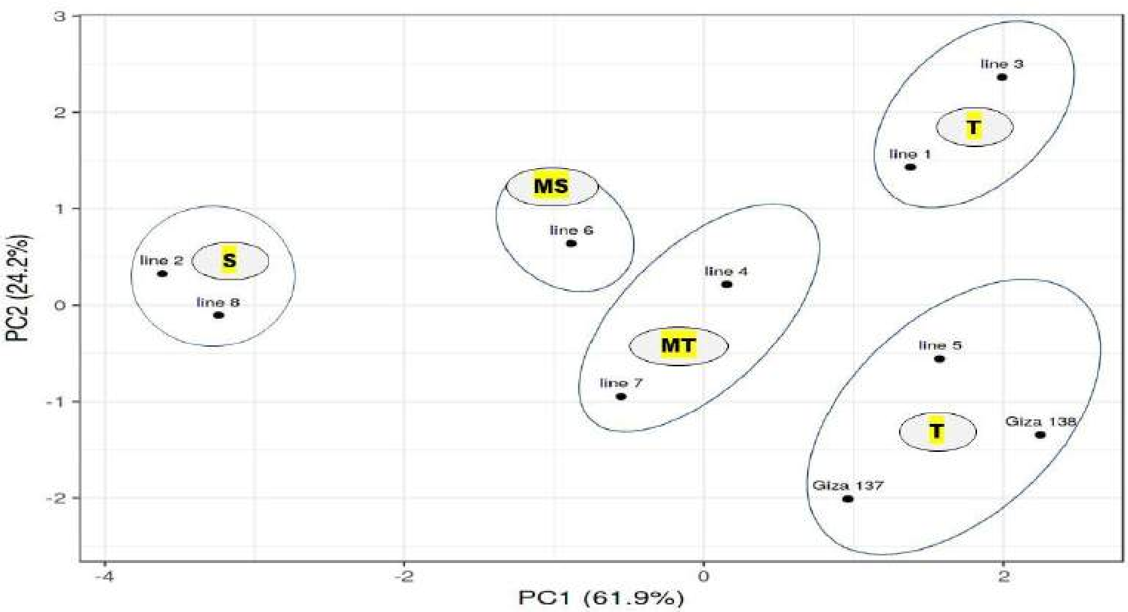

Figure 6: PCA scatter plot of all the ten barley genotypes based on studied traits

2.2.4. Heatmap Cluster Analysis

The heatmap cluster analysis were constructed to investigated the effect of different temperatures on morphological traits of ten barley genotypes which use Euclidean distance and average linkage by R software (Figure 7), which confidential all the genotypes and the traits in two chief dendrograms. Column dendrograms drawing all the morpho studied traits. Row dendrogram design the ten barley genotypes which the analysis clustered them into two main clusters, first cluster include the tolerance and moderated heat divided to sub cluster, first sub includes the heat tolerance barley genotypes (Giza 137, Giza 138, line 5, line 1 and line 3), second sub cluster consisted of moderated heat tolerant genotypes line 7. While Second cluster include the sensitive and moderated sensitive heat first cluster divided to sub cluster, first sub includes the heat sensitive barley genotypes (line 1 and line 8) and second sub cluster consisted of moderated heat sensitive genotypes line 4 and line 6. Our results were in good harmony with *(Mohamed, et al 2021, Mariey et al 2022 and Mariey et al 2023 a&b)* which They reported that heatmap cluster analysis had used positively in sympathetic the information of phenotypic evaluations of the barley genotypes as a significant factor using to helps the breeders to have good plan for their programs for specific environments using targeted traits

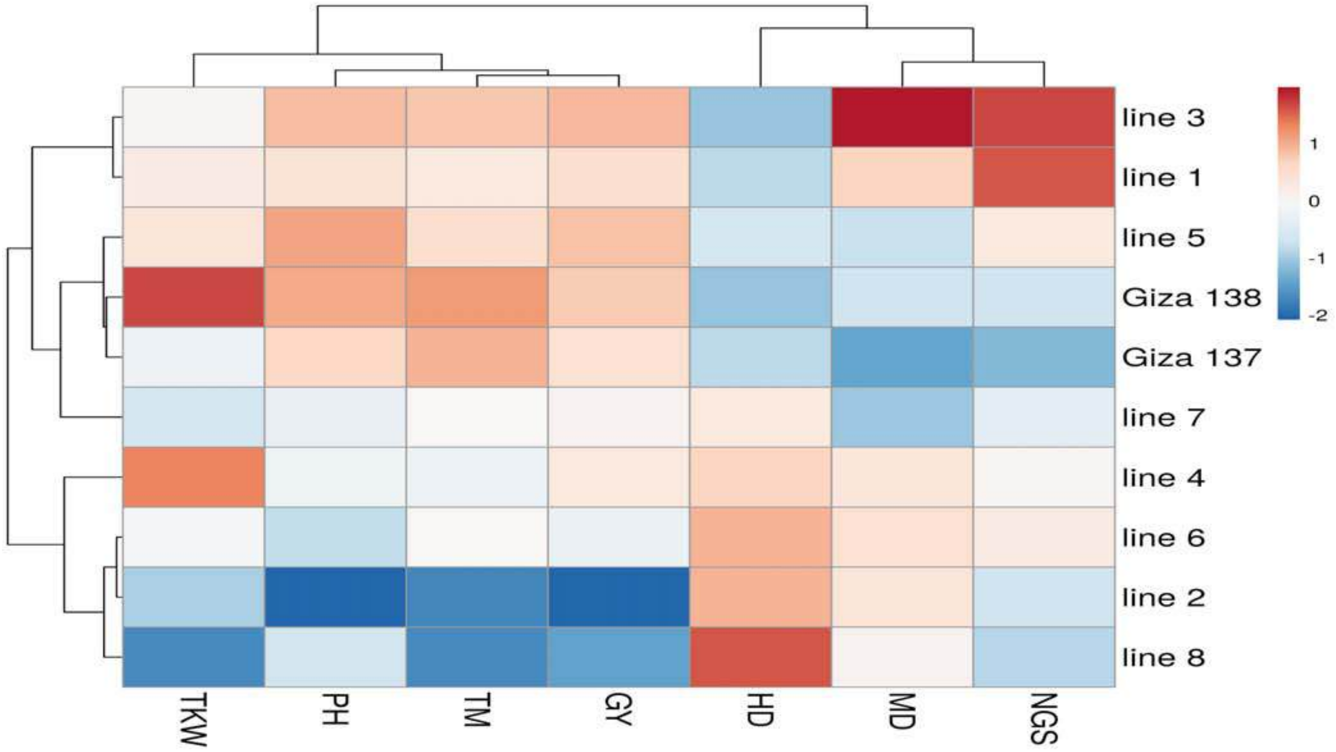

Figure 7: Multivariate heatmap illustrating the phenotypic diversity of ten barley genotypes , based morpho traits using the module of a heatmap of ClustVis, ,days of heading (HD) days to maturity MD, plant height (PH) numbers of tiller m2 (TM), number of grains spike,1 (NGS,1) , thousand kernel weight (TKW) and grain yield (GY)

IV. CONCLUSIONS

Ten barley genotypes were grown in three different field screening locations carried out at Sakha, Malawi and New-valley research stations, which studied their response to three different temperatures degrees by using phenotypic diversity and, multivariable analysis, which the resulted indicted that Giza 138, Giza 137. Line 1, Line 3 and Line5 we could consider them as heat tolerance genotypes and both of Line 2 and Line 8 were heat sensitive genotypes, these differences enable the breeders to use the tolerant at cultivars as a good parent in heat stress breeding programmers in Egypt due to increase the farmer's income.

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الملخص العربى

دراسة التحليل متعدد المتغيرات والتنوع المظهرى لبعض التركيب الوراثية من الشعير تحت الإجهاد الحراري

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الإجهاد الحراري هو واحد من أكثر الإجهاد البيئية الحيوية التي تؤثر على إنتاج الشعير. وهنا تم إجراء ثلاثة تجارب فى ثلاث مواقع بحثية مختلفة في محطات أمحطة بحوث سخا وملاوي والوادى الجديد ، للتعرف على استجابة عشرة تراكيب وراثية للشعير لدرجات الحرارة مختلفة باستخدام التنوع الظاهري، والتحليل متعدد المتغيرات خلال موسمين متتاليين 2020/2021 و2021/2022 تحت درجات درجات حرارة مختلفة. أدى

مؤشر الإجهاد الحراري (HI) إلى انخفاض جميع الصفات المدروسة حيث تراوح بين أدنى متوسط انخفاض في طول النبات بنسبة (5.43 و 20.37٪) إلى أعلى متوسط انخفاض في عدد الفروع متر مربع بنسبة (14.49 و 40.83٪) تحت محطة بحوث ملاوي و محطة بحوث الوادى الجديد على التوالي كمقارنة بمحطة بحوث سخا ، وكذلك درجات الحرارة العالية شجعت جميع التركيب الوراثية الى التبكير وتسريع عدد أيام النضج بمتوسط (7.24 و 8.35٪) تحت محطة بحوث الوادي الجديد. كما أظهرت كلا من عدد أيام التزهير والنضج علاقة ارتباط معنوية قوية وسلبية مع جميع السمات المدروسة . تحليل المكونات الرئيسية شكلت 86.1٪ PCA من إجمالي التباين ، والتي أوضحت المكون الثانى من التحليل 24.2٪ PCA2 من إجمالي التباين المتأثرة بعدد أيام التزهير والنضج حيث كان موقعها في الجانب الأيسر (سلبية). وأشارت Scatter plot ل PCA الى تصنيف جميعالتركيب الوراثية للشعير إلى أربع مجموعات حيث وجد ان التركيب الوراثية للشعير المصرية (الجيزة 137 والجيزة 138 وسلالة 5 وسلالة 1 وسلالة 3) كانت منفصلة عن التراكيب الوراثية الأخرى وتقع في الجانب الأيمن مع مجموعة تحليل المكون الرئسي الاساسى PCA1 بمسافة كبيرة , لذا يمكن اعتبارها هذة التراكيب الوراثية متحملة للحرارة. خريطة النسب الوراثية بناء على كل الصفات المدروسة اوضحت أن االتركيب الوراثية العشرة للشعير تم تجميعها في مجموعتين رئيسيتين ، الجيزة 137 ، الجيزة 138 ، سلالة 5 ، سلالة 1 وسلالة 3 كانت أكثر الأنماط الجينية قريبة معا بسبب تحملها للإجهاد الحراري بينما كان سلالة 2 وسلالة 8 أكثر االتراكيب الورلثية قريبة معا بسبب حساسيتهما للإجهاد الحراري . وبالتالي ، فإننا نستخدمها كمصدر لبرامج تربية الشعير المستقبلية للاجهاد الحرارى كخطوة مهمة للحصول على تراكيب وراثية جديدة ذات تحمل عالي للحرارة وإنتاجية عالية.

Experimental Determination of Suction Pressure using the filter Paper Method for Unsaturated Clay Soils in the Moscow Region

M.A. Novgorodova & D.N. Gorobtsov

University for Geological Prospecting

ABSTRACT

Introduction: In the world, practice of soil research there is ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper D 5298-16. However, unfortunately, in Russia there are no defining documents related to negative pore pressure, or matrix suction, or SWCC, yet.

Materials and methods. In this study, as a practical example of the effect of suction pressure on slope stability, the authors considered an object located in Zelenograd. Because of the experiment, SWCC was obtained and a geomechanical model was created.

Results. Thus, for cover loams, the value of the initial suction pressure is 17 kPa. The value of matrix suction is 199 kPa. For fluvioglacial loams, the value of the initial suction pressure is 14 kPa. The value of matrix suction according to the graph is 207 kPa.

Discussion and conclusion: Subsequently, calculations were performed in the Plaxis software without taking into account the suction pressure and with taking into account the suction pressure, while all other model parameters remained unchanged.

Keywords: matric suction; suction pressure; unsaturated soils; filter paper method; SWCC.

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I. FOR CITATION

In Russian engineering practice, the classical concept of soil mechanics assumes solving problems involving either fully saturated or fully dry soil, which are actually two limiting states of soil. In most cases, soils used as foundations for buildings and structures are in an unsaturated state and their degree of water saturation can vary from 0% to 100%. In this case, designers use the Van Genuchten-Mualem equation in their models, which is intended for unsaturated soils. Unsaturated soils are commonly found in many parts of the world, at a shallow depth (from 2-3 m to 10-15 m) from the surface, as well as in arid regions, where the natural groundwater level is usually at a greater depth. Common to all these soils is their negative pore water pressure, which plays an important role in assessing mechanical properties and also complicates their studies in the laboratory. The presence of air and water in the pore spaces between soil particles causes capillary action, which creates suction.

The relationship between soil suction pressure and water content is determined by SWCC and is an important tool for predicting and interpreting the behavior of unsaturated soils, including under load. SWCC is the relationship between matric suction (chemical potential) and water content (gravimetric or volumetric) or degree of saturation. As the soil passes from a saturated state to an unsaturated state, the distribution of mineral, water and air phases changes as the stress state changes. The relationships between these phases take different forms and affect the engineering properties of unsaturated soils.

In the world practice of soil research, there is ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper D 5298-16. But, unfortunately, in Russia there are no defining documents related to negative pore pressure, or matrix suction, or SWCC.

The advantages of the filter paper method are the ability to measure total suction, which is the sum of osmotic and structural suction, as well as technical simplicity, low cost and sufficient accuracy.

Based on ASTM D 5298, we conducted experimental studies to determine the characteristic curve. The tests were carried out using the filter paper method with the Whatman No. 42.

As a practical example of the effect of suction pressure on slope stability, we will consider an object located in Zelenograd.

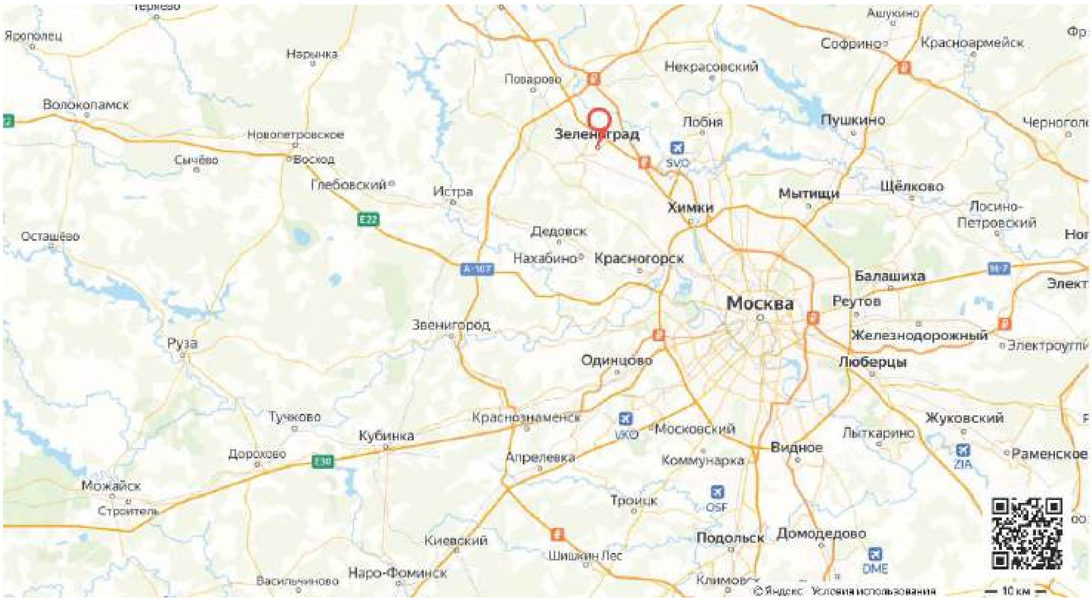

Figure 1: The city of Zelenograd (marked with a red dot).

The engineering-geological section is represented by:

- Modern technogenic (fill) accumulations (tQIV), represented by refractory loams; fine sands of medium density;
- Upper Quaternary cover deposits (prQIII) represented by gray-brown, heavy, semi-hard loams with ferrugination interlayers;
- Middle Quaternary fluvioglacial water-glacial and lacustrine-glacial deposits of the Moscow horizon (f,lgQIIms), represented by soft- and refractory loams, rarely peaty.

As a result of the experiment, the characteristic soil-water curves presented in Figures 2 and 3 were obtained. Thus, for the mantle loams, the value of the initial suction pressure, or air entry value (AEV), is 17 kPa. Before this point, there is a boundary effect zone. The value of matrix suction according to the graph is 199 kPa, and after it, there is a residual zone. Between the points of air entry and matrix suction values, there is a transition zone. For fluvioglacial deposits, the value of the initial suction pressure, or air entry value (AEV), is 14 kPa. Before this point, there is a boundary effect zone. The value of matrix suction according to the graph is 207 kPa, and after it, there is a residual zone.

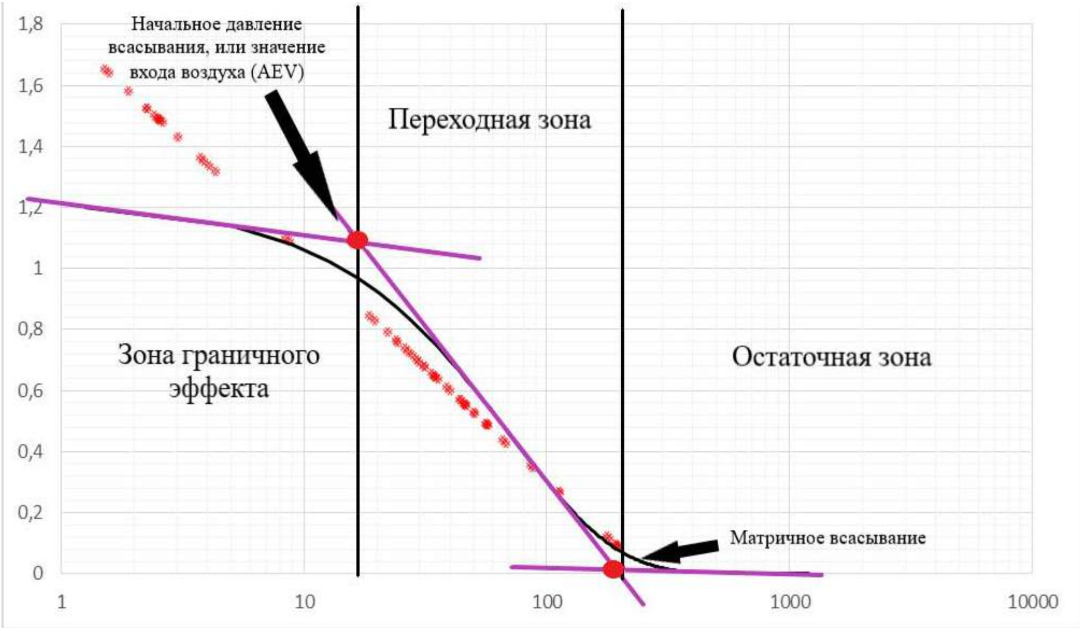

Figure 2: Results of constructing SWCC for cover loams

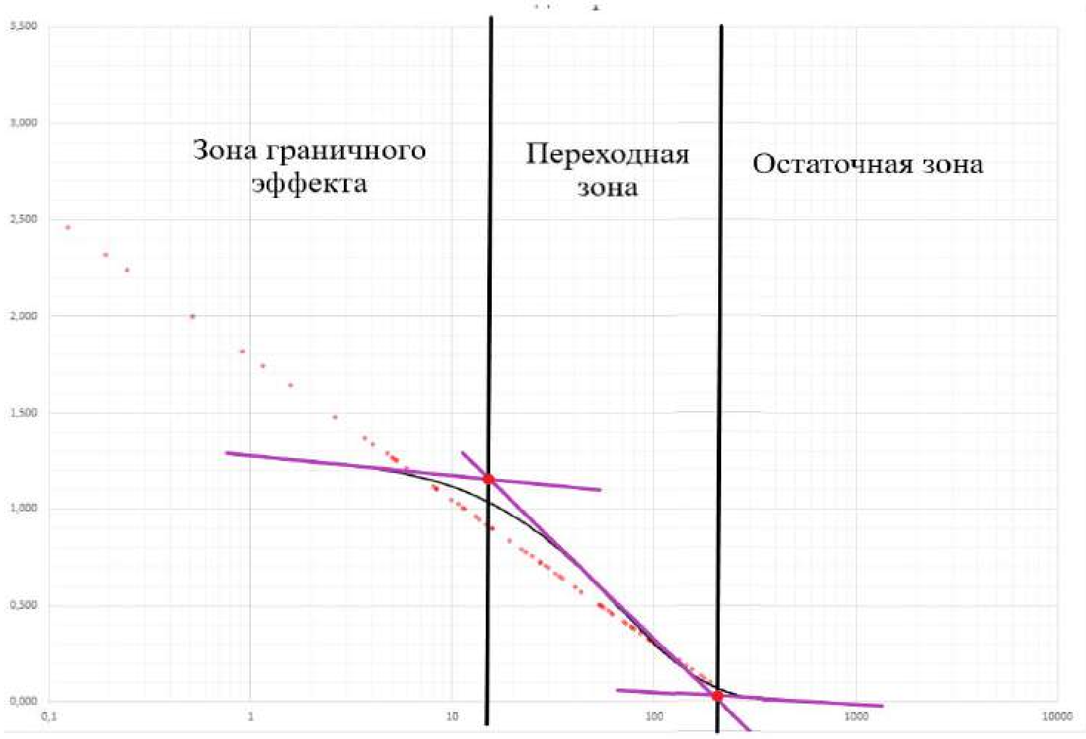

Figure 3: Results of SWCC construction for fluvioglacial loams

In addition, X-ray diffraction quantitative analysis was performed for qualitative assessment. The results of mineral analysis are presented in Table 2:

Sample	smectite*	illite	chlorite	kaolinite	palygorskite	quartz	calcite	Actinolite	Aragonite	dolomite	potassium feldspar (микроклин)	plagioclase (альбит)	Gypsum	Pyrite	Anatase
Sheet	8,7	23,8	10,7	4,0	-	17,2	2,2	-	-	20,1	8,7	4,6	-	-	-
Glacial	13,6	5,1	2,1	3,3	2,0	45,2	11,6	-	-	7,5	7,6	2,0	-	-	-

**smectite and mixed-layer illite-smectite minerals*

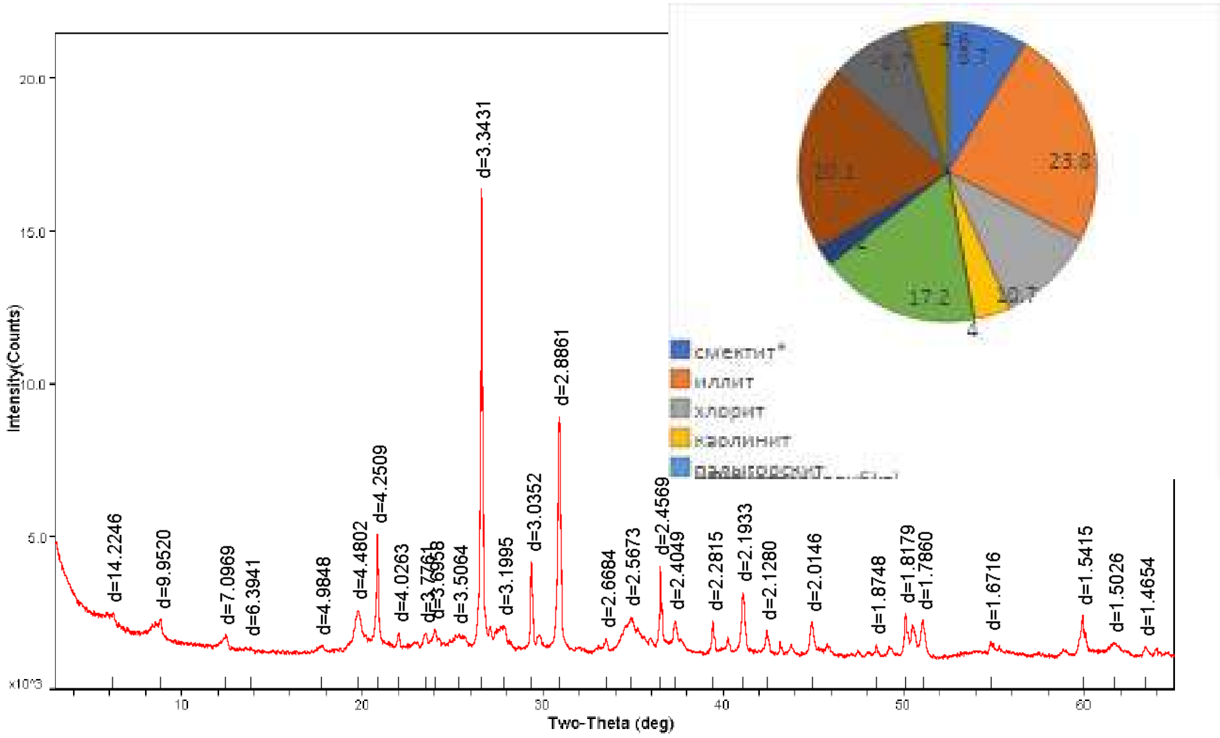

Figure 4: Results of mineral analysis for cover loams

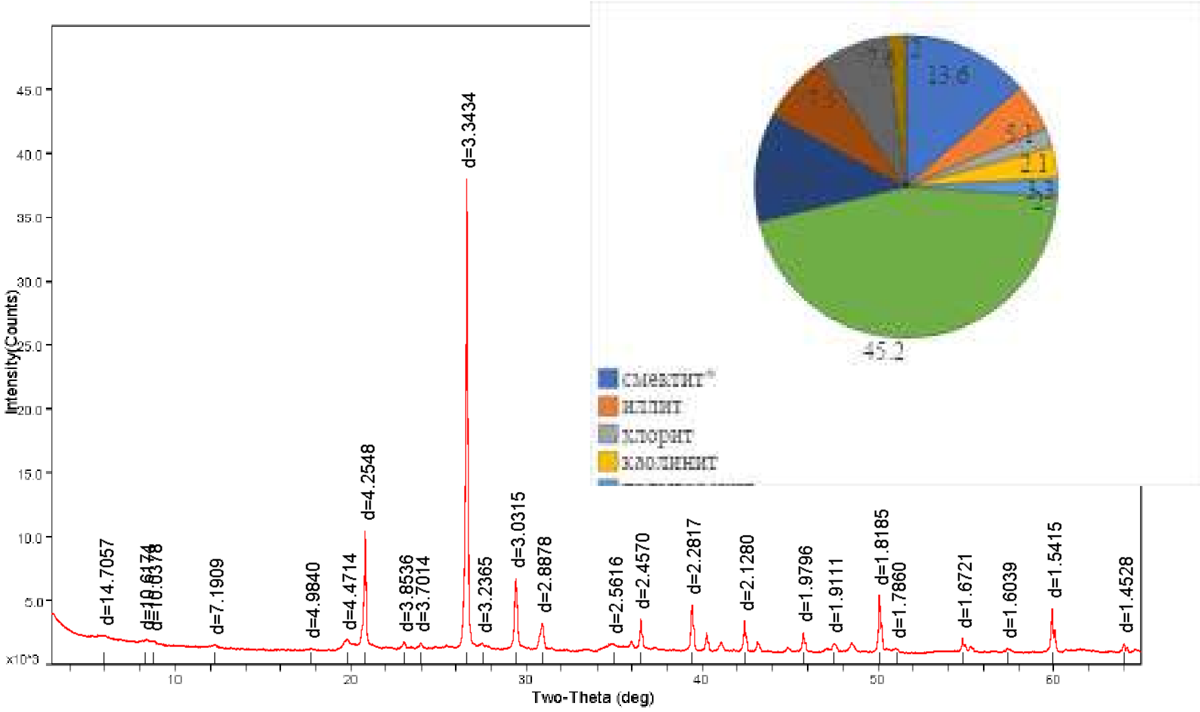

Figure 5: Results of mineral analysis for fluvioglacial loams

As mentioned above, suction pressure and capillarity can affect the physical and mechanical properties of soils, which in turn affect the value of the slope stability coefficient.

The solution of the Mualem-Van Genuchten model is presented in the Plaxis software for geotechnical calculations, but it is necessary to use the SWCC soil-water characteristic curve, constructed on the basis of laboratory tests or adopted according to recommendations for different types of soils.

Subsequently, calculations were performed in the Plaxis software without taking into account the suction pressure and with it, while all other parameters of the model remained unchanged.

The modeling results are presented in Figures 6-10.

The first calculation was performed for a preliminary assessment of the effect of suction pressure. For this purpose, a stability calculation was performed for a slope composed of one type of soil. The calculation results are shown in Figures 6 and 7. The modeling analysis showed that even if the slope is formed by one type of soil, there is a difference between the values of the stability coefficient without taking into account suction and with it. Without suction and capillarity, the stability coefficient was 1.21. With suction and capillarity, the stability coefficient was 1.38.

The analysis of the figures showed that the slope stability coefficient without taking into account suction and capillarity of the soil is 1.46 (Figure 8). With suction and capillarity, the stability coefficient increases to 1.67 (Figure 9). If only the suction pressure is taken into account in the modeling process without taking capillarity into account (Figure 10), the stability coefficient is 1.66.

In addition, the pore pressure was calculated, including the negative one. The maximum value was 39.94 kPa, and the negative value was -50 kPa according to the boundary condition. The maximum suction pressure was also 39.94 kPa.

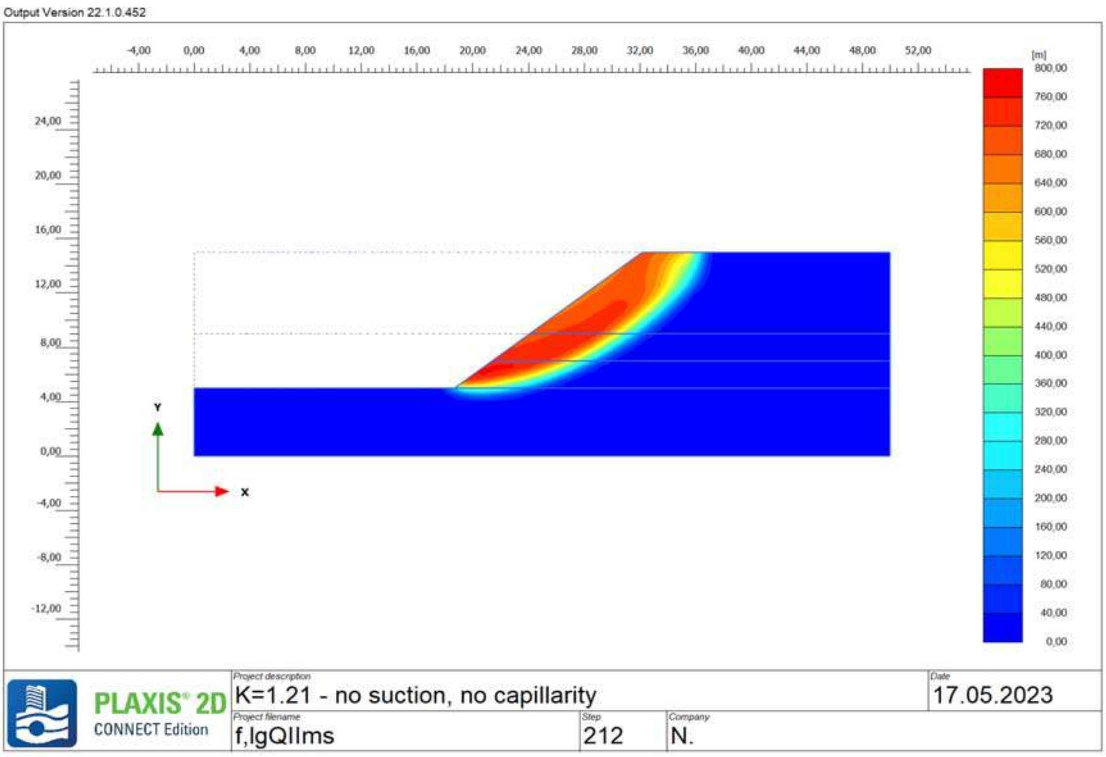

Figure 6

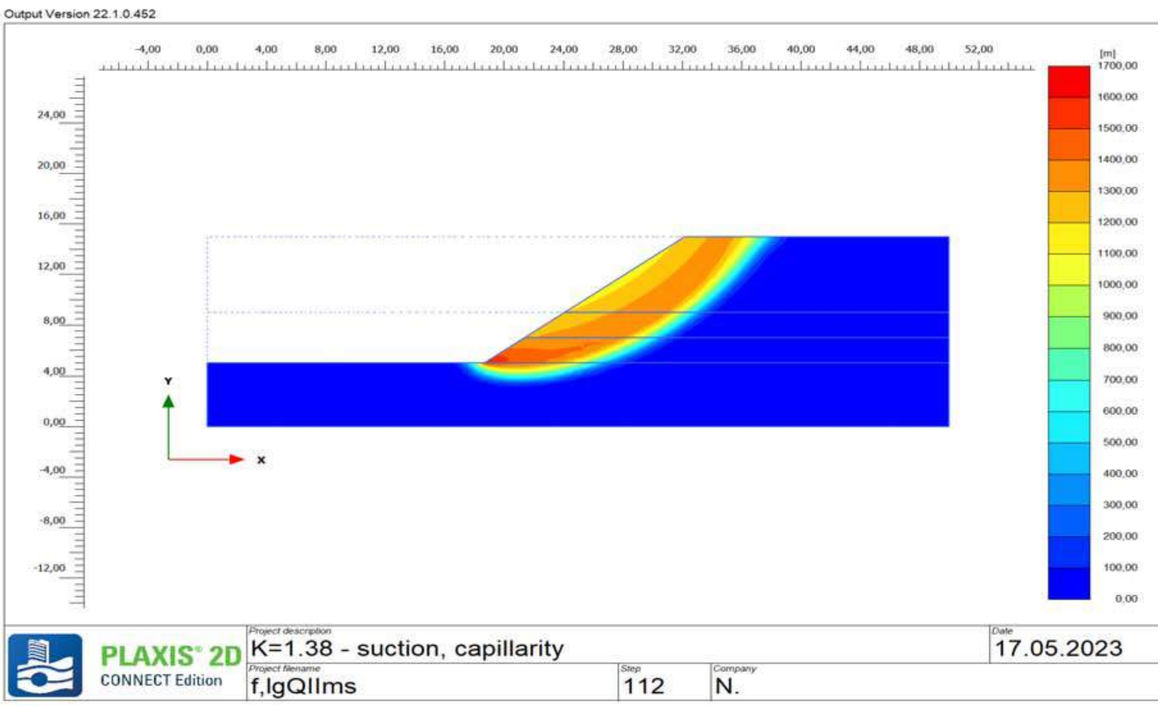

Figure 7

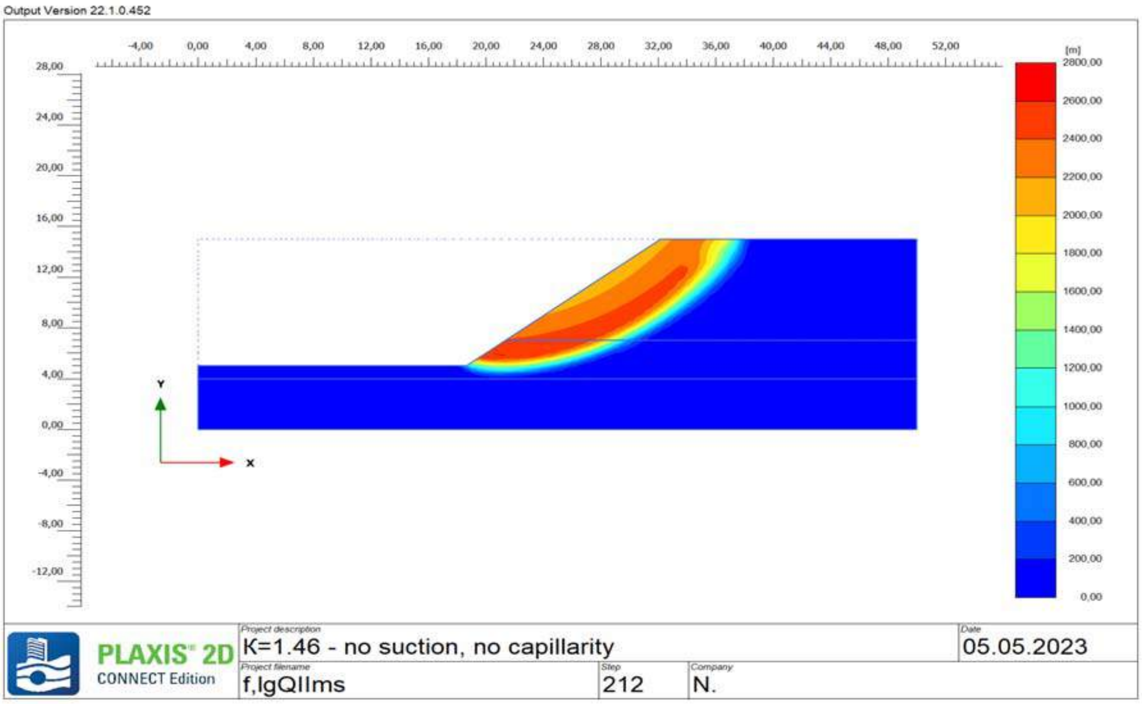

Figure 8

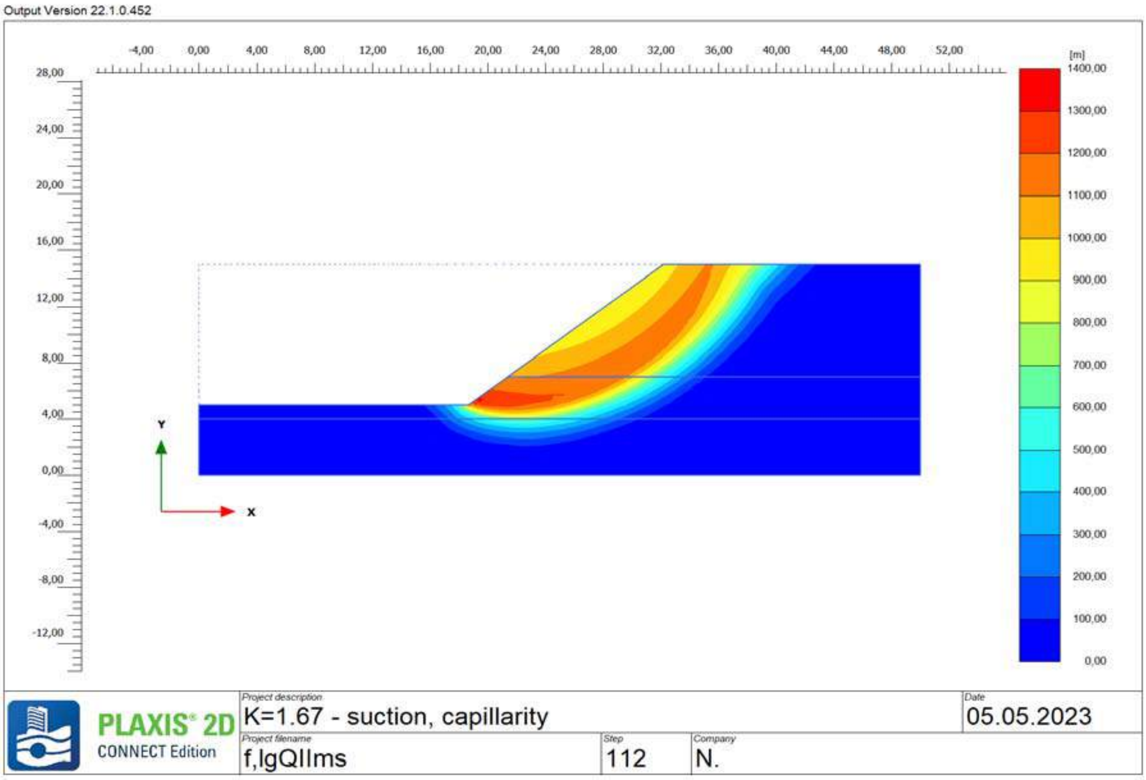

Figure 9

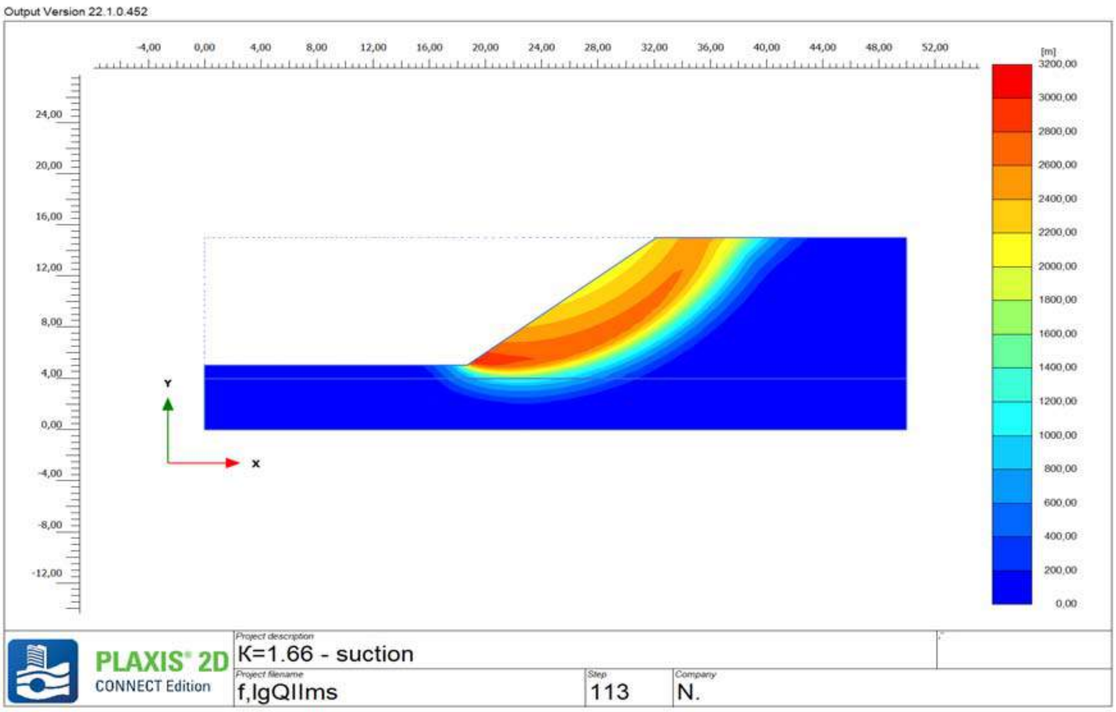

Figure 10

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Algebra of Ecology

Gulamov Muhammad Isakovich

Tver State University

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1. *Ecological factors are, first, diverse (potentially unlimited) changing natural forces; second, the adaptive responses of biological objects to the impact of ecological factors constitute their survivability; third, the influence of environmental factors on individuals within a population should be considered through the concept of survival functions; fourth, despite the potentially infinite variety of ecological factors, their corresponding survival coefficient functions can be classified into six types.*
2. *The interaction of environmental factors forms a kind of survivability hypervolume, created by the interaction of survival coefficient functions corresponding to the ecological factors of the environment.*
3. $\alpha(z) = a_{\{23\}} + a_{\{22\}} \cdot e^{\{-k\left|a_{\{11\}} \cdot z(t) - a_{\{13\}}\right|\}}$ *the invariance of the form of the survival coefficient function – corresponding to the environmental factors – indicates that all ecological factors – abiotic, biotic, and anthropogenic – are closely related with respect to their survival coefficient functions, i.e., they constitute different manifestations of the same function. This indicates an internal consistency among ecological factors, in other words, ecological factors are symmetrical with respect to each other.*
4. *The ecological quantity, the survival coefficient function of biological species in the habitat, defines a non-stationary scalar ecological survival field.*
5. *The fundamental basis of theoretical ecology is the study and research of the algebraic properties of ecological phenomena. One of the main ecological phenomena is the interaction of ecological factors and the formation of corresponding survival functions in biological objects in response to this interaction.*

All of this constitutes the initial steps of theoretical ecology.
The monograph is intended for researchers, graduate students, and students working in the field of theoretical ecology, and for anyone interested in this area of science.

I. INTRODUCTION

When choosing one or another scientific approach, we proceed mainly from the goal - the closest possible description of the object being studied with its natural characteristics. This is precisely the approach that can be traced in most natural science research. But there is another option, namely: what the phenomenon being studied should ideally look like. This option makes it possible to consider the phenomenon under study not as an independent manifestation, but as a particular manifestation of the whole. In other words, the second option makes it possible to describe a holistic manifestation based on the study of its particular phenomenon. This approach considers the object under study in isolation from reality, i.e. an idealized version is studied. Naturally, such an approach will probably be formalized. Nowadays, such formalized options are increasingly in demand.

Modern scientific and technological progress is proof that it occurs on the basis of the emergence and establishment of abstract terms and paradigms. This atmosphere of development dictates the algebraization of the methodology of the problems being studied. This requirement of modern scientific and technological progress is caused by the development of artificial intelligence.

In a broader sense, algebra is understood as a section of mathematics devoted to the study of operations on elements of sets of arbitrary nature; in other words, abstract algebra is nothing more than a natural development of the axiomatic method, which deals with the study of operations performed on certain elements. The power and beauty of the ideas and methods of modern abstract algebra are widely recognized, and its scope of application is expanding so rapidly that there is sometimes talk of an "algebraic plague" that has engulfed not only mathematics but also other sciences (Fried, 1979). In the second half of the 20th century, the rapid development of the theory of elementary particles is a classic example of the application of methods of abstract algebra to solving fundamental problems in this area. The design of new types of technology and their use is unthinkable without the use of modern algebra methods, for example: electronic computers are designed on the principle of finite automata; methods of Boolean algebra are used in the design of electronic circuits; modern programming languages for computers are based on the principles of algorithm theory; computer search systems use set theory; in pattern recognition problems category theory is used; coding and decoding of information is carried out using methods of group theory, etc.

One of the important aspects of applying algebraic methods to the study of solutions to natural science problems is that it is not at all necessary to reduce the formulation of the problem to a purely mathematical one. This is a very important property of abstract algebra, since it allows one to use its methods in cases where the physical laws necessary for the transition from a physical problem to a mathematical one are not yet known. This is precisely the situation observed in the theory of elementary particles (quantum chromodynamics) (Lyubarsky, 1986; https://ru.wikipedia.org › *wiki* › *Yang_Theory _—_Mi...*).

Genuine, living, meaningful theoretical foundations of the problem being studied are born on the basis of a combination of abstract algebra and concrete problems. Algebraic studies of natural phenomena always reveal the internal and external harmony of the object being studied, and this is its beauty (Kline, 1984).

Any natural (physical, biological) and artificial (for example, agroecosystem) phenomena are based on the process of interaction of environmental factors.

Environmental factors can be viewed as various natural forces. If so, then the interactions of environmental factors can be viewed as interactions of various natural forces. Formalization and research of the interaction of environmental factors gave rise to many functions of survival rates of biological species (Gulamov, 1982; 1986; 1989; 1994; Gulamov, Pasekov, 1985; Gulamov, Fayziev, 1990, 1992; Gulamov, Khoshimov, 1997; Gulamov, 2006; 2012).

Algebraic studies of the set of functions of survival coefficients and information models made it possible to describe and identify certain algebraic patterns of ecological and information phenomena, for example: the minimum survival space; invariance of the elements of the set of functions of survival coefficients relative to each other; symmetrical relationships of ecological factors relative to each other; symmetrical relationships of ecological niches of species; formal description of the ecological field of survival and generalized operations of a set of information models.

The results of the study on a set of functions of survival rates of biological species, conducted by M.I. Gulamov in the period from 1982 to 2021, were the basis that made it possible to formalize them into a scientific monograph entitled "Algebra of Ecology". Naturally, this monograph does not cover all aspects of algebraic questions of ecology, these are only the first steps in the algebraization of ecology.

II. THEORETICAL BACKGROUND OF ECOLOGICAL ALGEBRA

2.1 Basic Concept

To formalize the interaction of environmental factors and the consequences of this interaction, we must first create its "idealized" version in order to be able to subsequently describe its formal model. For this purpose, it is necessary to introduce some basic concepts, which are the "building blocks" with which one can build its formalized model. Such basic concepts must meet two conditions: 1) they must reflect the main characteristics of the object being studied and 2) they must allow for a formal description of the phenomenon being studied. But we know that "bricks" are also made from a certain material. In our case, such material is the concept of the environmental factor. It follows from this that the basic concept that we will introduce should reflect the various aspects of the nature of the concept of an environmental factor.

From the above description and analysis of the concept of the environmental factor it follows:

1. An ecological factor is a certain force that changes according to certain patterns $f(A)$ and manifests itself as a condition or element of the environment (A), capable of directly or indirectly influencing a living organism, at least at one of the stages of its individual development: $f(A)$ where $A_i = \{a_i^1, a_i^2, ..., a_i^n\}$, $i = 1,2,...$
2. Environmental factors are independent (meaning the absence of a linear relationship between factors) of each other:

$$f(A_i) \bigcap f(A_j) \neq \varnothing$$

3. A population is a minimal self-reproducing group of individuals of one species ($N(t)$), which over a long evolutionary period inhabits a certain space, forms a genetic system and creates its own ecological niche.
4. Survivability is the ability of an organism or population to withstand the impact of environmental factors. This is, in percentage terms, the survival of an organism or the proportion of surviving individuals in a population.
5. Survival function is a quantitative expression of survival, characterizing the impact of an environmental factor A_i :

$$\forall i \quad \alpha(A_i, t) : \mathbf{R} \rightarrow [0,1], \qquad i = 1,2,...$$

6. Optimality interval is an interval in the gradient of an ecological factor where the survival rate of an organism or individuals of a population is 100%:

$$[a_l, a_{l+k}] \in A_i \text{ where } \alpha(A_i, t) = 1,$$

7. The interaction of environmental factors can be represented as the interaction of the corresponding functions of survival rates:

$$\alpha(A_1, t) \bigcap \alpha(A_2, t) \bigcap ... \bigcap \alpha(A_n, t) \neq \varnothing$$

8. Environment – formulating and determining the states of interacting environmental factors.
9. Ecological factors of the environment affect the organism or individuals of the population simultaneously and jointly:

$$N(t+1) = \alpha(\vec{A},t) \cdot N(t),$$
$$\alpha(\vec{A},t) = \alpha(A_1, A_2, \dots, A_n, t)$$
$$0 < \alpha(\vec{A},t) \leq 1$$

Based on these basic concepts, research is conducted on the mechanisms of: survival of individuals in a population, interaction of environmental factors, calculation of ecological niches, as well as theoretical group research of the interaction of environmental factors.

2.2 Qualitative Study of Survival Mechanisms

Population analysis, Williamson (1975) notes, is one aspect of ecology and is currently made up of guesswork, intuition, and accumulated experience.

The discrepancy in the opinions of the majority of researchers on this issue and the lack of a clear picture of survival mechanisms have led to a revision of issues related to population dynamics. The purpose of this section is to study the mechanism of population dynamics based on the basic concepts that we have outlined above.

In natural conditions, environmental factors that determine the behavior of population dynamics are divided according to the nature of their impact into the following main groups:

1) external factors that do not depend on the size (density) of the population, characterized by their optimal values or optimal intervals in which the survival rate of individuals in the population is maximum (6th basic concept). Such factors include temperature, humidity, radiation, period of food availability, etc. (Gulamov, 1982; Gulamov, Pasekov, 1985; Shilov, 1985).

The qualitative behavior of the value of the survival function $\alpha(A,t)$ for the factors of this group is, for example, as in Fig. 1.

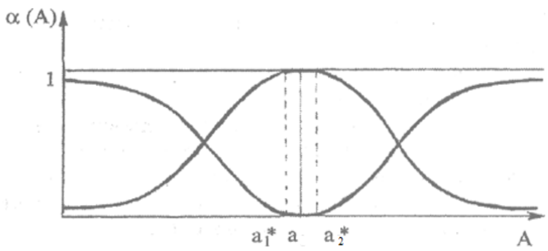


Fig. 1: Qualitative Behavior of the Survival Function Value $\alpha(A,t)$ in the Density-Independent Case

A quantitative dependence of this kind can be expressed, for example, as follows:

$$\alpha(A,t) = \begin{cases} \exp(-\gamma(a(t) - a_1^*)), & npu \quad a(t) < a_1^* \\ 1, & npu \quad a_1^* \leq a(t) \leq a_2^* \\ \exp(-\gamma(a_2^* - a(t))), & npu \quad a(t) > a_2^* \end{cases} \tag{1}$$

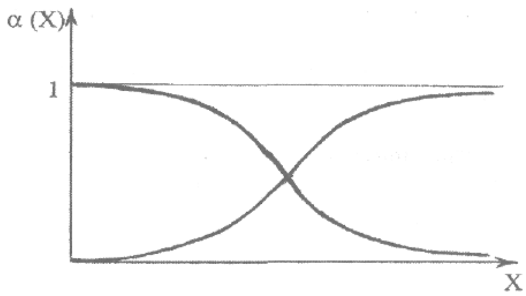
or

$$a(A,t)=\begin{cases}1-\exp(-\gamma(a(t)-a_1^*)), & npu \quad a(t)<a_1^* \\ 0, & npu \quad a_1^*\le a(t)\le a_2^* \\ 1-\exp(-\gamma(a_2^*-a(t))), & npu \quad a(t)>a_2^*\end{cases} \tag{2}$$

and in the case of the optimal value $a_1^*=a_2^*=a^*$. Where A is the density independent factor, $a_1^*,a_2^*\in A$ are the boundary values of the optimal interval, $a(t)$ is the value of the factor A at time t, $\gamma>0$ is the parameter responsible for the steepness of the survival function;

2) Factors whose effects change in direct, inverse or non-monotonic dependence on population density. Such factors include the density of the population itself, the density of parasites, predators, the intensity of the disease, etc. (Burov, 1968; Polivanova, Triselova, 1988; Semevsky, Semenov, 1982).

The qualitative behavior of the survival value for the factors of this group is, for example, as in Fig. 2.



Rice. 2: Qualitative behavior of the survival function value $\alpha(A,t)$ in the density-dependent case.

A quantitative dependence of this kind can be expressed, for example, as follows:

$$\begin{aligned}\alpha(x,t)&=\exp(-\theta(x(t-1)/x(t))),\\ \alpha(x,y,t)&=\exp(-\eta(y(t)/x(t))),\\ \alpha(x,t)&=1-\exp(-\theta(x(t-1)/x(t))),\\ \alpha(x,y,t)&=1-\exp(-\eta(y(t)/x(t))),\end{aligned} \tag{3}$$

where $x(t-1)$ and $x(t)$ are the host population densities at times $(t-1)$ and t, respectively, θ and η are parameters, $y(t)$ is the population density of parasites (predators) at time t (Viktorov, 1969; Kemp, Arms, 1989).

In addition, it is assumed that the survival rate of individuals in a population at any given moment in time is determined by the impact of a combination of factors that are density-dependent and independent (9th basic concept) (Viktorov, 1969; Gulamov, 1982; Shilov, 1985):

$$\begin{aligned}&\alpha(\vec{A},t)=\alpha(A_1,A_2,...,A_n),\\ &\alpha(A_i,t):\ \forall i\quad 0<\alpha(A_i,t)\le 1,\\ &N(t+1)=\alpha(\vec{A},t)\cdot N(t),\end{aligned} \tag{4}$$

where $(A_1,A_2,...,A_n,t)$ is a set of density-dependent and independent factors; $\alpha(\vec{A},t)$ is the resulting survival rate, characterizing the impact of a complex of environmental factors.

From the optimal interval (3 basic concept) and the formal notation (4) (9 basic concept) it follows that under optimal conditions of density-independent factors ($\alpha(A,t) \to \max$), the dynamics of the population size is determined only by density-dependent factors, i.e. $N(t+1) = \alpha(\vec{X},t) \cdot N(t)$, where $X(t) = \{X_1, X_2,..., X_n, t\}$ is a set of density-dependent factors.

In the future, based on these hypotheses, we analyze all possible levels of population dynamics using thought experiments.

Depending on the combination of factors of the first and second groups, populations of different sizes may exist in different species (Odum, 1986 a, b; Ricklefs, 1979). Although there are different values of population size, population sizes are limited by a lower and an upper bound. This is expressed more voluminously by Odum (1986 b): “There are certain upper and lower limits to population sizes that are observed in nature or that could theoretically exist for an arbitrarily long period of time.”

The impossibility of analyzing the entire spectrum of population levels forces us to select among them a set of levels, the analysis of which will give an approximate continuous change in the population dynamics as a whole. Based on these considerations, the spectrum of population size levels can be conditionally divided into three levels:

1) lower maximum permissible level; 2) average level; and 3) upper maximum permissible level.
Based on the above hypotheses, the population dynamics can theoretically be represented in the form of a graphical model in Fig. 3.

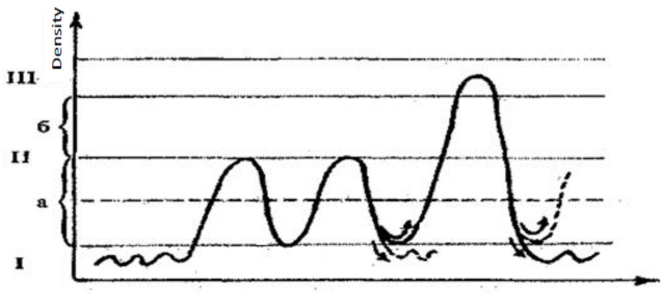


Fig. 3: Hypothetical Model of Population Dynamics

The lower permissible limit (Fig. 3, range I) is determined by the dominance of the role of one of the groups – density independent $(\alpha(A,t) \to \min)$ or dependent ($\alpha(X,t) \to \min$) factors. At this level, the species adapts and acquires certain traits that increase the survival rate and fertility of individuals under unfavorable living conditions. As E. Pianka (1981) explains, under such conditions animals experience weak competition and are subject mainly to r -selection. In such environmental conditions, the lower maximum permissible level is a potential basis for the transition to the average level of abundance.

At the average level (Fig. 3, range II), the regulation of numbers below the saturation level (Fig. 3, range II , a) is determined by density-dependent and independent factors (Rasnitsyn , Volkova, 1982). This density level is characterized by the same values of the survival function for both groups of factors ($\alpha(A) \approx \alpha(X)$). This combination of factors corresponds to the condition of the ecological optimum of I.A. Shilov (1985) - "this is the most favorable combination of all (or at least leading) ecological factors, each of which most often deviates somewhat from the physiological optimum." This combination of two groups of factors leads to stabilization of the population at this level, i.e. the smallest fluctuations in population occur within this level. $N(t) = N(t-1) = N^*$ In addition, a conditional stationary state is possible, that is, preservation over a short period of time. $\alpha(x^*) = \exp(-\theta)$ This state generates a stationary density-dependent factor, the value of which is characterized by (3). Such a stationary density-dependent factor is the cause of the violation of the stationary state at the next t +1 moment in time, which leads to an oscillatory regime. $\alpha(A,t)$ Deviation of the value from the average value $\alpha_{av}(A,t)$ leads to a decrease or increase in the number. When decreasing, the number falls into the lower maximum permissible level, which was discussed above, and when increasing, into the saturation zone (Fig. 3 range II , b), which is a prerequisite for moving to the upper permissible level (Fig. 3 range III). In the saturation zone (Fig. 3 range II, b), provided that the values $\alpha(A,t)$ are close to the physiological optimum, species can realize their maximum possible fertility (Isaev, Khlebopros, 1973).

This behavior of the species in the saturation zone leads to fluctuations in their numbers.

The upper maximum permissible level (Fig. 3 range III) of the population is reached when $\alpha(A,t) \to \max$, and the population level is in the saturation zone. Under such conditions, population fluctuations occur. This in turn increases the influence of density-dependent factors, that is $\alpha(X,t) \to \min$, which leads to a decrease in the population level.

The mental experiments we conducted to study the spectrum of population size levels allow us to draw the following conclusion:

$$\begin{gathered}\alpha(A,t) \to \min: \ N(t) \to \min, \ \alpha(X,t) \to \max; \\ \alpha(A,t) \to \alpha_{av}(A,t): \ N(t) \to N_{av}(t) \text{ and } \alpha(X,t) \to \alpha_{av}(X,t); \\ \alpha(A,t) \to \max: \ N(t) \to \max, \text{A } \alpha(X,t) \to \min,\end{gathered} \tag{5}$$

where $\alpha_{av}(A,t)$, $N_{av}(t)$, $\alpha_{av}(X,t)$ are average values.

From these relationships it follows: firstly, the levels of population dynamics are determined by density-dependent factors in conditions when $\alpha(A,t) \to \max$ the population size is at the upper limit of permissible level, in other cases the main role is played by density-independent factors; secondly, one-sided conditionality is obvious $\alpha(A,t) \Rightarrow \alpha(X,t)$, that is, behavior $\alpha(X,t)$ is determined by the very nature of the function $\alpha(A,t)$. Lindstrom Research J. at al (2001) is proof of this.

To summarize, it can be said that the decisive role in the dynamics of the population size is played by the factor for which the values of the survival function are closer to the minimum (Odum, 1986 b).

2.3 Interactions of Environmental Factors

Environmental factors affect the body simultaneously and jointly (1.1. 9 basic concept).

In studying the combined impact of environmental factors on a particular biological object, all factors that can be measured are taken into account. But here the main emphasis is not on identifying the main factors and studying their impact separately on the dynamics of the population, but on the interactions of factors with each other and determining the resulting, joint impact on the dynamics of the object (Sharov, 1985, 1989; Iogansen, 1964; Kemp, Arms, 1989; Gulamov, 1994, 1996).

In order to more clearly imagine the interaction of environmental factors with each other, we will use Hutchinson's concept of a fundamental ecological niche. He believed that an ecological niche should be defined by taking into account the entire range of physical, chemical, and biological environmental variables to which a given species must adapt, and under which a given population lives and renews itself indefinitely. Ideally, each such variable can be viewed as a gradient along which each species has its own range of activity or stability (Bigon et al., 1989; Giller, 1988; Hutchinson, 1991). n A comprehensive analysis of environmental variables (factors) corresponds to the consideration of an ecological niche as a certain set in n a -dimensional space.

If each such variable is considered as a gradient, on which each species has its own range of survival, then the corresponding n -dimensional hypervolume (ecological niche) of the species can be represented as a certain n -dimensional hypervolume of its survival. Then the problem of the complex impact of environmental factors on the survival of individuals in a population can be considered from the point of view of n the -dimensional hypervolume of species survival, and the complex impact of factors should be understood as a non-trivial form n of -dimensional set, different from the traditional parallelepiped of Hutchinson's fundamental niche.

Let us assume that we know some combination of environmental factors - $A^1, A^2, ..., A^n$ at time t and the corresponding survival functions $\alpha_1(A^1,t), \alpha_2(A^2,t), ..., \alpha_n(A^n,t)$, which we will consider to be continuous monotone curves defined in the corresponding one-dimensional spaces of factors and displaying them in the segments [0,1] (5th basic concept). The explicit form of these survival functions is presented above (Gulamov, 1982, 1989).

Let us assume that the factors are independent of each other (2nd basic concept), then the survival functions under consideration as functions over the entire space of factors generate “cylindrical” surfaces (the generators of which are parallel to the space of the remaining ($n-1$) factors):

$$H_1 = \{\alpha_1(A^1,t) | A^1, A^2, ..., A^n\},$$

$$H_2 = \{\alpha_2(A^2,t) | A^1, A^2, ..., A^n\},$$

$$...............................$$

$$H_n = \{\alpha_n(A^n,t) | A^1, A^2, ..., A^n\},$$

Where $A^i \in R$, $i = \overline{1,n}$. In this case, the interaction of environmental factors can be understood as the intersection, product, union or sum of these cylinders ($\mathrm{H} = \bigcap_n H_i$ $\mathrm{H} = \prod_n H_i$, $\mathrm{H} = \bigcup_n H_i$ or $\mathrm{H} = \sum_{i=1}^{n} H_i$), and the measure of the joint impact of the environment is the hypervolume of this intersection of the product, union or sum H . H clearly, this is the minimum or maximum possible hypervolume of survival at time t (Gulamov, 1991; Gulamov, Logofet, 1997). The survival value at any point on the surface of the formed hypervolume (H) at time t can be written, following the theory of fuzzy sets, as follows:

$$\begin{aligned}
&\alpha(A^1, A^2, ..., A^n, t) = \min\{\alpha_1(A^1,t), \alpha_2(A^2,t), ..., \alpha_n(A^n,t)\}, \\
&\alpha(A^1, A^2, ..., A^n, t) = \alpha_1(A^1,t) \cdot \alpha_2(A^2,t) \cdot ... \cdot \alpha_n(A^n,t), \\
&\alpha(A^1, A^2, ..., A^n, t) = \max\{\alpha_1(A^1,t), \alpha_2(A^2,t), ..., \alpha_n(A^n,t)\}, \\
&\alpha(A^1, A^2, ..., A^n, t) = \alpha_1(A^1,t) + \alpha_2(A^2,t) + ... + \alpha_n(A^n,t)
\end{aligned} \qquad (8)$$

Following the above reasoning, given in the section "Factorial Theory" and Liebig's principle, we can say that the measure of the joint influence of the environment is the hypervolume formed by the intersection of "cylindrical" surfaces - $H = \bigcap_n H_i$.

Thus, each point on the surface of such a hypervolume is determined by the minimum values of the survival function for individual factors, i.e., according to the Liebig principle. An example of the minimum survival hypervolume is the one considered in the work of G.E. Zaikova and others. (1991), the result of experimental observation is a diagram of the dependence of fish fry survival on the concentration of calcium (Ca) and aluminum (Al).

Let us now turn to the situation when it is necessary to take into account the nature of the interaction of environmental factors. Analysis of the works of Tingey and Reinert (Tingey, Reinert , 1975), Shin and others. (Shinn and al., 1976), Howe et al. (Hau and al., (1977) and Raneckles (1988) show that pollutants in atmospheric air are present in various combinations, as a result of which the effect of their combined or sequential exposure on individuals of a population differs from the effects of exposure to a single substance.Such combined effects can lead to antagonistic, additive, or synergistic effects (Streffer, Bucker, Consier, 2003).

1. Additivity - the effect of a mixture (i.e. the summation of individual substances) differs from the effect of the substances separately included in the mixture.
2. Synergism is the effect of a mixture of several substances that is greater than the effect of the sum of the effects of each of them.
3. Antagonism - the effect of the mixture is less than the combined effect of each substance.

(n) in the aggregate must necessarily be present. If we denote the survival function for each environmental factor in three cases of their interaction as: $\alpha_i(A^i, t)$ with additivity, $\alpha_i(A^i, t, n)$ with synergism, $\alpha_i(A^i, t, n)$ with antagonism, $i = 1, \ldots, n$, then the resulting values of the survival function in all considered types of interaction must satisfy the inequality

$$\alpha(A_{an}, t, n < \alpha(A_{add}, t) < \alpha(A_{syn}, t, n) . \quad (9)$$

In all three cases considered, the resulting values of the survival function are determined similarly (8 in the case of intersection), but the "cylindrical" surfaces of the independent case must give way to surfaces of a more complex nature – the corresponding families of single-factor survival functions, covering a known variety of other factors.

Based on the resulting values of the survival function (8), one can judge the stability of the nature of the interaction of environmental factors. For example, when $\alpha(\vec{A}, t) \to 1$ the considered nature of the interaction of environmental factors is stable, but $\alpha(\vec{A}, t) \to 0$ unstable.

Relation (9) can be used to verify the obtained approximations.

Based on the resulting values of the survival function (8), one can judge the stability of the nature of the interaction of environmental factors. The closer $\alpha(\vec{A}, t) \to 0$, the more stable the interactions of environmental factors according to the given nature of interaction, the closer $\alpha(\vec{A}, t) \to 0$, the less stable.

Taking into account the above-mentioned effects of interaction of environmental factors in the expressions of the exponents of the survival functions, given in the section "Qualitative study of the mechanisms of population survival", it is possible to write in a more general form, for example for i the -th factor: $\exp(-\gamma\Delta a_i(t))$.

Depending on the type and value of the factor $A^i(t)$ (whether it is optimal or not), $\Delta a_i(t)$ it can be: with a difference, equal to zero (0), or a ratio. Taking into account the type of interaction and the number of environmental factors, the following is clearly $\exp(-\gamma\Delta a_i(t))$ evident:

1. with additivity $\exp(-\gamma\Delta a_i(t))$,
2. in synergy $\exp(-\gamma\Delta a_i(t)\cdot n)$, (10)
3. in antagonism $\exp(-\gamma\Delta a_i(t)/n)$.

A comparative analysis of the factorial theory and the theory of interaction of environmental factors shows:

$$\prod_{i=1}^{n}\alpha_i(A^i,t) \leq \min\{\alpha_1(A^1,t),\alpha_2(A^2,t),\ldots,\alpha_n(A^n,t)\} < \sum_{i=1}^{n}\alpha_i(A^i,t) , \quad (11)$$

i.e. the approach we propose to determine the resulting survival function may be equal to or overestimated relative to the multiplicative form and strictly less than relative to the additive form of recording (Gulamov, 2004).

2.4 Formal Definition of an Ecological Niche

The approach to describing a niche proposed by Hutchinson (1959, 1978, 1991) has received the greatest number of adherents . Using set theory, he formalized the problem and defined a niche as the set of conditions under which a population lives and reproduces itself. Hutchinson called the entire set of optimal conditions under which a given organism can exist and reproduce itself a fundamental niche.

Hutchinson viewed a fundamental niche as a region of multidimensional space, or hypervolume, within which environmental conditions allow a population to persist indefinitely. Therefore, a fundamental niche is a hypothetical, imaginary niche in which the environment is optimal for individuals in a population and in which it does not encounter "enemies" such as competitors and predators. In contrast, the actual range of conditions for the existence of an organism, which is always less than or equal to the fundamental niche, is called the realized niche.

The realized niche of most organisms changes in time and space depending on changes in the physical and biological environment. Temporal changes in a niche can be considered at two levels: 1) at the level of short-term changes (on the scale of ecological time), usually occurring during the life of an individual or, at most, several generations; 2) at the level of long-term changes occurring on the scale of evolutionary time and affecting many generations. Thus, the realized niche can be considered as a constantly changing subset of the fundamental niche or, in terms of multidimensional space, a pulsating hypervolume, which is limited by the hypervolume corresponding to the fundamental niche (Pianka, 1981).

The dimensions of the multidimensional space describing the environment are the gradients of environmental factors. If the environment is described by n factors, then the ecological niche can be described in terms of the corresponding n -dimensional space (Gulamov, 2002).

For example, in the case of inactive factors, the ecological niche can be represented as follows. Let for each factor X_j of the ecological space E_n there exist a tolerance interval of the population $[x_{j1}, x_{j2}]$ such that the well-being function $f_j(X_j)$ for the factor X_j is equal to one inside this interval and zero outside it. The set of all points $(x_1, x_2, ..., x_n)$ n -dimensional space, all coordinates of which are within the corresponding tolerance intervals, i.e. satisfy the conditions

$$\begin{gathered} x_{11} \leq x_1 \leq x_{12} \\ x_{21} \leq x_2 \leq x_{22} \\ \dots\dots\dots \\ x_{j1} \leq x_j \leq x_{j2} \\ \dots\dots\dots \\ x_{n1} \leq x_n \leq x_{n2} \end{gathered} \tag{12}$$

forms *an* n -dimensional parallelepiped

$$[x_{11}, x_{12}] \times [x_{21}, x_{22}] \times \ldots \times [x_{n1}, x_{n2}] \tag{13}$$

with sides parallel to the coordinate axes $X_1, X_2, ..., X_n$ n -dimensional space. For example, for the case $n = 3$, this is a normal three-dimensional parallelepiped

$$[x_{11}, x_{12}] \times [x_{21}, x_{22}] \times [x_{31}, x_{32}]$$

Of course, in the case of dependent factors, *the* n -dimensional region corresponding to the ecological niche may have a more complex configuration.

Hutchinson's definition of a niche had a revolutionary impact on the development of ecological theory. This is connected, firstly, with the fact that Hutchinson's niche can be described in a formal language and operated with mathematically. Secondly, the niche was determined by him, first of all, based on the properties of organisms and their relationships in the community. At the same time, Hutchinson emphasized that in the process of evolution of organisms or communities the situation can change, and a niche previously occupied by one species can be divided between several more specialized species (Hutchinson, 1978; Nikolaikin et al., 2003). Thirdly, the concept of the fundamental niche developed by Hutchinson was of great importance for the development of the concept of the ecological niche. The fundamental niche is sometimes called the pre-competitive or potential niche, and the realized niche is called the post-competitive or actual niche. This approach gave rise to a direction associated with the predominant study of the role of competition in the division of environmental resources. These works include (MacArthur, 1961, 1968, 1970, 1972; MacArthur, Levins, 1967; MacArthur, Pianka, 1966, MacArthur, Wilson, 1967).

A careful analysis of the definitions of ecological niche presented by Grinnell, Elton, Odum, Pianka and Hutchinson shows that their essence coincides. The differences lie rather in the use of different terminology, and also in the fact that they reflect different stages in the development of this approach, culminating in the definition given by Hutchinson.

However, although the modern concept of ecological niche is based on the multidimensional niche model developed by Hutchinson, ecological niche has increasingly come to be identified with the spectrum of resource use along just one or more of the most important (or most easily measured) dimensions of the niche. The reason for this is that the model of a niche as *an* n -dimensional hypervolume is too abstract. Usually, most parameters are given implicitly and are defined only qualitatively; for example, the explicit form of the well-being function is not defined. When the number of environmental factors is more than three, description (13) loses its visual meaning, etc. Therefore, usually several of the most important dimensions are distinguished, along which, one way or another, the division of resources and the divergence of species into niches occurs: food, space and time.

From the above it follows that the concept of an ecological niche is given mainly a qualitative meaning and this approach is difficult to apply to solving applied problems. True, the works of Yakhontov (1969), Pianka (1981), Odum (1986b), Giller (1988) provide examples of the application of the concept of an ecological niche to the solution of a number of specific ecological problems concerning the size of a habitat, the rate of population growth, fitness, *r- and* K - *selection* , time distribution, morphological features (such as, for example, the thickness and length of a bird's beak), etc. However, all these applications are qualitative in nature, i.e. they lack quantitative assessments that could be used for comparative analysis of ecological niches of different species.

True, such problems at present, in the context of the rapid development of information technology, can be solved quantitatively. As an example, we can consider the works:

Peterson (2001) considers predicting the geographic distributions of species based on ecological niche modeling for the purpose of studying biodiversity. This study mainly uses the ecological niche models of species development based on artificial intelligence algorithms, GARP and GIS and projects them onto geography to predict species.

Sibly and Hone (2002) consider the formation of an organism's ecological niche from the resulting impact of environmental factors and the rate of population growth. In this paper, the authors make more realistic conclusions that an ecological niche is the result of the interaction of a complex of environmental factors and the rate of population growth. Although the authors do not specify the "resulting impact" of environmental factors.

Anderson et al. (Anderson at al., 2002) study the geographic distribution of mice (Heterde australis and H. anomals) in South America by constructing their ecological niches based on GIS modeling. The basis of such modeling is the idea that the matrix of a complex of favorable factors in relation to the species under consideration is formed and then checked using certain methods, a comparative analysis is made with these geographic locations. Although the results of the study do not confirm the actual situation, they nevertheless provide an opportunity and are a good basis for testing this kind of scientific hypotheses through field and laboratory research.

Hirzel et al. (Hirzel at al., 2002) consider ecological niche as a factorial analysis. According to Hutchinson's concept, an ecological niche is constructed in the area of multidimensional ecological factors, the distribution of neighborhoods, where the distribution of varieties is observed according to a set describing the entire area of study. The first thing that will be given is the fact of maximizing the probability of densification of a variety, defined as the ecological distance between the optimal densifications of varieties and the average ecological niche within the areas under consideration. This, in turn, provides a habitat suitability map.

Lim et al. (Lim, Paterson, Engstrom, 2002) construct an algorithm for modeling a sustainable ecological niche for mammals in Guyana. This model is used to solve real environmental problems such as the distribution of diversity in the open spaces and forests of Guyana. In this work, genetic algorithm for rule set prediction (GARP) is used.

Peterson et al. (Peterson, Ball, Cohoon, 2002) test the utility of the genetic algorithm rule set prediction (GARP) modeling program for ecological niche modeling to make accurate geographic distribution predictions for 25 Mexican bird species. The authors conclude that the trial was a success (78-90%), indicating that ecological niche modelling approaches such as GARP provide a promising tool for investigating a wide range of questions in ecology, biogeography and conservation.

The above-mentioned modern works on modeling ecological niches are mainly practical in nature and differ from the work of V.V. Yakhontov (1969), Yu. Odum (1986), E. Pianka (1981) and P. Giller (1988) in that, thanks to the capabilities of modern computers, they can operate with a large number of ecological factors. The presented modern models of ecological niches are mainly based on Hutchinson's multidimensional ecological niche. Basically, the principle of operation of these models of ecological niches (GIS, GARP) is based on the method of matrix comparison, i.e. the data table for a species (organism or population) is compared with geographic data tables. Based on this comparison, appropriate conclusions are made regarding distribution, biodiversity, etc.

The theoretical basis of the presented works on modeling ecological niches remains the same at the level of Hutchinson's multidimensional ecological niche.

Although the work of E.I. Khlebasolov (1999) was published several times earlier than the above-mentioned modern works, nevertheless his conclusions have not lost their relevance: “Despite the significant volume of factual data and theoretical generalizations, it has not yet been possible to fully determine the essence of understanding ecological niches. Modern models based on Hutchinson's multidimensional niche theory attempt to understand the nature of species interactions in a community by measuring the width and degree of niche overlap in one or more of the most important indicators. At the same time, little attention is paid to the study of the properties and parameters of the niche itself. Therefore, the modern approach contains a number of limitations that hinder the further development of niche theory and understanding of the mechanisms of formation and functioning of ecosystems. This serves as a reason for a pessimistic assessment of the state of research in the field of community ecology and encourages biologists to search for alternative approaches to studying the problem of niches."

2.5 Generalization of Hutchinson's Definition

An ecological niche is defined by Hutchinson as a set of environmental factors in space. In turn, one of the ways to define a set in space is to use characteristic or indicator functions (Mathematical Encyclopedic Dictionary, 1988). The indicator of a set is a function that is equal to one at points in the set and zero at points that do not belong to the set. In fact, such an indicator for one dimension is the above -mentioned binary (i.e. taking only two values) well-being function $h_j(X_j)$ for factor X_j. Fig. 4 illustrates this situation (Gulamov, Terekhin, 2004).

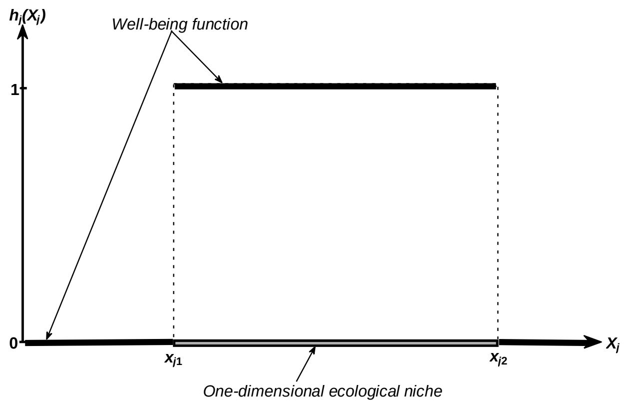


Figure 4: An example of a one-dimensional Hutchinson ecological niche $[X_{j1}, X_{j2}]$ given by a binary (taking only two values) indicator function of well-being $h_j(X_j)$

If the environment is described by two factors X_j and X_k, then the indicator function of well-being will be a function of two variables - $h_{jk}(X_j, X_k)$. In Fig. 5 shows an example of a two-dimensional indicator function for a situation where factors X_j and X_k do not interact within the boundaries of their one-dimensional niches, i.e., a one-dimensional niche for factor X_j does not depend on the value of factor X_k, as long as its values do not go beyond the niche for this factor, and a one-dimensional niche for factor X_k does not depend on the values of X_j within their niche boundaries. In this case, the two-dimensional indicator function (and, consequently, the niche itself) can be reconstructed from the known one-dimensional indicator functions of the non-interacting factors X_j and X_k either by multiplying them or by taking their minimum, i.e.

$$h_{jk}(X_j, X_k) = h_j(X_j) \times h_k(X_k)$$

or

$$h_{jk}(X_j, X_k) = \min\{h_j(X_j), h_k(X_k)\}$$

which leads to the same result, illustrated in Fig. 5. We cannot, however, say that in this situation the interaction of factors is completely absent: if the value of one of the factors goes beyond the boundaries of its niche, then the one-dimensional niche for the other factor becomes an empty set.

In more complex situations, the one-dimensional niche for factor X_j may depend on the value of factor X_k, and the one-dimensional niche for factor X_k - on the value of X_j. So, in Fig. 6 shows the situation when one-dimensional niches by factor X_j, corresponding to two different values x_{ka} and x_{kb} of the factor X_k, do not even overlap. To find a two-dimensional niche in such cases, one must know either one-dimensional niches by factor X_j for all values of factor X_k, or one-dimensional niches by factor X_k for all values of factor X_j.

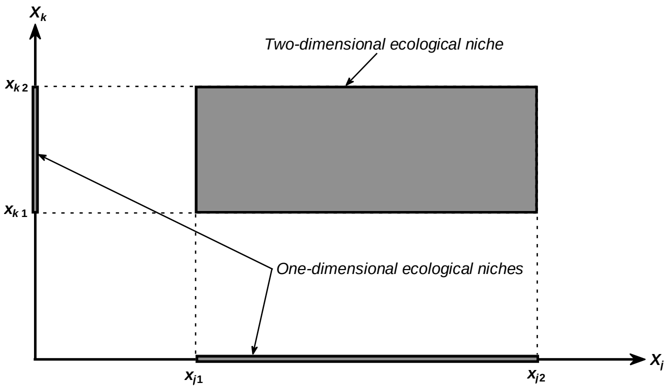


Figure 5: An example of a two-dimensional ecological niche of Hutchinson, given by a binary indicator function of well-being $h_{jk}(X_j, X_k)$, equal to one above the shaded area and zero outside this area (the case of factors not interacting within the boundaries of the niches)

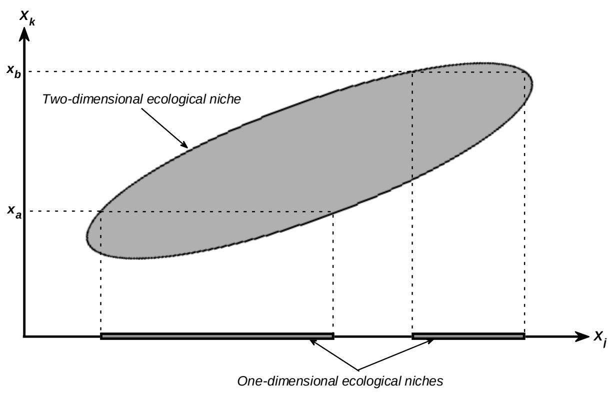


Figure 6: An example of a two-dimensional Hutchinson ecological niche given by a binary indicator function of well-being $h_{jk}(X_j, X_k)$ equal to one above the shaded region and zero outside this region (the case of significantly interacting factors)

P.A. Zade (1976) generalized the concept of a set by introducing the concept of a fuzzy or blurry set (fuzzy set). If in the classical sense a set can be defined by a binary indicator function, which for each point in space indicates its membership or non-membership in the set, then a fuzzy set is defined by an arbitrary non-negative indicator function, the value of which at any point characterizes the degree of membership of this point in the set. For example, the probability density function of a random variable can be considered as a special case of such an indicator function.

By replacing the term "set" in Hutchinson's definition of an ecological niche with a "fuzzy set", i.e. allowing non-binary well-being functions as indicator functions, we obtain a definition of a generalized, one might say, "fuzzy", ecological niche. This generalization seems to us as natural as it is to use softer, gradual functions of well-being instead of contrasting binary ones. It is obvious that fuzzy ecological niches can more adequately describe real situations. In Fig. 7 shows an example of a one-dimensional non-binary indicator function, which is taken as the density function of the normal distribution.

In Fig. 8 gives an example of a fuzzy ecological niche defined by a non-binary indicator function of well-being $g_{jk}(X_j,X_k)$ for the case of multiplicatively interacting factors X_j and X_k .. The degree of belonging of points of space to a niche, i.e. the magnitude of the values of the indicator function, is represented by the saturation of the gray color. The values of the function $g_{jk}(X_j,X_k)$ are obtained by multiplying the values of two one-dimensional indicator functions, which are the normal distribution density functions $g_j(X_j)$ and $g_k(X_k)$.

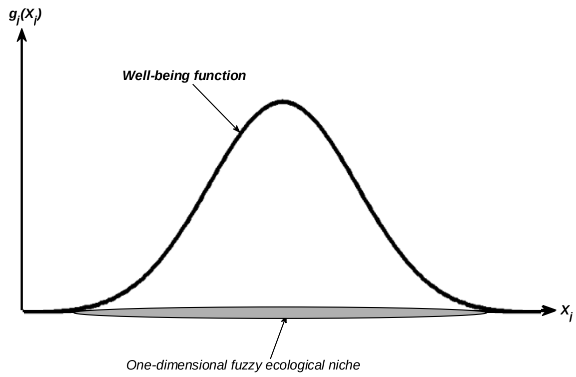


Figure 7: An example of a one-dimensional fuzzy ecological niche $[X_{j1},X_{j2}]$ defined by a non-binary indicator function of well-being $g_j(X_j)$ (the density function of the normal distribution is taken)

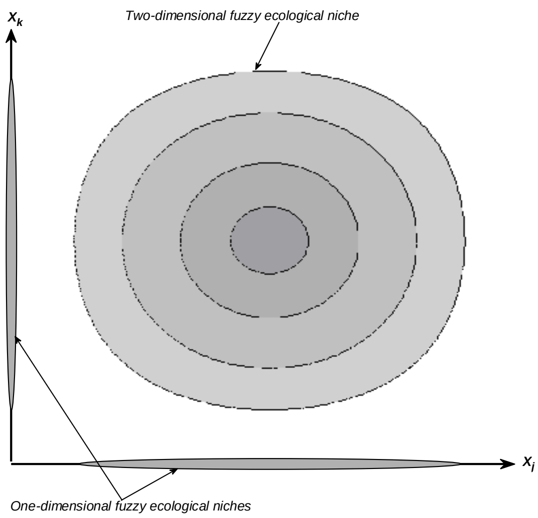


Figure 8: An example of a two-dimensional fuzzy ecological niche defined by a non-binary indicator function of well-being $g_{jk}(X_j,X_k) = g_j(X_j) \times g_k(X_k)$ (the case of multiplicatively interacting factors). The magnitude of the values of the function $g_{jk}(X_j,X_k)$, characterizing the degree of belonging of points in space to the niche, is represented by the saturation of the shading.

In Fig. 9 shows another example of a blurred ecological niche. In this case, the two-dimensional indicator function of well-being $g_{jk}(X_j,X_k)$ is obtained by taking the minimum of the values of two one-dimensional indicator functions $g_j(X_j)$ and $g_k(X_k)$ (this type of interaction of factors can be called limiting). Unlike Fig. 8, where the two-dimensional indicator function has a bell-shaped form, and its level lines are concentric circles or ellipses, in the situation of Fig. 9 The two-dimensional indicator function resembles a square or rectangular pyramid, and its level lines are squares or rectangles.

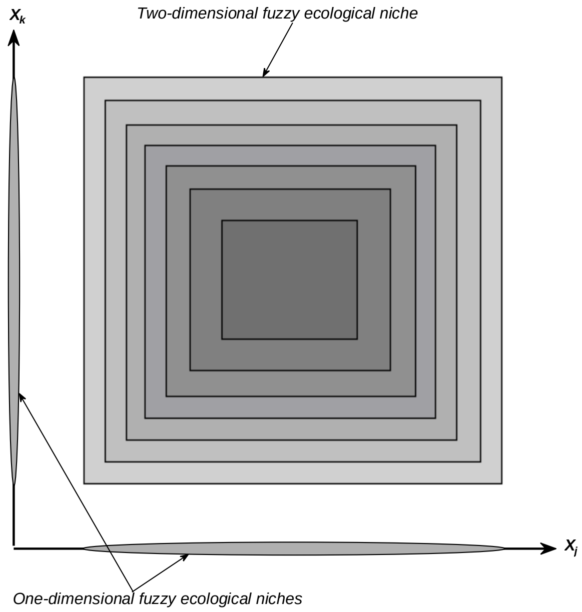


Figure 9: An example of a two-dimensional fuzzy ecological niche defined by a non -binary well-being indicator function $g_{jk}(X_j,X_k) = \min\{g_j(X_j), g_k(X_k)\}$ (the case of non-interacting factors). The magnitude of the values of the function $g_{jk}(X_j,X_k)$, characterizing the degree of belonging of points in space to the niche, is represented by the saturation of the shading.

2.6 Practical Aspects of using the Apparatus of Fuzzy Ecological Niches

Using the above definition of a blurred ecological niche and introducing additional hypotheses about the nature of the interaction of ecological factors (Gulamov, 1994; Gulamov, Logofet, 1997), one can obtain a practically working apparatus for the quantitative description of the environment.

Let $g_k(X_k, x, y, t)$ is a one-dimensional survival function of a certain species with respect to factor X_k, $k = 1, 2, ..., n$, at a geographic point (x, y) at time t. In the case of simple interactions of multiplicative or limiting type, the multivariate survival function can be reconstructed from one-dimensional functions.

Let's consider the limiting interaction. In this case, the multivariate survival function describing the joint action of factors will be a surface over the space of all factors, defined by the formula

$$g_{1,...,n}(X_1, ..., X_n, x, y, t) = \min\{g_1(X_1, x, y, t), ..., g_n(X_n, x, y, t)\}$$

i.e. each point of the multidimensional survival surface is determined by the minimum survival for all factors, i.e. according to the Liebig principle (Odum, 1986 b). As an illustrative example, we can refer to the results of an experimental determination of the dependence of fry survival on the concentrations of calcium (Ca) and aluminum (Al), given in the work of G. E. Zaikov et al. (1991).

To carry out calculations to find fuzzy niches, we created a special computer program (Gulamov, 1982; 1989). By setting the values of factors with a certain time step at different points of the area, it can be used to obtain the dynamics of a niche for a given area over a certain time interval. Some of its practical applications are described in (Gulamov, Logofet, 1997; Gulamov, 1995).

Thus, the proposed approach to formalizing an ecological niche makes it possible to find real ecological niches of populations and species and allows for their comparative analysis for different biological objects. In addition, it allows us to solve a number of problems in applied ecology, such as predicting the distribution of a species across its range, ecological zoning of a region relative to the values of environmental factors of interest to us, and obtaining ecological assessments of the area.

III. GROUP-THEORETIC RELATION OF THE SURVIVAL RATE FUNCTION

The application of the group-theoretical approach to the study of biological phenomena is caused by some features of these problems. What are these features?

Before answering this question, it is necessary to present those features of physical problems that served as the basis for the application of group-theoretical approaches in the study of solutions to these problems. Such a feature is the symmetry properties of physical problems. For example, the symmetry of space and time plays a fundamental role in physics. Its manifestations are varied. In its most general form, it is expressed in the fact that all inertial reference systems are physically equivalent. It follows from this that all physical laws have the same form in all non-rational reference systems.

To apply the methods of group theory to the study of solutions to a particular physical problem, it is not at all necessary to bring the formulation of the problems to a purely mathematical level. This is a very important property of group theory, since it allows one to use its methods even in cases where the physical laws necessary for the transition from a physical problem to a mathematical one are not yet known. This is precisely the situation that is currently observed in the theory of elementary particles (Lyubarsky, 1986).

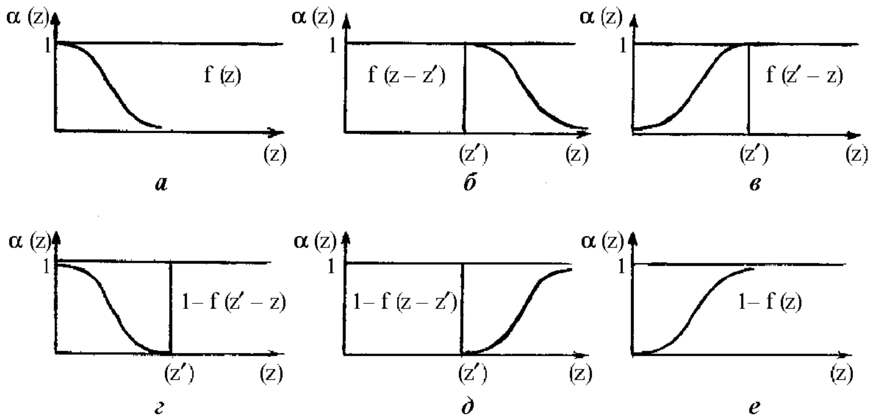
Now let's answer the question posed above. This is, firstly, the identical symmetrical structure of the gene structures of most living beings; secondly, the presence of clear classification structures (taxonomy) in biology, which reflect the relationships between organisms, and, thirdly, the predominantly qualitative, rather than quantitative, nature of the concept of biological patterns. All these features, such as symmetry, classification and the qualitative nature of biological concepts, may be the features that would allow the application of the group-theoretical approach to the study and solution of a number of biological problems.

Applying the above to the problem we are studying, we can note, firstly, the presence in the general case of only qualitative characteristics of the survival function (see Fig. 1 and 2), and, secondly, the study of the relationships between two types of factors (density-dependent and independent) of the survival of individuals in an insect population showed that the survival functions for these types of factors can differ from each other by only one or two parametric transformations (parameters mean slope and shift). All this speaks to the entirely appropriate application of the group-theoretical approach to studying the solution to the problem of interaction of environmental factors (Gulamov, Fayziev, 1990; 1992).

3.1. Group-Theoretical Study of the Survival Rate Function

This section is a study of the relationship between two types of density-dependent and density-independent factors with respect to their survival function using the mathematical apparatus of group theory. This approach does not seek to identify any one main dominant type of factor, but rather attempts to identify the relationship between these two types of factors. We proceed from the assumption that the cause of fluctuations in population size is all possible combinations of density-dependent and independent factors. Viktorov (1971) said it very well on this matter: "...it is more likely that an analysis of the factors of population dynamics can help to clarify the causes of fluctuations in productivity than that a quantitative characteristic of the flow of matter and energy through a population will provide the key to understanding the causes of its dynamics."

In a qualitative study of the mechanisms of population survival, we came to the conclusion: the behavior of the survival function as a function of density-dependent factors ($\alpha(X)$) is determined by the very nature of the determination of the survival function as a function of density-independent factors ($\alpha(A)$).



Rice. 10: Qualitative behavior of the survival function under the operations translation, rotation π and transformation $1-\alpha(z)$.

To analyze the relationship between the two types of factors described above, we will proceed as follows. We abstract from specific types of factors, replacing them with some generalized set of factors Z , the components of which are density-dependent and independent factors:

$$Z = \{d_1, d_2, ..., d_k, l_1, l_2, ..., l_r\} .$$

For simplicity, we will first consider the values of the survival function $\alpha(k \cdot z)$ with the steepness parameter $k=1$ (i.e.) and for all possible values of z , where $z \in \mathbf{Z}$. $\alpha(z)$ Let some behavior of the survival function ($\alpha(z) = e$) of the type shown in Fig. 10 be given. $f(z)$ 10 a, we will denote it by.

$f(z)$ By shifting (translating) to the point $z^{'}$ we obtain: $f(z-z^{'}) = x$ (Fig. 10b), then we rotate π around the axis (perpendicular to the axis Z) passing through the points $z^{'}$, and obtain: $f(z^{'}-z) = A$ (Fig. 10 c), i.e. mirror reflection of the function $f(z-z^{'})$. Let's perform transformations $1-\alpha(z)$ on the function $f(z^{'}-z)$ and obtain: $1-f(z^{'}-z) = B$ (Fig. 10 g). $1-f(z^{'}-z)$ For the function, we once again apply the rotation operation by π , as a result we get $1-f(z-z^{'}) = Y$ (Fig. 10 d). $1-f(z-z^{'})$ Shifting to the origin, $1-\hat{f}$ we get $1-f(z) = C$ · $1-f(z) = \hat{f}$ Once again transforming , as $1-\hat{\hat{f}}$, we obtain $1-(1-f(z)) = f(z) = e$ (Fig. 10 a). $\alpha(z)$

All possible options $\alpha(z)$ shown in Fig. 10, were obtained from the state (Fig. Z 10 a) by successive action of the operation: shift (translation) along the abscissa axis $f(z)$, rotation by π , relative to (perpendicular to the axis Z), passing through some fixed point $z^{'}$ and transformation $1-\alpha(z)$.

Let **T** denote the set of monotone functions (such as in Fig. 10), mapping **R** into the segment [0,1], and let **S (T)** be the symmetric group of the set **T** , i.e. the group of all one-to-one mappings **of T** onto itself, where the product of two mappings φ denotes ψ their superposition.

For each $z^{'} \in \mathbf{R}$ denote by $G(z^{'})$ the subgroup of **S (T)** generated by the set $P(z^{'}) = \{x(z^{'}), y(z^{'}), a(z^{'}), b(z^{'}), c\}$ (the elements x, y, a, b, c are X, Y, A, B, C respectively, and note that the element does $c \in P(z^{'})$ not $z^{'}$ depend on), where the elements of $P(z^{'})$ act on the set **T** as follows (the result of the action of the element $g \in$ **S (T)** on the function $\varphi \in$ **T** is denoted by φ^{g}):

$$1)\ \varphi^{x(z'0}(z)=\varphi(z-z') \qquad 2)\ \varphi^{a(z')}(z)=\varphi(z'-z)$$
$$3)\ \varphi^{b(z')}(z)=1-\varphi(z'-z) \qquad 4)\ \varphi^{y(z')}(z)=1-\varphi(z-z') \tag{14}$$
$$5)\ \varphi^{c}(z)=1-\varphi(z)$$

It is easy to verify that from formulas (14) it follows that if $z' \neq 0$, That $a^2(z')=b^2(z')=c^2=e$, and the elements $x(z')$ and $y(z')$ have an infinite order, if then $z'=0$, it is obvious that $x^2(z')=y^2(z')=e.$

Let $z' \neq 0$. the study of the structure of the group $G(z')$ show that in terms of generating and defining relations $G(z')$ it has the following form:

$$G(z')=\prec a(z'),x(z'),y(z')\big|x^2(z')=y^2(z'),x(z')y(z')=y(z')x(z'),a^2(z')=e,$$
$$a(z')x(z')a(z')=x^{-1}(z'),a(z')y(z')a(z')=y^{-1}(z')\succ.$$

Let us denote by G the subgroup of the group **S (T)** generated by the set $\{G(z')\big|z' \in \mathbf{R}\ \} = Q$. Q - is the union of all subgroups $G(z')$, where z' it runs along the entire real line **R.**

For any $k>0$ we define in **T** subset M_k as follows: $M_k=\{a^g(k\cdot z)\big|g\in G\}$. Let **M** = $\bigcup_{k\in R^+} M_k$, for $\varphi \in$ **M** and $k \in$ **R** $^+$ we define the function φ_k, assuming $\varphi_k(z)=\varphi(k\cdot z)$. It is clear that $\varphi_k \in$ **M.** It can be verified that for any $\varphi \in$ **M**, $k \in$ **R** $^+$ and $g \in G$ the following equality holds:

$$(\varphi^g)_k=(\varphi_k)^g. \tag{15}$$

Let **K** be the group of positive real numbers under multiplication and KxG **is** a direct product of groups **K** and G . For any $k \in$ **K**, $g \in G$, $\varphi \in$ **M** we set

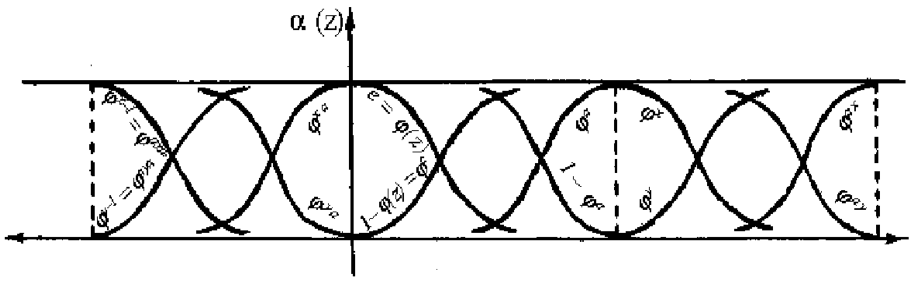
$$\varphi^{(k,g)}=(\varphi_k)^g \tag{16}$$

From (15) it follows that equality (16) determines the actions of the group KxG on the set **M.**

Moreover, it can be verified that KxG acts on the set **M** transitively, i.e. for any φ u ψ from **M** $\exists g \in$ **TO** $\times G$ such that $\varphi^g=\psi$.

3.2 Biological Interpretation of the Group Structure G

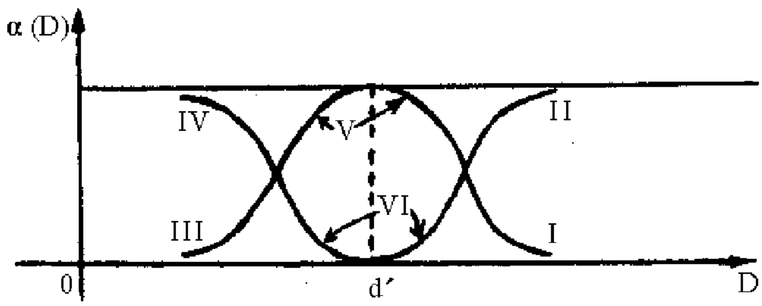
Let G $f\in M_k$. Let us denote the image of the function f when it is affected by an element g from the group . G Let us consider several monotone curves formed using group elements f^g (Fig. 11).



Rice. 11: Qualitative Behavior of the Survival Function Formed by the Elements of the Group G

From this it is clear that all elements $H(z')$ are subgroups of the group G generated by the set $\{x(z'), y(z')\}$ (it is easy to see that $H(z')$ is a normal divisor of the group $G(z')$), when acting on the element e ($e = f(z)$) curves are formed that lie to the right of the axes passing through fixed points z' ($z' \in \mathbf{R}$), including the ordinate axis, $z' = 0$, and the elements of the cosets (aH) give us their mirror image. Taking into account the hypotheses we have previously put forward (Gulamov, 1989), these monotonic curves (right-hand and their mirror images, as well as the corresponding combinations of right-hand curves with their mirror images), shown in Fig. 11, give us all possible curves of survival functions as a function of density-dependent ($\alpha(x)$) and independent ($\alpha(a)$) factors.

The analysis of monotone curves in Fig. 11 gives us only six varieties of the survival function as a function of density-dependent factors relative to some fixed value of $d'(d' \in \mathbf{Z}^+$ - the set of positive integers) (Fig. 12), and two varieties of the survival function as a function of density-independent factors relative to some fixed value of $l'(l' \in \mathbf{R})$ (Fig. 13).



Rice. 12: Six Varieties of the Survival Function as a Function of Density-Dependent Factors

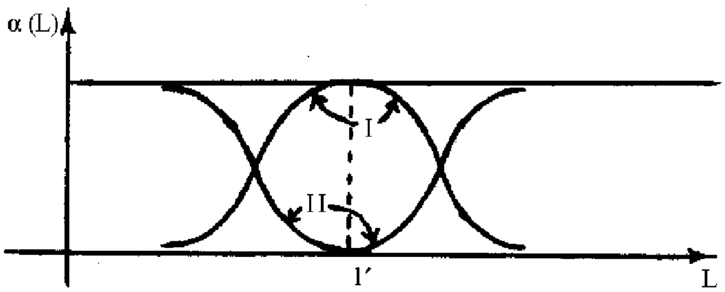


Fig. 13: Two Types of Survival Function as a Function of Density-Independent Factors

Of the six possible curves of the survival function as a function of density- dependent factors, two (Fig. 12, V and VI) completely coincide in shape with two types of curves of the survival function as a function of density-independent factors (Fig. 13, I and II).

3.3. Intersection in n -dimensional space

Let the volume formed by the intersection of the corresponding cylinders in n -dimensional hyperspace be given:

$$H_1 = \{\alpha_1(A_1, t, n) | A_1, A_2, \ldots, A_n\}$$

$$H_2 = \{\alpha_2(A_2, t, n) | A_1, A_2, \ldots, A_n\} \qquad (17)$$

....

$$H_n = \{\alpha_n(A_n, t, n) | A_1, A_2, \ldots, A_n\}$$

The intersection of these cylinders in a given hyperspace is given as follow:

$$\mathrm{H} = \bigcap_{i=1}^{n} H_i = \{\min[\alpha_1(A_1, t, n), \alpha_2(A_2, t, n), \ldots, \alpha_n(A_n, t, n)] | A_1, A_2, \ldots, A_n\}. \qquad (18)$$

In general, such intersections can be infinite in n the -dimensional hyperspace belonging to the set of monotone curves

$$\mathbf{M} \times \mathbf{M} \times \ldots \times \mathbf{M} = \mathbf{M}^{\mathrm{n}}, \text{ i.e.}$$

$$\mathrm{H} = \bigcap_{i=1}^{n} H_i \in \mathbf{M}^{\mathrm{n}} \qquad (19)$$

If $K \times G$ a group acting on a set of monotone curves M, we denote it by T (K $\times G$ = T), then on the set M $^{\mathrm{n}}$ the group will act

$$\mathbf{T} \times \mathbf{T} \times \ldots \times \mathbf{T} = \mathbf{T}^{\mathbf{n}}.$$

Using the results of the group-theoretical study of the population survival function and the above, we can write that for any two intersections $\bigcap_{i=1}^{n} H_i$ and $\bigcap_{j=1}^{n} H_j$ from $\mathbf{M}^{\mathrm{n}}$ $\exists t \in \mathbf{T}^{\mathrm{n}}$ is such that

$$(\bigcap_{i=1}^{n} H_i)^t = \bigcap_{j=1}^{n} H_j. \qquad (20)$$

Since the group $\mathbf{T}^{\mathrm{n}}$ acts on the set $\mathbf{M}^{\mathrm{n}}$ transitively, the elements of the set $\mathbf{M}^{\mathrm{n}}$ (different intersections) are symmetrical relative to each other with respect to the action of the elements of the group $\mathbf{T}^{\mathrm{n}}$.

An ecological niche can be considered as an intersection of volumes such as (17) in n -dimensional hyperspace, i.e. as a certain n -dimensional hypervolume of survival $\mathrm{H} = \bigcap_{i=1}^{n} H_i$. From the assumption and relation (20) it follows that the various intersections (18) in the set $\mathbf{M}^{\mathrm{n}}$ are symmetrical relative to each other. This means that the survival hypervolumes of different species are symmetrical relative to each other. Moreover, this conclusion may be supported by another piece of evidence of pulsating niches.

3.4. On the Symmetry of the Survival Rate Function

This section deals with the symmetric relationship of the survival functions to each other. At the same time, I would like to briefly dwell on the concept of symmetry, and what exactly we have when we study the symmetrical relationship of environmental factors among themselves.

The concept of symmetry in a popular form is covered in the works of G. Weyl (1968), L. Tarasov (1982), A. Migdal (1983, 1989), I. Shafransky (1985), A. Sonin (1987), R. Feynman (1987) and S. Petukhov (1988).

The famous mathematician G. Weyl (1968) suggested a wonderful definition of symmetry, according to which an object is called symmetrical if it can be changed in some way, resulting in the same thing you started with. It is in this sense that we speak of the symmetry of the laws of physics. This means that physical laws or the ways of representing them can be changed in such a way that this does not affect their consequences (Weyl, 1968; Feynman, 1987). Symmetry is understood in this sense in this work.

Those operations of change, the consequences of the impact of which are not reflected in the nature of physical or biological laws, constitute the essence of the concept of symmetry (Alekseev, 1978; Achurkin, 1978).

The principles of symmetry (invariance) are divided into geometric and dynamic. In this case, geometric symmetry is the symmetry that can be directly seen (rotations in space, shifts in time, translation). Unlike geometric principles, dynamic principles of symmetry are formulated in terms of the laws of nature and relate to certain types of interactions rather than to any correlations between events. An example of dynamic symmetry can be group-theoretical transformations. In confirmation of this, we will cite what Sonin (1987) said: "Thus, in classical mechanics, symmetry has lost its visual geometric meaning. Now it appears in an abstract form as a condition under which the control describing this or that physical law does not change its form. In this case, the conditions themselves must form a group in the mathematical sense."

From the above it follows that the study of symmetry is the conduct of classification in the corresponding field of research. For this purpose, we conducted a group-theoretical study of the survival function. For this purpose, the behavior of the survival function of the species shown in Fig. 10a was chosen. $\alpha(z) = e$

By performing the following operations on these curves: shift (translation), rotation π around the axis (perpendicular to the Z axis) and transformations $1-\alpha(z)$ the appropriate number of times, we can obtain all kinds of survival functions (Fig. 10). Group-theoretical study of these operations showed that they form a group (see 14 - 16 formulas). Moreover, the group $K \times G$ found acts on the set **M** (the set of all possible functions of survival rates) transitively, i.e. for any φ and ψ from **M** $\exists g \in K \times G$ such a thing that $\varphi^g = \psi$. It follows from this that the various survival functions **(M)** considered by us (see Fig. 1, 2 and 10) relative to the action of the elements of the group $K \times G$ are in a symmetrical relationship with each other, i.e. any element of the set **M** can be obtained from any other element of the set **M** using the corresponding symmetry operation from the group $K \times G$. A clear example is shown in Fig. 11.

From the above it follows that, knowing the survival function for one type of environmental factor, we can obtain survival functions for other environmental factors using the elements of the transformation group $K \times G$, i.e. group $K \times G$ plays the role of the law of transformation of the survival function of environmental factors. Moreover, this law of transformation operates regardless of the type of factors.

The symmetry of the survival function also indicates that all environmental factors: abiotic, biotic and anthropogenic, with respect to their survival functions, are closely related, i.e. different manifestations of the same function (Gulamov, Fayziev, 1992). This also speaks of the internal consistency of environmental factors among themselves.

The logical conclusion about the symmetry of various intersections of the survival function in n the -dimensional hyperspace, i.e. about the symmetry of the hypervolumes of species survival, is fully justified. This conclusion indicates the symmetry of ecological niches of different species.

3.5. About some of the group's ideas G

In the work of Gulamov and Fayziev (1992) and in other previously published works of ours (Gulamov, 1989, 1994, 2012 (a); Gulamov and Fayziev, 1990), the different nature of the survival function and their transformations were studied. Research has shown that all varieties of the survival function are essentially different modifications of the function:

$$\alpha(z) = \exp(-k|z(t)|), \tag{21}$$

where k is the steepness coefficient ($k \in \mathbf{R}$) and $z(t)$ is the value of any environmental factor ($\mathbf{Z}$) at a given time t ($z(t) \in \mathbf{Z}$).

Knowing one law of change of the survival function (21), it is possible to derive any other using the appropriate transformation from the group G . The abstract nature of the group structure does not allow for transformations for applied purposes. In this regard, a problem arises: the need to study various G representations of the group G that would allow this theoretical premise to be applied in solving applied problems.

First, about the structure, nature and morphisms – objects to the study of which any group-theoretical research is ultimately reduced. The group G in terms of forming and defining relations has the following form (Gulamov, Fayziev, 1992; Gulamov, 1994):

$$G =< a,x,y|a^2 = e, xy = yx, x^2 = y^2, axa = x^{-1}, aya = y^{-1} > . \tag{22}$$

Studies of the behavior of the elements of the group G have shown that each of the generators of the group (4.9) forms a cyclic subgroup:

$$D = \{a,e\}; \quad X = \{\dots,x^{-1},x^0,x^1,\dots\}, \quad Y = \{\dots,y^{-1},y^0,y^1,\dots\}.$$

Let us consider a subset of the set of elements of a group H of the following form: $C = \{x^{2n+1}y^{-(2n+1)}, x^{-(2n+1)}y^{2n+1}\}$, where $n \in \mathbf{Z}^+$, $x = f(z-z')$, $y = 1-f(z-z')$ (Gulamov, Fayziev, 1992; Gulamov, Khoshimov, 1997). Research shows that $\forall n \in \mathbf{Z}^+$

$$x^{2n+1}y^{-(2n+1)} = x^{-(2n+1)}y^{2n+1} = 1-f(z) = c, \quad c^2 = f(z) = e \, .$$

Moreover, a subgroup c of a group G is its center, i.e. $c = Z(G)$ From the above it follows that: $H =< x,y,|xy = yx, x^{2n} = y^{2n}, x^{2n+1}y^{-(2n+1)} = x^{-(2n+1)}y^{2n+1} = c, c^2 = e, cx = y, cy = x, n \in Z^+ \succ$ Taking into account the properties of the element, c group G (22) can be rewritten as:

$$G_M =< a,c,x | a^2 = c^2 = e, ac = ca, xc = cx = y, axa = x^{-1}, xax = yay = a > \qquad (23)$$

$C =< c | c^2 = e >$; $C = \{c,e\}$; $G/H =< a | a^2 = e >$; $H/C =< x >$;

$G/C =< a,x | a^2 = e, axa = x^{-1}, xax = a >$; $C/1 =< 1 >$; $G/H \cong D$ - Abelian; $G/C \cong Z_2 \cdot \mathbf{Z}$ = $\{x^n, ax^n\}$, where is Z_2 the type of the group C and D, $Z = \{x^n | n \in \mathbf{Z}\}$ $\Rightarrow H/C \cong Z$, $G/C = D \times H/C$.

Analysis of the structure (23) shows that, firstly, the group G_M is structurally much simpler than the group G, and secondly, $G \cong G_M$. This allows us to further study the group G_M instead of G.

With respect to the generating elements of group G_M (23), we assume: let be x a parallel transfer (P); a be a symmetric transformation (S_1) with respect to the perpendicular axis passing through the points z^* of the abscissa axis; c be a symmetric transformation (S_2) with respect to the horizontal (parallel to the abscissa axis) line passing through the middle of the interval [0,1] of the ordinate axis. In other words $x - P$, $a - S_1$, $c - S_2$.

Now let us consider transformations in a more general form - this transformation on the projective plane is defined in the following form:

$$\begin{aligned} x' &= a_{11}x + a_{12}y + a_{13}t \\ y' &= a_{21}x + a_{22}y + a_{23}t \\ t' &= a_{31}x + a_{32}y + a_{33}t \end{aligned} \qquad (24)$$

The matrix of such a transformation has the form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (25)$$

Let the elements of this transformation matrix satisfy the conditions: $a_{11} = 1, a_{12} = 0, a_{13} = z^*, a_{21} = 0, a_{22} = 1, a_{23} = 0, a_{31} = 0, a_{32} = 0, a_{33} = 1$. Then we obtain the matrix of parallel translation along the axis z. Such a matrix has the form:

$$P = \begin{bmatrix} 1 & 0 & z^* \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (26)$$

The entry $a_{13} = z^*$ means that the origin of coordinates is shifted to a point z^* along the abscissa axis. Symmetric transformations S_1 for S_2 matrix (25) have the form:

$$S_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Here the matrices S_1, S_2, P are the essence of the elements a, c, x in the group G_M (23), respectively. Research has shown that matrices form a group with the usual matrix multiplication operation. Let us designate this group as follows: S_1, S_2, P

$$SM(3,R) = \{A \in GL(n,R) | \det A = \pm 1\} \quad (27)$$

Elements of the group $SM(3,R)$ satisfy all the properties of the group G_M : $S_1^2 = S_2^2 = E$, $S_1S_2 = S_2S_1$, $PS_2 = S_2P$, $S_1PS_1 = P^{-1}$ And $PS_1P = S_1$. It follows from this that The obtained results allow us to write: $G \cong G_M \Rightarrow G \cong SM(3,R)$. This result satisfies the above statement of the problem $k \geq 1$. The following material is presented under the assumption that in (21). For this purpose, we rewrite equation (21) taking into account the coefficients of the transformation matrix (25) as follows: $G_M \cong SM(3,R)$.

$$\alpha(z) = a_{23} + a_{22} \exp(-|a_{11}z(t) - a_{13}|) \quad (28)$$

Thus, the survival functions can be transformed depending on the impact of the group elements $SM(3,R)$ as follows:

$$\begin{gathered} \alpha(z)^E = \exp(-|z(t)|), \quad \alpha(z)^P = \exp(-|z(t) - z^*|), \qquad \alpha(z)^{S_1P} = \exp(-|z^* - z(t)|), \\ \alpha(z)^{S_1} = \exp(-|-z(t)|), \quad \alpha(z)^{S_2} = 1 - \exp(-|z(t)|), \quad \alpha(z)^{P^{-1}} = \exp(-|z(t) + z^*|), \\ \alpha(z)^{S_1PS_2} = 1 - \exp(-|z^* - z(t)|), \quad \alpha(z)^{PS_2} = 1 - \exp(-|z(t) - z^*|), \text{ etc.} \end{gathered} \quad (29)$$

The superscripts of these survival functions denote the effects of the group elements $SM(3,R)$ on the function of the type (21) according to the law (28). Such impacts transform the survival functions into appropriate forms – what needed to be solved (Gulamov, Khoshimov, 1997).

Depending on the value of the survival function in the optimal intervals for the corresponding environmental factors, one or another type of survival function curve can be selected, for example:

1. For the optimal interval $[z_1^*, z_2^*]$, where $\alpha(z) = 1$ corresponds to S_1P and P the transformation.
2. For the optimal interval $[z_1^*, z_2^*]$, where $\alpha(z) = 0$ corresponds to S_1PS_2 and PS_2 the transformation.
3. For the origin of coordinates, where the $\alpha(z) = 1$ transformation S_1 corresponds E .
4. For the origin of coordinates, where the $\alpha(z) = 0$ transformation S_2 corresponds S_1S_2 .

In this fourth point, the indicated transformations are sufficient to calculate the value of the survival function for all environmental factors of any type.

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To summarize the above, it can be stated that in a group $SM(3, R)$ there will always be a transformation that allows us to move from one type of survival function to another according to the ecological factors of interest to us, which do not require preliminary research on this factor.

IV. ON THE SET OF INFORMATION MODELS

This section is devoted to a qualitative study of the nature of information models and their set DIMIFN (All possible mentally permissible diversity of information models of objects of the ideal and physical nature) (Gulamov, 2017; 2018; 2021 a).

It should be noted that the concepts of information and information models are associated with fundamental issues of deep concepts of natural and humanitarian sciences (Shileiko A., Shileiko T., (1983).

We begin our study of this issue with an analysis of scientific and technical publications that currently exist (Internet materials : ru.wikipedia.org ; wiki . vspu . ru › users / wodolazov / model / index ; best - exam . ru › znakovie - modeli /).

1. An information model (in a broad, general scientific sense) is a set of information characterized by the essential properties and state of an object, process, phenomenon, as well as its relationship with the outside world.
2. *Information models cannot be felt or visualized; they have no material embodiment because they are built only on information*. Information models are models created in a formal language (i.e. scientific, professional or specialized). Examples of formal models are all types of formulas, tables, graphs, maps, diagrams, etc. The information model is thus - *a general scientific concept meaning both an ideal and a physical object of analysis*.
3. The verbal information model is obtained as a result of human mental activity and is presented in verbal form; it is symbolic, i.e. it can be expressed in drawings, diagrams, graphs, formulas, etc.

In modern scientific research, the term "information model" is used for various scientific concepts, such as: the structure of an atom, DNA, quark, gluon, the chemical structure of hydrogen, circle, plane, n -dimensional space, Higgs field, electric field, etc.All this can be generalized under the term information model. And the set of information models (DIMIFN) can be represented as a forming and reflecting space-time continuum (Gulamov, 2018).

“Information” is the essence of physical or abstract quantities, it is a more general concept relative to the term “information model” (Shileiko A , Shileiko T., (1983). An information model is any logical, structured, informational-semantic, abstract expression (Gulamov, 2017; 2018).
Examples of information models: *mathematical expressions, functions and formulas:*

$$= a\cdot b,\ E = mc^2,\ \vartheta = \frac{S}{t},F =- F,\ F = am,\ \frac{dy}{dx} = \left(\frac{\Delta y}{\Delta x}\right),\ \ I(a, b) = \int_a^b y(x)dx,\ \text{ etc.;}$$

physical and chemical structures (http://images.myshared.ru/4/319832/slide_1.jpg):

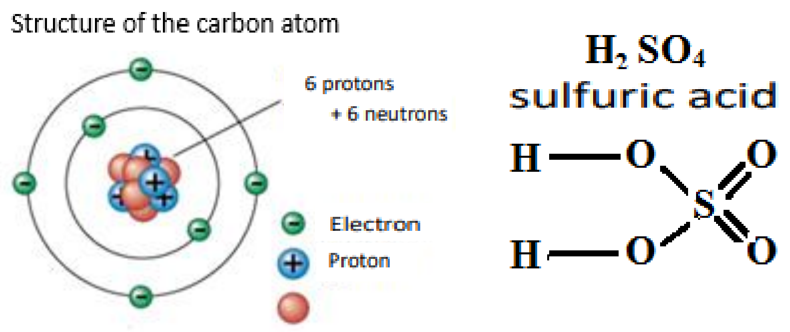


Fig. 14: Various Information Models of the Structure of the Carbon Atom and Sulfuric Acid

In the first and second chapters of this monograph, where the issues of interaction of environmental factors and the theoretical-group relationship of the function of survival rates were considered, the set of functions of survival rates (M) ($\forall i \quad \alpha(A_i,t): \mathbf{R} \rightarrow [0,1], \quad i=1,2,...$), which is, in turn, an information model reflecting the interaction of an environmental factor on a biological object (Fig. 14).

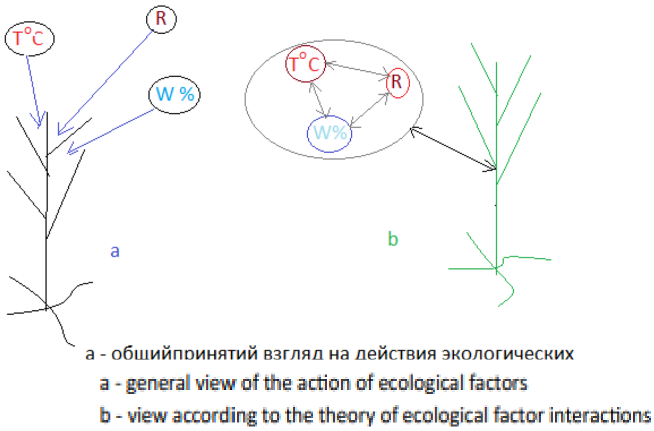


Fig. 15: Various Hypothetical Information Models Reflecting the Impact of Environmental Factors on Plants

It would also be possible to cite many examples from the fields of biology and astronomy to information models, but within the framework of this work we will limit ourselves to references to Internet materials (berl.ru/article/kletka/dnk/..; medicalplanet.su/genetica/27. html; collectedpapers. com . ua / ru /... ;). In a word, information models are any mathematical, physical, chemical, biological formalisms that have a hidden structure and dynamics that manifest themselves in the space-time continuum (Gulamov, 2018) and in theoretically conceivable ideal objects (mathematical formalisms and objects of a spiritual nature).

Information models have different kinds of variants. In mathematics, physics and chemistry, the same object can be represented by different information models in the form of a mathematical formula, graph, table or diagram. A more concrete and illustrative example can be given from biology: the number of genes in plant and animal species is in the thousands, each of which, through mutations, can produce dozens of alleles. Let us consider a simplified situation of the code in a haploid set, where there are only 1000 genes, each of which is capable of producing only 10 alleles through mutations. In this case, the number of gene combinations reaches 10^{1000} (each combination is an independent information model), that is, it reaches a huge value, which is greater than the number of electrons and protons in the Universe. If we translate all this into real species existing in nature and try to imagine the possible number of gene combinations, we will get infinity to the highest degree! This is the real unimaginable power of nature's biodiversity (Gulamov, 2016).

If we try to imagine various options of all possible conceivable and inconceivable mathematical, physical, chemical and biological information models, then we can definitely say that the set DIMIFN forms a set of infinite order of the power of the continuum.

In a word, the elements of the DIMIFN set are all possible variants of reflection of objects of the space-time continuum and abstract theoretical representations of their interaction.
From the above it follows that there are two types of information models:

a) information models of objects of the physical world;
b) information models of an abstract (ideal) nature.

Item a) includes mathematical, physical, chemical, and biological descriptions of various natural phenomena and objects, for example: the phenomena of electricity, magnetism, gravity, elementary particles, and others. Item b) includes mathematical objects: arithmetic, algebraic, geometric operations, functions and the Holy Scriptures.

The nature of information models of any objects of the physical world is an immaterial entity that contains the information laws of the existence of physical objects. The nature of information models, regardless of the real existing natural objects, is given. Therefore, information models do not disappear, are not destroyed, are not lost, i.e. they are outside of space-time. An information model is like an immaterial beginning of the life of physical objects or a kind of hidden treasure of the physical world.

Information models are knowledge about physical and abstract (ideal) objects, processes and phenomena. If this assumption is taken as a basis, then with a high probability it can be said that any mathematical operations are specified in DIMIFN.

An information model, unlike information, only makes sense to humans. Any new knowledge can be obtained on the basis of operating with elements of the DIMIFN set. It is appropriate here to quote from the work of A. Shileiko, T. Shileiko (1983): “They say that I. Newton was once asked how he managed to discover the law of universal gravitation. "I thought about it!" was the answer." This means that I. Newton operated with the corresponding information models and eventually discovered a new information model – the law of universal gravitation!

The elements (M_i, где $i \in N$ – sets of natural numbers) of the DIMIFN set are not interconnected, but at the same time they can have the ability to combine with each other and with themselves in any quantity, thereby creating new information models of physical and ideal objects. In this case, an important condition for the combination is reasonable mutual correspondence. For example, the elements of the periodic table: they can be found in nature independently or in combinations in the form of some chemical or biological molecule.

The term "combinations" refers to certain structured mathematical, physical, chemical and biological formalisms.

Let us try to carry out some formalization of the set DIMIFN , taking into account some natural scientific generalizations:

1. The invariant nature of physical laws.
2. Transitions from a particular case to a general one and vice versa.
3. Elements of the periodic table and all kinds of chemical descriptions of physical and biological substances.
4. Genetic structures and descriptions of living objects, etc.

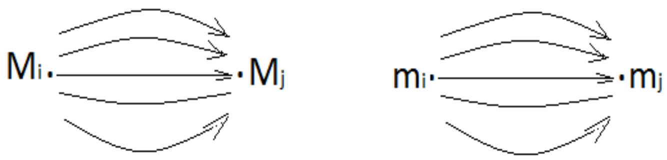
Considering the abstract nature of the elements of the infinite-dimensional set DIMIFN of the continuum cardinality, it is possible to carry out generalized algebraic formalization of the set DIMIFN (Gulamov, 2018):

$$DIMIFN = \{M_1, ..., M_i, M_{i+1}, ..., M_n, ...\},$$

each element of this set is a subset, that is,
$M_i = \{m_1, ..., m_i, m_{i+1}, ...\}$, where $m_i \cap m_k \neq \emptyset$, $i, k \in N$, (there is a combination between the elements of the subset M_i). The elements of the set DIMIFN (M_i) can be of different dimensions $l(l = \overline{1, n})$. Naturally, the elements of the set DIMIFN, interacting with each other, can generate new all sorts of different elements: $M_i \cap M_j \neq \emptyset$, perhaps, there are such elements of DIMIFN that do not interact: $M_r \cap M_k = \emptyset$ (there is no combination between the elements).

Considering that the DIMIFN sets include all sorts of scientific and technical and abstract (ideal) sets and are to a high degree a superset, we can assume with high probability:

a) that there are subsets of the set DIMIFN $\left(M_i, M_j\right)$, where $Mor\left(M_i, M_j\right)$and are given $Mor\left(m_i, m_j\right)$(Fig. 16):



Rice. 16: All possible morphisms are a subset of the set DIMIFN $\left(M_i, M_j\right)$.

b) in many cases for DIMIFN elements $\left(M_i, M_j M_k\right)$and the elements of its subsets $\left(m_i, m_j m_k\right)$ satisfy the law of composition (Fig. 17):

$$Mor\left(M_i, M_j\right)\times Mor\left(M_j, M_k\right)\rightarrow Mor\left(M_i, M_k\right) \text{ and}$$

$$Mor\left(m_i, m_j\right) \times Mor\left(m_j, m_k\right) \rightarrow Mor\left(m_i, m_k\right)$$

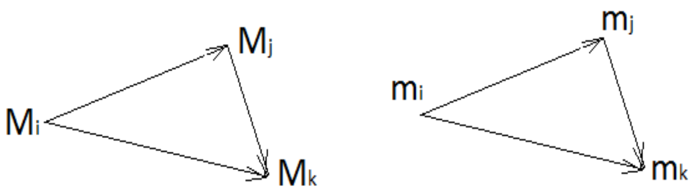


Fig. 17: Schematic reflections of the composition of DIMIFN elements $\left(M_i, M_j M_k\right)$.

From the theory of interaction of environmental factors it follows that many elements of $\left(M_i, M_{i+1}, ...\right)$ the DIMIFN set are symmetrical with respect to the corresponding transformations (Gulamov, Krasnov, 2009; Gulamov, 2012 (b)).

All kinds of combinations and permutations of information models reflect new phenomena of the physical or ideal world that were previously unknown.

It is undeniable that the diversity of elements of the DIMIFN set is of an unimaginably high order.

4.1 On Generalized Set Operations Information Models

This section is a continuation of our previous works and is devoted to a qualitative study of the nature of the execution of generalized operations on elements of a set of all kinds of information models, mentally admissible for the diversity of objects of ideal and physical nature DIMIFN (All possible mentally permissible diversity of information models of objects of the ideal and physical nature) (Gulamov, 2017; 2018; 2020). For convenience, in the future we will simply call the DIMIFN set the set of information models (MIM).

The nature of the terms “information”, “information model” and partially “set of information models” was analyzed in our previous works (Gulamov, 2018; 2020). In this paper, the execution of generalized operations on MIM elements will be formally described, to the extent possible, with appropriate examples. If we represent a set of information models (Gulamov, 2020):

$$\text{DIMIFN} = MIM = \left\{... IM_i, IM_{i+1}, ..., IM_k, ...\right\}, \qquad (1)$$

then each element of this set represents a certain information model (IM). In turn, each IM consists of a certain number of information components (IK). It follows that each MIM element is a subset consisting of the corresponding information components, that is:

$$IM_i = \{IK_1^i,\ IK_2^i, \ldots, IK_n^i\}.$$

Depending on the level of complexity of the phenomenon being described, the number IK_n^i in each IM_i may be different.

Let us consider generalized operations: combinations, differentiations and recompositions over MIM elements. The generality of the operation of combination, differentiation and recomposition lies in the fact that they can be represented in the form of any mathematical operations or their combination, taking into account the observance of the principle of logical-constructive and logical-informational correspondence. The meaning of the terms of the operations we consider in this work is understood as follows:

A combination is a collection of parts together, one alongside the others in some kind of unity, coherence.

To differentiate means to dismember, to distinguish the separate from the particular when considering the whole.

Reassembly is the creation of a whole from many parts in a new way, relative to the original form.

MIM elements produces a specific type of information model. Differentiation of the corresponding elements of the MIM set generates a limited number of information components. By rearranging the results of the differentiation operation, it is possible to obtain a limited number of information models. These three operations are the basis of any research, knowledge, analysis and synthesis in the process of studying the environment. The degree to which the simulation models we have identified are reflected in reality depends on the successful application of the operations of combination, differentiation and recomposition. An instructive example is the hero of Arthur Conan Doyle's stories (1986), namely the actions of Sherlock Holmes, where he, by combining, differentiating, and recombining the collected information, is able to create the integrity of the completed event (a new information model).

I. Combinations: the action of this operation is that, by combining a certain number of elements M I M in logical-constructive and logical-informational correspondence, it is revealed $IM_r \in MIM$. The result may be previously known to us or new to us IM

The property of the combination operation in (1):

1. $\exists\, IM_{i+r}, IM_{k+l} \in MIM\colon IM_{i+r} * IM_{k+l} \neq \emptyset$, information models IM_{i+r}, IM_{k+l} are combined in a logical-constructive combination.
2. $\exists\, IM_{j+r}, IM_{h+l} \in MIM\colon IM_{j+r} * IM_{h+l} = \emptyset$, information models IM_{j+r}, IM_{h+l} are not combined in a logical-constructive combination.
3. $\exists\, IM_i,\ IM_{i+r}, IM_l \in MIM\colon \left(IM_i * IM_{i+r}\right) * IM_l = IM_i * \left(IM_{i+r} * IM_l\right) = IM_n \in MIM$, the combination is associative.
4. $\exists\, IM_l,\ IM_{l+r}, IM_k \in MIM\colon \left(IM_l * IM_{l+r}\right) * IM_k \neq IM_l * \left(IM_{l+r} * IM_k\right)$, the combination is not associative.
5. $\exists\, IM_i, IM_{i+r} \in MIM\colon IM_i * IM_{i+r} = IM_{i+r} * IM_i = IM_j \in MIM$, the combination is commutative.
6. $\exists\, IM_t, IM_{t+r} \in MIM\colon IM_t * IM_{t+r} \neq IM_{t+r} * IM_t$, the combination is not commutative.

Examples of the Combination Operation: Information models can be combined or not combined depending on the principle of their logical-constructive and logical-informational correspondence. More illustrative examples of compatible and incompatible things IMcan be given from the fields of mathematics, physics, chemistry and biology. IM functions of the form $x^2 + 1 = 0$and Newton's second law $(F = ma)$are not compatible because these two $IM$are not in logical-constructive and logical-informational correspondence. kxHooke's law (reflecting the action of elastic forces on a metal ball), the second $(F = ma)$ and third laws (F = - F) of Newton are compatible IM. IMThe result of such a combination is a type $m\ddot{x} =- kx$- a mathematical model of elastic vibrations of a metal ball. IM, characterizing the combination of chemical elements: sodium $1123Na\,)_2)_8)_1\; 1S^2, 2S^2, 2p^6, 3S^1$ and chlorine $1735Cl\,)_2)_8)_7\; 1S^2, 2S^2, 2p^6, 3S^2, 3p^5$ combine, and this combination is commutative. The result of this combination is $NaCl$. $24He\,)_2\; 1S^2$Helium and neon $1020Ne\,)_2)_8\; 1S^2, 2S^2, 2p^6$are incompatible according to the principle of logical-constructive and logical-informational correspondence. In the language of chemistry, this is explained as the completeness of the outer electron shells of the elements helium and neon. One can give many examples from the field of biology and ecology to the above-described properties of the combination operation, but the examples given quite convincingly reflect the idea.

II. Differentiation: the action of this operation is that most elements of MIM can be divided into a limited number of information components (IK_n). There are some MIM elements that cannot be differentiated because they are not divided into information components, that is, they are considered a single whole IM. Fundamental world constants (physical, mathematical: the speed of light, Planck's constant, Boltzmann's, ..., π...) just relate to non-differentiable elements MIM. Non-differentiable elements of MIM are by their nature more compatible with other elements of MIM, thus they participate in the identification of new information models previously unknown to us.

Property of the Differentiation Operation in (1):

1. $\exists\, IM_i \in MIM$: $IM_i = \{IK_1, IK_2, ..., IK_n\}$, IM_i $-$ differentiable
2. $\exists IM_k \in MIM$: $IM_k \neq \{IK_1, IK_2, ..., IK_m\}$, IM_k – non-differentiable, that is, it consists of a single information $-$ component IM_k.

Examples of the operation of differentiability. Differentiation of chemical structures of elements of the periodic table into electrons, protons and neutrons; differentiation of the ecosystem into ecological components; differentiation of DNA and RNA into nucleic acid sequences, etc.

III. Re-Arrangement: The effect of the recomposition operation is that the result of the differentiation operation $IM_i = \{IK_1, IK_2, ..., IK_n\}$ can be recomposed in such a way that it can generate other IM, different from the original form IM_i. Various rearrangements and different information models are possible.

Properties of the Reflow Operation (1):

Let be $\{IK_1, IK_2, ..., IK_n\}$a set of information components. Different rearrangements of this set of information components can produce a variety of information models. As a result, it is possible to obtain the same or new, previously unknown ones IM. $\{IK_1, IK_2, ..., IK_n\}$ There are also options where this differentiation is not recomposed in any other way than its original form.

Examples of the Recomposition Operation: Let us be given the information components: S −distance, t − time to travel this distance S, and v − speed to travel this distance. Various logical-constructive and logical-informational correspondences of these information components (S, t, v) give us different information models: $S = v{\cdot}t$; $t = S/v$ and $v = S/t$. IMThus, we got different distances, times and speeds. All kinds of nucleic acid sequences can give rise to different DNA and RNA. Here all possible sequences correspond to a variety of combinations.

These operations are the basis of a generalized algorithm for cognition, research, analysis and synthesis of environmental studies and artificial intelligence.

IM that we are considering is not limited; they can be applied as much as necessary. The optimal execution of the above operations should be based on the principle of logical-constructive and logical-informational correspondence, that is, spatio-temporal correspondence to the imaginary reality. The above operations are the mechanism that generates all kinds of information models.

The operations of combination, differentiation and recomposition are the basis of any cognitive, scientific research and gaming processes, in other words, the basis of any intellectual processes. In these processes, the order in which the named operations are performed does not matter. The sequence of execution depends on the nature of the problem being studied, the relevant moment in time and location.

The object of research and study of human intelligence is information models of the surrounding world. The process of exploration and study of the world around humanity is not limited in the past, present or future. Therefore, the power of MIM is potentially infinite.

MIM elements do not impose any restrictions on MIM; on the contrary, they provide it with an infinite variety of continuum power. In a word, the MIM set is a universal set of all information models.

The set of survival rate functions with respect to the operations of superposition (translation (shift), rotation π and transformation $1-\alpha(z)$), multiplicativity, and taking a minimum does not form an algebraic structure of a ring. Our analysis, taking into account the above-mentioned operations on the subject of the algebraic structure of the ring, showed that in MIM, firstly, it does not fulfill commutativity and, secondly, there is no inverse element.

V. ECOLOGICAL FIELD OF SURVIVAL

Before moving on to the concept of the "ecological field of survival" it is advisable to briefly explain the meaning of the term "physical field". In the work of V.I. Smirnova (1974) defines a physical field: "If some physical quantity has a certain value at each point in space or part of space, then the field of this quantity is determined in this way. If a given quantity is a scalar (temperature, pressure, electric potential), then the field is called scalar. If a given quantity is a vector (speed, force), then the field determined by it is called a vector field." Ya. B. Zeldovich and M. Yu. Khlopov (1988) explain this same term as follows: "Let a certain quantity be defined in space. This means that we can say what this quantity is at each point in space. For example, we know what the temperature is in a particular place. In this case, they say that the field of this value is given. In our example, it is a temperature field. If a rectangular coordinate system X, Y, Z is introduced into space , so that each point in space is characterized by the values of its coordinates, then the field is a function of the coordinates of each point and formally represents a function of three variables x,y,z . If the quantity whose field we are considering changes over time, then the field of this quantity depends on time and is called non-stationary. If a quantity does not depend on time, its field depends only on spatial coordinates and is called stationary" (Zeldovich, Khlopov, 1988).

Currently, there are various names for physical fields: gravitational, electromagnetic, strong and weak nuclear, quantum and Higgs fields. More detailed information about physical fields is given in Internet materials (https://ru.wikipedia.org › wiki › 2021).

To reveal the concept of "ecological field of survival", we need to reveal such concepts as ecological factor, survival function, habitat (Gulamov, 1982; 1994; 1997; 2012 (a); Gulamov, Logofet, 1997).

An environmental factor is a certain physical and/or biological force that changes according to certain patterns $f(A_i)$and manifests itself as a condition or element of the environment A_i, which is capable of directly or indirectly influencing a living organism, at least at one stage of its individual development (Gulamov, 1982; 2012 (b)):

$$f\left(A_i\right) \text{ где } A_i = \left\{a_i^1, a_i^2, \ldots, a_i^n\right\},\ i = \overline{1, n}\ .$$

Environmental factors interact with biological objects, and biological objects respond with adaptive reactions, acting as biological factors, and this, in turn, gives grounds to speak about the interaction of environmental factors.

In nature, environmental factors affect biological objects in a complex manner. The nature and significance of the complex impact depends on the nature of the interaction of environmental factors with each other.

When environmental factors interact, a certain state always arises, which is called the environment. If we assume that the environment is a formative and determining process, then it becomes clear that environmental factors act on biological objects simultaneously and jointly, that is, in a complex manner.

Adaptive reactions of biological objects to the impact of environmental factors is the survival of these objects. If the specific adaptive response of biological objects is understood as their survival, then we can talk about the maximum or minimum survival of biological objects in connection with the impact of environmental factors. It follows that for each environmental factor there are certain intervals of optimality where the survival rate of biological objects is maximum. Survival is the ability of an organism or population to withstand the impact of an environmental factor. The impact of environmental factors on individuals of a population should be taken into account through the concept of the survival function. We represent the survival function as a quantitative expression of the survival of organisms or individuals of a population, which characterizes the impact of an environmental factor and is a scalar value. The survival function can be chosen as a monotone function mapping R (sets of real numbers) onto a segment [0, 1]using the example of an environmental factor A_i:

$$\forall_i\ \alpha\left(A_i, t\right):\ R\rightarrow[0, 1],\ i = \overline{1, n} \tag{1}$$

In reality, the survival of an organism or individuals of a population at any given moment in time $t$is determined by the influence of a complex of factors:

$$\alpha\left(\vec{A}, t\right) = \alpha\left(A_1, A_2, \ldots, A_n, t\right),$$
$$\alpha\left(A_i, t\right):\ \forall i\ \ 0 < \alpha(A_i, t)\leq 1, \tag{2}$$
$$N(t + 1) = \alpha(\vec{A}, t)\cdot N(t),$$

Where $N(t)$ and $N(t + 1)$ – the number of individuals in the population at time t and $t + 1$ respectively.

Each biological species differs from each other in its characteristic ecological value – the values of the survival function, $\left(\alpha\left(\vec{A}, t\right)\right)$ respectively. It follows that each point of the survival space (habitat) at each moment in time (x, y, z, t) is characterized by a set of values of the survival function of the corresponding biological species:

$$(x, y, z, t):\ \left\{\alpha_1\left(\vec{A}, t\right), \alpha_2\left(\vec{A}, t\right), \ldots, \alpha_m(\vec{A}, t)\right\}, \tag{3}$$

Where (x, y, z, t) – coordinate of the survival space at the corresponding moments of time, $\left\{\alpha_1\left(\vec{A}, t\right), \alpha_2\left(\vec{A}, t\right), \ldots, \alpha_m(\vec{A}, t)\right\}$ – corresponding values of the survival function $m$of biological species. Since biological species are different, it is natural that the values of their survival function differ from each other.

Against the background of the above definitions and examples of physical fields, we can say that the ecological value we have described – the survival function (1,2) – determines *the ecological field of survival* of a population or biological species in the survival space (habitat). *The ecological field of survival,* by the nature of the survival function (1), is a non-stationary scalar field (Gulamov, 2021 (b)).

Depending on the type of combination of environmental factors and their quantity at different points in the survival space (habitat), the values of the survival function of individuals in the population will be different. If there are biological species $m$in this survival space, then each point of the survival space under consideration is characterized by a different value $m$of the survival function pieces (3).

Each point of the ecological field of survival, unlike other physical fields, at each moment in time is characterized by a multiple value (3) of the survival function of the corresponding biological species, otherwise known as a multidimensional survival function (Gulamov, Terekhin, 2004).

The environment-forming ecological factors, the number of species of biodiversity and the presence of corresponding trophic chains at different points of the habitat vary over time, hence the “tension” (value m) (3) *of the ecological field of survival,* which is also changeable. All this points to the complex nature of the “pulsation” of the non-stationary scalar *ecological field of survival.*

The uneven distribution of biological species in the survival space (in the habitat) is explained by the variability of the structure of the ecological survival field (3).

VI. CONCLUSION

This monograph studies the issues of theoretical prerequisites of ecological algebra: basic aspects of the theory of interaction of ecological factors; group-theoretical relations of the survival rate function, information models, generalized operations over a set of information models and ecological survival fields. This was the first time that the problem was formulated in this way.

Summarizing the theoretical research on the above aspects in this monograph, the following conclusions can be drawn.

According to the theory of interaction of environmental factors:

Ecological factors: firstly, these are diverse (potentially unlimited) variable natural forces; secondly, the adaptive reactions of biological objects to the impact of ecological factors is their survival; thirdly, the impact of environmental factors on individuals of a population should be taken into account through the concept of the survival function; fourthly, despite the potentially infinite varieties of ecological factors, the functions of survival coefficients corresponding to them can be divided into six varieties.

The interaction of environmental factors is a certain hypervolume of survival, formed by the interaction of the function of survival coefficients of the corresponding environmental factors among themselves.

By replacing the term "set" in Hutchinson's definition of an ecological niche with a "fuzzy set", i.e. allowing non-binary well-being functions as indicator functions, in other words, replacing ecological factors with survival functions, we obtain a definition of a generalized, one might say, "fuzzy" ecological niche. This generalization seems to us as natural as the use of softer survival functions corresponding to them instead of ecological factors. It is obvious that blurred ecological niches can more adequately describe real situations.

Our group-theoretical studies of six varieties of the function of survival coefficients of ecological factors show that: firstly, all possible monotonic functions of survival coefficients (**M**) differ from each other only in the parameters of steepness and shift. And with respect to the translation operation (shift), rotation by π and transformations $1-\alpha(z)$ form a non-Abelian infinite-dimensional group $K \times G$; secondly, the invariance of the form of the survival rate function is

$$\alpha(z) = a_{23} + a_{22} e^{-k\left|a_{11} z(t) - a_{13}\right|},$$

the corresponding ecological factors indicates that all ecological factors – abiotic, biotic and anthropogenic – are closely related in relation to their functions of the survival coefficient, i.e. they constitute different manifestations of the same function. This indicates the internal consistency of environmental factors with each other, in other words, environmental factors are symmetrical relative to each other.

According to research on simulation models:

An information model is any logical, structured, informational-semantic, abstractly expressed physical or immaterial (ideal) object.

All kinds of combinations of information models generate a variety of conceivable or inconceivable information models of physical and ideal objects that have not been previously identified.

Knowledge of the nature of the physical and ideal world is based on the analysis and synthesis of their various information models and on the identification of corresponding new information models.

The set of survival coefficient functions is a simulation model of the survival of biological objects, and is therefore a subset of the sets of simulation models.

The set DIMIFN of the power of the continuum.

The set DIMIFN contains all kinds of mathematical operations: morphisms, homomorphisms, compositions, etc.

On the study of generalized operations on a set of simulation models (MIM):

Any information model is the result of applying the operations of combination, differentiation and recomposition. The order in which the above operations are performed does not matter. The main thing is to comply with the principle of logical-constructive and logical-informational correspondence.

The mechanism of any cognitive process is the operations of combination, differentiation and recomposition.

MIM is an open set with respect to the operation of combination, differentiation, and recomposition.

Non-differentiable elements MIM can be taken as IMan information component.

The implementation of variants of equally possible combinations and reconfigurations is dictated by dissipative, bifurcation, attractor, synergetic and other nonlinear states of the structure of spatio-temporal structures.

The subset of the survival rate function, the MIM set, with respect to superposition operations (translation (shift), rotation on π and transformation $1-\alpha(z)$), multiplicativity and taking a minimum is an open subset.

According to research on the ecological field of survival:

The ecological value, a function of the survival rate of biological species in the environment, determines the non-stationary scalar ecological field of survival.

Each point of the ecological field of survival, unlike other physical fields known in science, is characterized by several values of the survival function.

Variability over time, environment-forming ecological factors, biodiversity and trophic chains form the "pulsating" nature of the non-stationary scalar ecological field of survival.

The reason for the heterogeneity of ecological systems is the variability of their biotic components over time, in other words, the "pulsating" nature of the ecological field of survival.

The fundamental basis of theoretical ecology is the study and research of the algebraic properties of ecological phenomena. One of the main ecological phenomena is the interaction of ecological factors and the formation of corresponding survival functions in biological objects for this interaction.

All the beauty in nature is when the human "voice" is in unison with the great "symphony" of nature.

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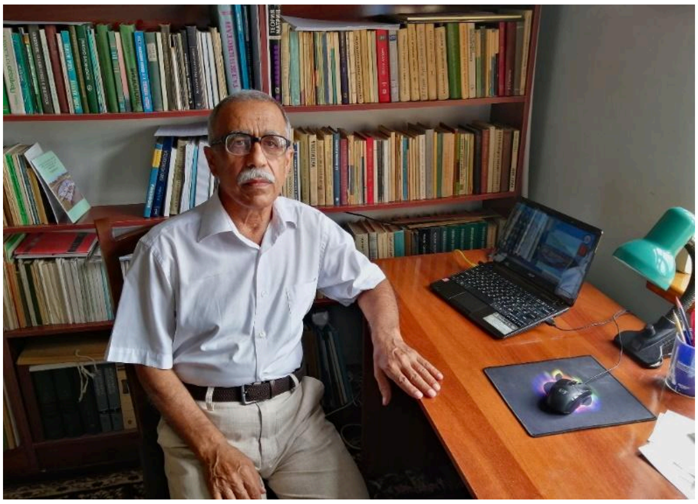

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Currently serving as an independent researcher, I specialize in various aspects of ecological science and mathematical modelling. My primary research interests include the mathematical modelling of ecological factor interactions and ecological niches, the algebra of ecology, biodiversity theory, the biophysics of ecosystems, and the development of information models.

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Professional Engagement:

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