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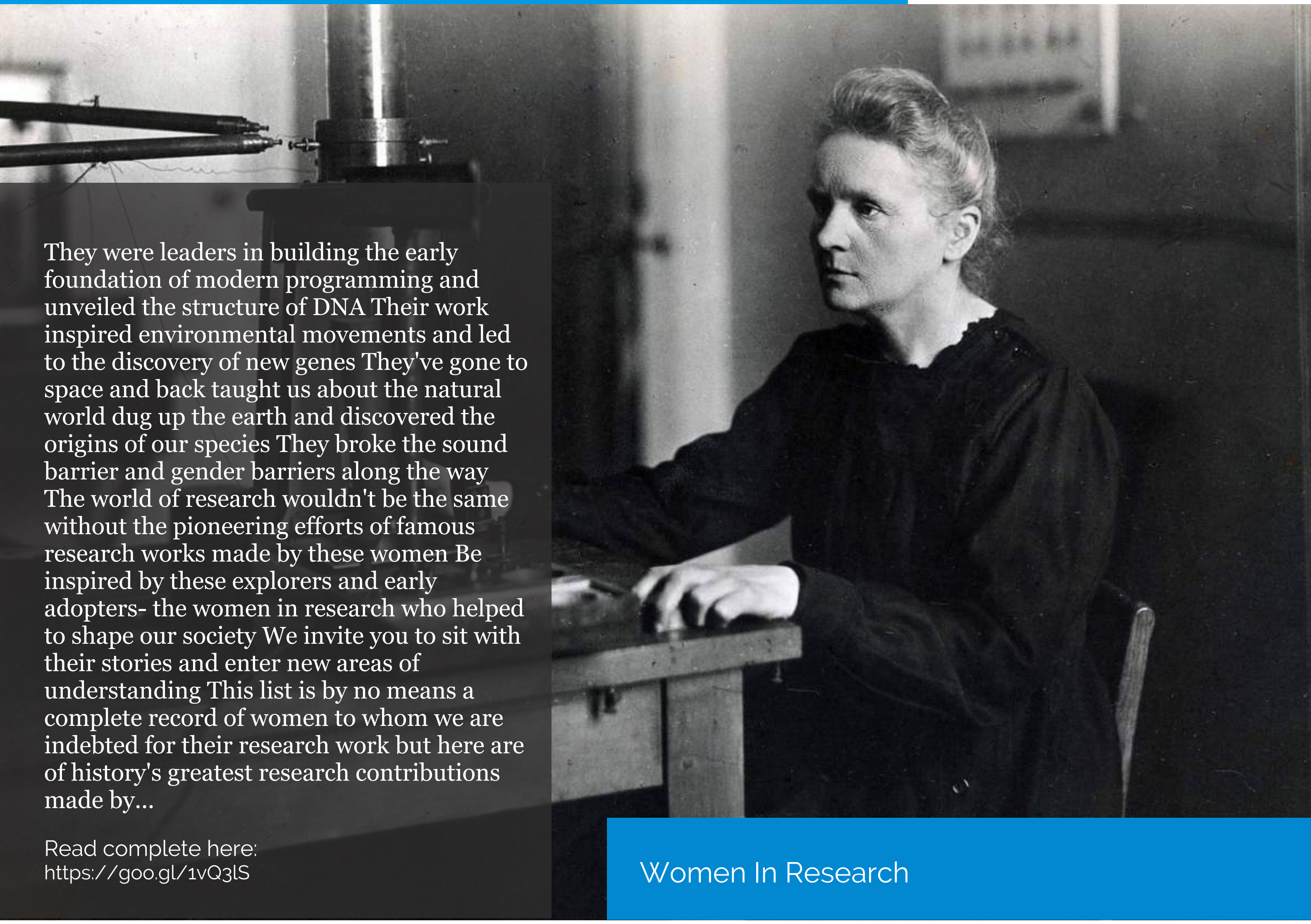

They were leaders in building the early foundation of modern programming and unveiled the structure of DNA Their work inspired environmental movements and led to the discovery of new genes They've gone to space and back taught us about the natural world dug up the earth and discovered the origins of our species They broke the sound barrier and gender barriers along the way The world of research wouldn't be the same without the pioneering efforts of famous research works made by these women Be inspired by these explorers and early adopters- the women in research who helped to shape our society We invite you to sit with their stories and enter new areas of understanding This list is by no means a complete record of women to whom we are indebted for their research work but here are of history's greatest research contributions made by...

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Theory of Dark Energy

Friedhelm M. Jöge

ABSTRACT

The presented article provides a theory of dark energy that appears to have been developed in two complementary ways. On the one hand, this theory is based on physics and mathematics and, on the other hand, it is developed on the basis of available data. This corresponds to the discovery of the laws of planetary motion in elliptical planetary orbits by JOHANNES KEPLER in the past. He developed his laws from a large amount of data. Later it was theoretically substantiated more thoroughly by ISAAK NEWTON. The focus is on deriving a formula for the equivalence of energy and time (1), page 2. Precursors to the presented „Theory of Dark Energy" were published in the articles [1-11].

The derivation of the formula for the equivalence of energy and time provides new theoretical insights and applications in theoretical terms. These are listed in the "Application" and „Future research fields" sectors; five applications are listed in the „Application" sector.

This derivation leads to the discovery of a new law of nature. This is explained in section „Conclusion".

A formula for calculating dark energy was developed in a previous article published in the International Journal of Physics and Astronomy [1].

Keywords: dark energy, planck time, law of nature, age of the universe, fundamental oscillations of a cosmic space, cosmology, theoretical physics.

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Theory of Dark Energy

Friedhelm M. Jöge*

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A formula for calculating dark energy was developed in a previous article published in the International Journal of Physics and Astronomy [1].
It is:

$$E_d = (h/t_p^2) \cdot t_u \tag{1.2}$$

The „The Foundations of a Dark Energy Theory" was first mentioned in my previous work "Commentary about Calculation of Dark Energy and Dark Matter", published in the Journal of Physics and Astronomy [2]. The presented article „Theory of Dark Energy" is now completed and a physical-mathematical and theoretical derivation of the formula (1.2) is provided.

Keywords: dark energy, planck time, law of nature, age of the universe, fundamental oscillations of a cosmic space, cosmology, theoretical physics.

First way

I. PHYSICAL-MATHEMATICAL AND THEORETICAL DERIVATION

The derivation of the formula (1), page 2 for the „Equivalence of Energy and Time" [3] requires only the assumptions that the PLANCK time t_p is an oscillation period τ and dark energy satisfies the PLANCK / EINSTEIN formula

$$E = h\,\nu \tag{1.1}$$

Oscillations are fundamental oscillations of a cosmic space [4 pg.15]. THOMAS GÖRNITZ says: „Structural quanta emerge from a quantum-theoretical description of „oscillation states" of a system

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around its ground state. They produce many effects. The AQIs of protyposis are also structural quanta and not particles. One can interpret them as the „fundamental oscillations of the cosmic space“.

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for the equivalence of energy and time then this leads to:

With $\nu = 1/\tau$, you get

$$E = h/\tau$$

With $\tau = t_p$, you get

$${}_pE = h/t_p \quad \text{for Energy in PLANCK time}$$

$${}_1E = (h/t_p^2) \quad \text{for Energy in 1 s}$$

$$E = (h/t_p^2) \cdot t \quad (1) \; \textit{Equivalence of Energy and Time}$$

For the age of the universe t_u , you get

$$E_d = (h/t_p^2) \cdot t_u \quad (2) \; \textit{Equivalence of dark Energy and age of the universe}$$

Second way

II. DERIVATION WITH DATA

In my article „Calculation of Dark Energy and Dark Matter“ [1] you can find on page 2 the derived formula:

$$E_M = c^5 / (8^{1/2}\, G\, H_o) = 5.61 \cdot 10^{69}\ \text{J} \qquad (2.1)$$

This formula did emerge from the BECKENSTEIN HAWKING entropy and the HAWKING temperature, see my article [1], pg.2.

In formula (1.2): (h/t_p^2) is $= 2.2802 \cdot 10^{53}\ \text{Js}^{-1}$ (a)

$$E_d = 5.61 \cdot 10^{69}\ \text{J} \cdot 70 / 4 = 0.982 \cdot 10^{71}\ \text{J}$$

With $t_u = 4.3056 \cdot 10^{17}$ s , you get

$$E_d/t_u = 0.982 \cdot 10^{71}\ \text{J} / 4.3056 \cdot 10^{17}\ \text{s} = 2.2807 \cdot 10^{53}\ \text{Js}^{-1} \qquad (b)$$

The numerical values calculated using formulas (a) and (b) correspond to a high degree.

This means that formula (1.2) is validated and correct. It should be acknowledged as a law of nature, so as KEPLER's laws of planetary orbit descriptions have been confirmed and acknowledged as correct from the large amounts of data available.

The available data has been published by the MAX PLANCK Institute for Radio Astronomy.

III. APPLICATION

Applications of the formula (1.2) as natural law for experimental research or practical applications have not yet been carried out. The reasons for this are explained in „Future research fields“ in the next section. However, applications to answer open questions in Theoretical Physics can be made.

The following publications show how formula (1.2) can be used to answer open questions and give concrete examples of such applications.

In addition to the four applications previously described in the article „Time is quantized“ [5], „The Universe – an Open System“ [6], „Dak Energy is not constant“ [7], „Equivalence of Information and Squared Energy“ [8] the present article “Theory of Dark Energy” also contains an application of formula (1.2).The statement of Prof. Dr. Alexandre Tkatchenko from the University of Luxemburg also contains a possible application of formula (1.2).The application in the present article „Theory of Dark Energy“ should be highlighted.

The possible application in this case consists in that what Prof. Dr. Alexandre Tkatchenko says: „Accurate calculating the value of Dark Energy could be helps bring together two of the largest fields in physics: Quantum Field Theory (QFT) and the General Theory of Relativity (ART) developed by ALBERT EINSTEIN.

Future research fields

Future experimental applications of formula (1.2) are hardly to be expected, as dark energy is not yet experimentally accessible. In addition, dark energy cannot be observed directly and is diffusely distributed throughout the universe and is therefore not easy to detected.

However, applications of formula (1.2) could be made to answer open questions in Theoretical Physics: Since the dark energy is relative [9] and dark energy is not constant [7], the energy on Earth is different than the energy at the edge of the universe. What this means for the development of the universe from Big Bang to today must be researched. That doesn't matter for the Earth, but whether the linear function of dark energy depending on the age of the universe (see diagram [7]) is still valid and the exact calculating of dark energy is still correct must be reconsidered.

Another application of the formula (1.2), which was already mentioned in the „Application“ section, is given by Prof. Dr. Alexandre Tkatchenko.

Research into possible interdisciplinary applications of formula (1.2) could, for example, be applied in areas outside of physics, such as in cosmology or in the interdisciplinary modeling of physical systems, in future research.

Expanding the possible scope of application could open up exciting avenues for further research.

IV. CONCLUSION

The formula (1.2) is theoretically justified and validated based on available data.

It should be acknowledged as a natural law. „KEPLER's“ laws of planetary motion were theoretically founded by ISAAK NEWTON („NEWTON's“ law of gravitation), which he discovered and which represents a law of nature. The situation is similar when generalizing the formula for the „Equivalence of Dark Energy to the age of the universe“ to the „Equivalence of Energy and Time“ [3]. That is, when I say in all modesty: „This formula also represents a law of nature“.

The article „Calculation of Dark Energy and Dark Matter“ [1] was the first to accurately calculate the value of Dark Energy. Accurate calculating of this value could help bring together two of the largest fields in physics: Quantum Field Theory (QFT) and the General Theory of Relativity (ART), developed by ALBERT EINSTEIN. That's what Prof. Dr. Alexandre Tkatchenko says. This is also a possible application.

Definition of symbols used in formulas

E = Energy
E_d = dark Energy
E_M = Energy that corresponds to the visible baryonic matter
t_u = age of the universe = 13.75 billion years with 1 year = 365, 2422 days (Google).
$t_u = 4{,}30557 \cdot 10^{17}$ s [12]
t_p = PLANCK time
h = PLANCK constant of action $\hbar = h/(2\pi)$
G = constant of gravitation
Ho = HUBBLE constant
ν = frequency
τ = period of oscillation

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9. JÖGE, F.M.: Energy is Relative. Advances in Theoretical & Computational Physics, 09. December 2024, Volume 7, Issue 1.
10. JÖGE, F.M.: Essay on Calculation of Dark Energy and Dark Matter. Advances in Theoretical & Computational Physics 2024, Dec. 05, Vol.7, Issue 4, pg.1-2.
11. JÖGE, F.M.: Advances in Exporation of Dark Energy. European Journal of Applied Sciences, Vol.12,No.4, pp.460-462.
12. This is the result of a new detailed study. Researchers from the University of Bonn evaluated images from the HUBBLE Space Telescope together with colleagues from the US- universities of Stanford and California.

The highlight:

your calculation takes more factors into account than previous studies. Their value for the age of the universe is therefore particularly close to reality. The results will soon be published in the trade magazine „Astrophysical Journal".

A General Method for Construction of Bivariate Stochastic Processes Given Two Marginal Processes

Jerzy K. Filus & Lidia Z. Filus

Northeastern Illinois University

ABSTRACT

Given two arbitrary univariate stochastic processes $\{Y_t\}$, $\{Z_t\}$, assumed to only share the same time t. When considered as describing (time dependent) random quantities that are physically separated (the baseline case), the processes are independent for every time epoch t. From this trivial case we move to the case where physical interactions between the quantities make them, at any moment t, stochastically dependent. For each time epoch t, we impose stochastic dependence on two "initially independent" random variables Y_t, Z_t by multiplying the product of their survival functions by a proper 'dependence factor' $\varphi_t(y_t,z_t)$, obtaining in this way a universal ("canonical") form of any (!) bivariate distribution (in some known cases, however, this form may become complicated thou it always exists). This factor, basically, may have the form $\varphi_t(y,z) = \exp[-\int_0^y \int_0^z \psi_t(s;u)\, dsdu]$ whenever such a function $\psi_t(s;u)$ exists, for each t. That representation of stochastic dependence by the functions $\psi_t(s;u)$ leads, in turn, to the phenomenon of change of the original (baseline) hazard rates of the marginals, similar to those analyzed by Cox (1972) and, especially Aalen (1989) for single pairs (or sets) of, time independent, random variables.

Keywords: bivariate survival functions, bivariate stochastic processes' constructions, dependence functions, biomedical applications, econometrics, bivariate Wiener and Pareto stochastic processes construction.

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The bivariate constructions are illustrated by examples of some applications in biomedical and econometric areas. Reliability applications, associated with the considered "micro shock ☐ microdamage" paradigm, obviously may follow.

Keywords: bivariate survival functions, bivariate stochastic processes' constructions, dependence functions, biomedical applications, econometrics, bivariate Wiener and Pareto stochastic processes construction.

Author: Dept. of Mathematics and Computer Science, Oakton College.
Dept. of Mathematics, Northeastern Illinois University.

I. INTRODUCTION

Suppose two random variables Y, Z are given, with some "real life" interpretation.

Mainly we consider a pair Y, Z to have bio-medical, reliability or econometric interpretation. For example, Y, Z may be (stochastically dependent) levels of some chemicals (hormones, for example) in a human or an animal body. In particular, one may consider Y to describe random level of "bad" cholesterol and Z an accompanying sugar level (both measured at the same time). Apparently, the two (random) quantities are stochastically dependent on each other. Another tandem of random quantities Y, Z can be daily salt consumption Y and blood pressure Z. There are a countless number of such pairs of random quantities that have biomedical meanings. They mostly are stochastically dependent. This

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means that a measured value z' of Z has an influence on the probabilities $P(Y \geq y_o)$ (if, for a fixed value z, the random event $Z \geq z$ occurs and z', such that $z' \geq z$, is its elementary realization). In particular, y_o may be considered a 'critical value' of Y (for example, the critical cholesterol level y_o) so that occurrence of the random event $Y \geq y_o$ means occurrence of a disease.

Again, the probability of occurring of the so defined disease may depend on the measured value z' of the random quantity Z when, for a given measurement z', we observe that $z' \geq z$, for some fixed value z. Here notice that, in this framework, one may disregard an obtained by a measurement particular value z' and only notice if $z' \geq z$ or otherwise. The more essential value, in this case, is z.

In the general case of two random variables Y, Z we will be interested in finding analytical formulas for all the conditional probabilities $P(Y \geq y \mid Z \geq z)$ for any arbitrary value y of Y and not only for a critical one y_o. To achieve this goal we will outline methodology for construction of the joint probability distribution of the random variables Y, Z under various situations.

A similar but different development of methodology of construction of bivariate probability distributions based on Cox and Aalen ideas (see, [2] and [1]) can be found in [3].

The constructions so far rely on building bivariate probability distributions of random vectors. However, the main goal of this work is to extend the obtained methods to methods where the random variables Y, Z are replaced by t-dependent stochastic processes $\{Y_t\}$, $\{Z_t\}$. Thus, given two marginal (originally independent) univariate stochastic processes, we provide tools for constructing a class of bivariate processes $\{(Y_t, Z_t)\}$ such that the "initial" processes $\{Y_t\}$, $\{Z_t\}$ and their probability distributions, for each t, remain as the marginals within the constructed joint. The obtained methods of the constructions turn out to be quite general and are governed by some (linear, in particular) integral equations.

As an example of the construction of processes with continuous time we consider construction of some classes of "bivariate Wiener processes" based on two given Wiener univariates. In applications, the underlying univariate Wiener processes may have econometric meanings. For example, one of the two marginal processes may describe the stock market level time evolution while the other the accompanying gross national product rate at time t. In another application, a process describes the level of inflation at a given time and the other processes the level of employment at the same time. In all cases one is interested in the stochastic dependence between the two marginal processes. That need motivates constructions of the 'joint stochastic processes'. In general, the marginal processes, as applied in econometry, are normal. However, among all the mutual stochastic dependencies at any given time epoch t we choose not to be those described by the classical bivariate normal models, since such dependencies are very well known. Our method provides a large variety of stochastic dependencies, some possibly overlooked in literature.

In another example we consider discrete time bivariate processes associated with first Pareto distributions. All the bivariate stochastic processes considerations are included in section 4.
Random Vectors, A General Approach

2.1 Suppose we are given two arbitrary random variables Y, Z with known marginal survival functions $S_1(y) = P(Y \geq y)$ and $S_2(z) = P(Z \geq z)$. Our aim is to provide a general methodology for constructions of various joint survival functions $S(y,z) = P(Y \geq y, Z \geq z)$ such that all their marginal survival functions are invariant with respect to the performed constructions, always remaining the same as the originally given $S_1(y)$ and $S_2(z)$. In general, we restrict our considerations to the cases where both the probability distributions, given by the survival functions $S_1(y)$, $S_2(z)$, possess probability densities so that the corresponding hazard rates $\lambda_1(y)$, $\lambda_2(z)$ exist too. In this case, according to the common rule, we can express the considered marginal survival functions as follows:

$$S_1(y) = \exp [- \int_o^y \lambda_1(t)\, dt] , \quad S_2(z) = \exp [- \int_o^z \lambda_2(u)\, du] . \qquad (1)$$

In the simplest case, i.e., when the random variables Y, Z are independent, their joint survival function S*(y, z) satisfies:

$$S^*(y, z) = S_1(y)\, S_2(z) = \exp [- \int_o^y \lambda_1(t)\, dt - \int_o^z \lambda_2(u)\, du] . \qquad (2)$$

To impose a dependence structure for the above "initially" independent random variables Y, Z , we multiply the right hand side of formula (2) by a "dependence factor" $\varphi(y,z)$. This factor we propose to call the "Aalen factor".

2.2 At this point realize that for any joint survival function $S(y,z)$ having the same fixed marginals $S_1(y)$, $S_2(z)$ there exist unique Aalen factor $\varphi(y, z) = S(y,z) / S_1(y) S_2(z)$. However, in most situations we may encounter, we do not know in advance the joint survival function S(y,z) but rather we have the marginals $S_1(y)$, $S_2(z)$. We aim to choose a proper Aalen factor $\varphi(y, z)$ in order to construct the corresponding S(y,z) . To give some more intuitive meaning to the Aalen factor we express it in exponential form:

$$\varphi(y,z) = \exp [- \alpha(y,z)] = \exp[- \int_o^y \int_o^z \psi(t;u)\, dudt] .$$

The function $\alpha(y,z)$ we call the "joiner" of the two marginal survival functions $S_1(y)$, $S_2(z)$.

Of course, the joiner $\alpha(y,z)$ is uniquely determined by a given Aalen factor $\varphi(y,z)$ since we have $\alpha(y,z) = - \log \varphi(y,z)$. However, the joiner $\alpha(y,z)$ not always is represented by the integral $\int_o^y \int_o^z \psi(t;u)\, dudt$. The 'integral representation' of a joiner is only valid when such a function $\psi(t; u)$, considered as a *kind of* an "additional hazard rate", exists. Existence of $\psi(t; u)$ together with the existence of the hazard rates $\lambda_1(t)$, $\lambda_2(u)$ involves existence of the joint probability density of the considered random vector (Y,Z) . Using the function $\psi(t; u)$ we define the conditional hazard rate of $Y \mid Z = u$ to be equal to:
$\lambda_1(t \mid Z = u) = \lambda_1(t) + \psi(t; u)du$,

and the conditional hazard rate of $Z \mid Y = t$ as:

$$\lambda_2(u \mid Y = t) = \lambda_2(u) + \psi(t; u)dt .$$

For the meaning of the differentials $\psi(t;u)du$ and $\psi(t;u)dt$ see section 2.4 and the associated "(microshok ☐ microdamage) ☐ microchange in hazard rate" paradigm. (These infinitesimal "microchanges" correspond to the above considered differentials.)

Since we are mostly dealing with the survival functions (not with probability densities), we will rather be interested in the conditional hazard rates of $Y \mid Z \geq z$ which involve the form:

$$\lambda_1(Y = t \mid Z \geq z) = \lambda_1(t) + \int_o^z \psi(t;u)\, du , \qquad (3)$$

and in the conditional hazard rates of $Z \mid Y \geq y$ using the form

$$\lambda_2(Z = u \mid Y \geq y) = \lambda_2(u) + \int_o^y \psi(t;u)\, dt . \qquad (3^*)$$

The non-negativity assumption for the function $\psi(t;u)$ is not mandatory but simplifies many considerations. More general and also necessary assumption for the class of functions $\psi(t; u)$ are given by the inequalities:

$$\lambda_1(t) + \int_o^z \psi(t;u)\, du \geq 0 \text{ , and}$$

$$\lambda_2(u) + \int_o^y \psi(t;u)dt \geq 0 ,$$

for any nonnegative t, u . As we have already defined it, the left hand sides of the above inequalities represent the (conditional) hazard rates, so they must be non-negative.

Also recall that $\lambda_1(t)$, $\lambda_2(u)$ are always nonnegative.

Restricting the function $\psi(t;\ u)$ to nonnegative values implies considering positive stochastic dependences only between the considered random variables Y, Z .

2.3 Assuming existence of the underlying hazard rates, let us analyze the general form of the joint survival function under the investigation:

$$S(y,z) = P(Y \geq y\ ,\ Z \geq z) = \exp[\ -\int_0^y \lambda_1(t)\, dt\ \ - \int_0^y \int_0^z \psi(t;u)\, dudt - \int_0^z \lambda_2(u)\, du\ ] \qquad (4)$$

with $\psi(t;\ u) \geq 0$.

First realize that upon setting on the right hand side of (4) $z = 0$, the expression reduces to the marginal $\exp[\ -\int_0^y \lambda_1(t)\, dt\ ] = S_1(y)$. Likewise, upon the setting $y = 0$ in (4) turns the expression to the marginal $\exp[- \int_0^z \lambda_2(u)\, du\ ] = S_2(z)$.

Second, upon dividing both sides of (4) by $S_2(z)$ one obtains the conditional survival function

$$S_1(y \mid z) = P(\ Y\geq y \mid Z \geq z\ ) = \exp[\ -\int_0^y \lambda_1(t)\, dt\ \ - \int_0^y \int_0^z \psi(t;u)\, dudt\ ]\ . \qquad (5)$$

Also dividing (4) by $S_1(y)$ yields the conditional survival function

$$S_2(z \mid y) = P(Z \geq z \mid Y\geq y\ ) = \exp[\ -\int_0^z \lambda_2(u)\, du\ \ - \int_0^y \int_0^z \psi(t;u)\, dudt\ ]\ . \qquad (6)$$

2.4 Let us explain more the structure of the conditional hazard rate $\lambda_1(y \mid Z \geq z)$, given "initially" the marginal (baseline) hazard rate $\lambda_1(y)$. The case of $\lambda_2(z \mid Y \geq y)$ can be understood in an analogous way.
Example 1. Suppose we have two different objects O1, O2 whose "behaviors" are exhibited by the random quantities Y, Z respectively. Take, at first, object O1 as the object "of main interest", characterized by the quantity Y. Now consider the "activity" of object O2 , measured by the random quantity Z . The activity Z is regarded as "stress" that the object O1 is subjected to.

(Such an "activity" may be understood as temperature, for example.) In short, the stress Z is an explanatory variable for the variable of the main interest Y. In the *physical* absence of the object O2 the variable Y is independent on Z, so the corresponding hazard rate for Y is simply equal to $\lambda_1(t)$, and the (unconditioned) survival function of Y equals

$$S_1(y) = P(Y\geq y) = \exp[\ -\int_0^y \lambda_1(t)\, dt\ ]\ .$$

(The following 'micro-shocks ☐ micro-damages' mechanism that yields the stochastic dependence is described in more detail in [4].)

When we consider the case where the object O2 is *physically* accompanying ("connected to") object O1, its activity "produces" at each time epoch '*physical*' micro-shock on O1 . These micro-shocks result in corresponding micro-damages in the physical structure of O1 which in turn result in micro-changes in the hazard rate of the corresponding random quantity Y. Every such a micro-change (as related to a quantity u) at a given moment t relies on adding to the Y's hazard rate the infinitesimal quantity $\psi(t;\ u)du$. This change is, actually, too small to be recognized by any physical measurement, but mathematically we can express it as an infinitesimal differential $\psi(t;u)du$ as proportional to the given "intensity" $\psi(t;\ u)$. The microchanges cumulate as u (for Z = u) "runs" through the interval [0,z] .

This phenomenon of the cumulation of the micro-changes (as corresponding to an 'elementary' random events $Y = t$) results in the conditional hazard rate

$$\lambda_1(t \mid Z \geq z) = \lambda_1(t) + \int_0^z \psi(t;u)\, du \qquad (7)$$

which, evidently, is different from the "original" (baseline) $\lambda_1(t)$.

The foregoing integration relates to the transfer from an elementary event $Z = u$ to the random event $0 \leq Z < z$ [Here realize that since we apply, as the analytic tool, the survival functions, instead of the distribution functions, we eventually will '*calculate*' the probability of its complement, i.e., the probability of $Z \geq z$].

Integrating (7) with respect to "time" t over the interval [0,y] and applying formula (5), we obtain the conditional survival function $P(Y\geq y \mid Z \geq z)$. The latter, once multiplied by the marginal distribution in the form $P(Z \geq z)$, results again in the joint survival function of the random vector (Y,Z) .

Setting $\int_0^z \psi(t;u)\, du = \varphi(t,z)$ we obtain the formula

$$\lambda_1(t \mid Z \geq z) = \lambda_1(t) + \varphi(t,z) , \qquad (7^*)$$

which corresponds to the Aalen additive model [1] being the modification of the famous Cox proportional hazards model [2] .

Remark 1: In a more general case one may consider the second differential $\psi(t;u)dtdu$ of the joiner $\alpha(,)$ as a microchange (corresponding to both elementary events ["actions"] t and u [or u at the time epoch t]) in both hazard rates caused by mutual interaction of the "micro-shocks" described by t and u, caused by activities of the objects O1 and O2. Realize that neither t nor u needs to represent time (they may, for example, be levels of hormones in a human body).

In this framework, that interaction may be better understood if the intensity $\psi(t; u)$ is chosen to have separated variables, i.e., $\psi(t; u) = \psi_1(t)\, \psi_2(u)$ (see examples in below). Notice also that the secondary differential $\psi(t;u)\, dtdu$ is added to both the original hazard rates $\lambda_1(t)$ and $\lambda_2(u)$, which agrees with the idea of *mutual* interactions.

On Representation of Bivariate Survival Functions

3.1 The Representation

Write formula (4) in a slightly more general form:

$$S(y,z) = P(Y \geq y, Z \geq z) = \exp[-\int_0^y \lambda_1(t)\, dt - \alpha(y,z) - \int_0^z \lambda_2(u)\, du . \qquad (4^*)$$

We now depart from the Aalen-like model towards a more general paradigm. We even may drop the assumption on the existence of the hazard rates $\lambda_1(y)$, $\lambda_2(z)$, and the function $\alpha(y,z)$ need not to be expressed by the integral $\int_0^y \int_0^z \psi(t;u)\, dudt$. Instead of formulas (4) and (4*) for the joint survival function of (Y,Z) we may use the following, more general, formula:

$$S(y, z) = S_1(y)\, S_2(z) \exp[-a(y,z)], \qquad (8)$$

where $\alpha(y,z)$ is assumed to be a real function, defined for $y \geq 0$, $z \geq 0$.

We will require that it satisfies the conditions specified as follows. Thus the function $\alpha(y,z)$ is:

(1) continuous, for each z, with respect to y and for each y with respect to z,
(2) nondecreasing in y and z,
(3) $\alpha(0,z) = \alpha(y,0) = 0$.

From the last condition we directly obtain the following Property.

Property 1: If the latter condition (3) is satisfied, then both the marginal probability distributions of the joint distribution given by formula (8) are preserved in the sense that they are the same as the baseline distributions $S_1(y)$, $S_2(z)$ originally given.

If the marginal and the joint probability densities of the considered random variables Y, Z exist, the concern is on non-negativity of them. While the marginal densities, say, f(y) , g(z) are non-negative due to the common assumption, we need some special condition for $\alpha(y,z)$ to assure non-negativity of the corresponding joint density $h(y,z)$.

For the joint density $h(y,z)$ to exist, the function $\alpha(y,z)$ must have continuous partial derivatives of first order $\alpha_y(y,z)$, $\alpha_z(y,z)$ and the continuous second order mixed partial derivative $\alpha_{y,z}(y,z) = \psi(y,z)$.

An additional condition that must be satisfied by $\alpha(y,z)$ has the form of the following inequality:

$$[\ \lambda_1(y) + \alpha_y(y,z)\ ] \cdot [\ \lambda_2(z) + \alpha_z(y,z)\ ] \geq \alpha_{y,z}(y,z) = \psi(y,z) . \tag{9}$$

This follows from the form of the joint density (if it exists):

$$h(y,z) = \partial^2/\partial y \partial z\ S(y,z)$$

$$= \{\ [\ \lambda_1(y) + \alpha_y(y,z)\ ]\ ^x\ [\ (\lambda_2(z) + \alpha_z(y,z)\ ] - \alpha_{y,z}(y,z)\ \} \exp[- \Lambda_1(y) - \Lambda_2(z) - \alpha(y,z)\ ] ,$$

which must be nonnegative. Here, $d/dy\ \Lambda_1(y) = \lambda_1(y)$, and $d/dz\ \Lambda_2(z) = \lambda_2(z)$, and $\alpha_{y,z}(y,z) = \psi(y,z)$.
A simpler condition than (9) for the non-negativity of $h(y,z)$, which is sufficient and necessary too, for the existence of the joint survival function (4*) is the following, obtained from (9):

$$\lambda_1(y)\ \lambda_2(z) \geq \alpha_{y,z}(y,z) = \psi(y,z) . \tag{9*}$$

The condition (9) or (9*) together with the conditions (1), (2), and (3) are sufficient and necessary for "connecting" the two survival functions $S_1(y)$, $S_2(z)$ by a given joiner $\alpha(y,z)$ into the bivariate model $S(y, z)$.

As it was pointed out above, there may be a numerous of such models.

3.2 Particular Cases of the Bivariate Models

A. Obviously, when $\alpha(y,z)$ reduces to zero for all y , z , then model (8) describes independent random variables.
B. If the baseline hazard rates exist and are constant, we call this model "exponential".
In this case we may choose $\psi(y,z) = a$ = constant to obtain the following special case of model (4*):

$$S(y,z) = \exp[\ - \lambda_1 y\ - ayz - \lambda_2 z\ ] , \tag{10}$$

where $0 \leq a \leq \lambda_1 \lambda_2$.

This, apparently, represents the first bivariate exponential Gumbel probability distribution (see [5]).

C. One also obtains the following 'Weibullian version' of the above bivariate Gumbel:

$$S(y,z) = \exp [- \lambda_1 y^{\gamma_1} - a y^{\gamma_1} z^{\gamma_2} - \lambda_2 z^{\gamma_2}] , \qquad (11)$$

where γ_1 and γ_2 are positive reals.

The latter two models are special cases of representation (4*).
We only need to check when condition (9*) is satisfied, i.e.,

$$\lambda_1(y)\, \lambda_2(z) \geq \psi(y,z) . \qquad (11^*)$$

To check for that, we properly differentiate the terms of the expression $- \lambda_1 y^{\gamma_1} - a y^{\gamma_1} z^{\gamma_2} - \lambda_2 z^{\gamma_2}$ ($\lambda_1 y^{\gamma_1}$ over y and $\lambda_2 z^{\gamma_2}$ over z and $a y^{\gamma_1} z^{\gamma_2}$ over y and z) , and set inequality (11*) in the form:

$$\lambda_1 \lambda_2 \gamma_1 \gamma_2 \; y^{\gamma_1 - 1} z^{\gamma_2 - 1} \geq a \gamma_1 \gamma_2 \; y^{\gamma_1 - 1} z^{\gamma_2 - 1} . \qquad (12)$$

It holds, for every nonnegative y and z , whenever

$$a \leq \lambda_1 \lambda_2 .$$

Thus, if the latter condition is satisfied then model (11) is properly defined.

Remark 2: As for the above given Weibullian version (11) of the Gumbel exponential model (with $a \leq \lambda_1 \lambda_2$) , this can be 'partially' generalized by the following model:

$$S(y,z) = \exp [- \lambda_1 y^{\gamma_1} - a y^{\delta_1} z^{\delta_2} - \lambda_2 z^{\gamma_2}]. \qquad (13)$$

Now inequality (12) takes the form:

$$\lambda_1 \lambda_2 \gamma_1 \gamma_2 \; y^{\gamma_1 - 1} z^{\gamma_2 - 1} \geq a\, \delta_1 \delta_2 \; y^{\delta_1 - 1} z^{\delta_2 - 1} . \qquad (14)$$

The necessary condition for (14) to hold for all nonnegative values y , z is that

$$1 \leq \delta_i \leq \gamma_i \; , \text{ for } i = 1, 2, \qquad (15)$$

together with

$$a \leq \lambda_1 \lambda_2 . \qquad (16)$$

However, condition (15) makes inequality (14) not true for $0 \leq y , z < 1$. The remedy for this is to shift the variables y , z by imposing the additional conditions:
$1 = c = y$, $1 = c = z$ for some real c.

Now model (13) is well defined with $1 \leq \delta_i \leq \gamma_i$.

Remark 3: The form of model (8) and Property 1 allows us to construct numerous of bivariate probability distributions. Namely, realize that:

1. For any given fixed 'joiner function' α(y,z, 'any' pair of two probability distributions [not necessarily both from the same class of the distributions], determined by $S_1(y)$, $S_2(z)$ (such that inequality (9) or (9*) is satisfied), may be "connected" by formula (8) to "become" the bivariate survival (distribution) function in which they remain the marginals.

2. For any fixed pair of probability distributions $S_1(y)$, $S_2(z)$, there is a wide class of possible joiners α(y,z) [determined by the conditions (1), (2), (3) from section 3.1 together with inequality (9) or (9*)] such that the two distributions can be "connected" into the bivariate model (8) in as many ways as there are possible proper functions α(y, z) .

Finally notice that the above methods for "connecting" *any* two probability distributions into bivariate distributions, resembles the idea of the copula methodology [6]. It is, however, different. For more remarks on that see [3]. •

II. ON CONSTRUCTION OF BIVARIATE STOCHASTIC PROCESSES GIVEN ANY TWO UNIVARIATE MARGINAL PROCESSES WHICH SHARE THE SAME TIME

Suppose the (marginal) stochastic processes $\{Y_t\}$, $\{Z_t\}$ are completely defined. So, for every time epoch $t \in T$ (T is a nonempty set) two random variables Y_t, Z_t have known in advance survival functions
$S_t(y) = P(Y_t \geq y)$,
$R_t(z) = P(Z_t \geq z)$.

If the corresponding hazard rates $\lambda_1(y, t)$, $\lambda_2(z, t)$ exist, then, according to what was pointed out in previous sections, for any joiner function $\alpha_t(y,z) = \alpha(y,z,t)$ [satisfying, for each t, inequality (9) with respect to the $\lambda_1(y, t)$, $\lambda_2(z, t)$ as well as conditions (1) – (3) from section 3.1] there exist a unique joint survival function which is also a function of time t, given by:

$$S_t(y,z) = P(Y_t \geq y, Z_t \geq z) = S_t(y) R_t(z) \exp[-\alpha(y,z,t)].$$

Our intention is to consider, for each $t \in T$, the function $S_t(y, z)$ as the joint survival function (or joint distribution) of the bivariate stochastic process $\{(Y_t, Z_t)\}_{t \in T}$. At this point recall that the marginals $S_t(y)$, $R_t(z)$ [as functions of time t] are assumed to define completely the processes $\{Y_t\}$, $\{Z_t\}$ respectively.

Suppose time t is continuous. If both survival functions $S_t(y)$ and $R_t(z)$ are continuous functions of the time then we will postulate the joiner $\alpha(y,z,t)$ to be continuous, as a function of t, as well. Now consider two examples of bivariate stochastic processes whose constructions are based on known initial univariate (marginal) processes.

Example A: Consider two Wiener stochastic processes $\{Y_t\}$, $\{Z_t\}$ with the, given in advance, for each t, survival functions

$$P(Y_t \geq y) = 1 - \varphi((y - bt) / \sigma_1\sqrt{t}) \quad \text{and} \tag{17}$$

$$P(Z_t \geq z) = 1 - \varphi((z - ct) / \sigma_2\sqrt{t}), \tag{17*}$$

where $\varphi(x) = (1/\sqrt{2\pi})\int_{-\infty}^{x} \exp[-w^2/2]\,dw$, and b, c are the real parameters of the considered Wiener processes.

For all time epochs s, t such that $0 < s < t$, all the distributions of the random vectors (Y_s, Y_t) and (Z_s, Z_t) are described by the usual bivariate formulas associated with Wiener processes.

We seek, for every time epoch t, a joiner $a(y,z,t)$ for the two survival functions (17), (17*) which analytically is nice and simple enough, also as a function of t. It needs to satisfy conditions (1) – (3) [section 3.1] as well as inequality (9) or (9*) for each t. If found, then, for each $t > 0$, the joint survival function of the stochastic process $\{(Y_t, Z_t)\}$ would have the form (8) which, in this case, is:

$$S_t(y,z) = S(y,z,t) = P(Y_t \geq y) P(Z_t \geq z) \exp[-\alpha(y,z,t)]. \tag{18}$$

Of course, in the vast majority of cases, (18) is different from *classical* bivariate normal distributions even if the marginals are normal.

Realize that conditions (1) – (3) from section 3.1 are usually easily checked to be satisfied, so that we really need to check condition (9).

Recall, in the case of stochastic processes, this amounts to checking if:

$$\partial^2 /\partial y\partial z\, \alpha(y,z,t) = \alpha_{yz}(y,z,t) = \Psi(y,z,t)$$
$$\leq [\lambda_1(y,t) + \alpha_y(y,z,t)] \times [\lambda_2(z,t) + \alpha_z(y,z,t)] . \quad (19)$$

This inequality needs to be satisfied for each t.

At this point notice that**:**

$$\alpha_y(y,z,t) = \int_{-\infty}^{z} \Psi(y,v,t)\, dv \ \text{ and}$$

$$\alpha_z(y,z,t) = \int_{-\infty}^{y} \Psi(u,z,t)\, du .$$

These quantities, upon the additional assumption $\Psi(u,v,t) = 0$ for all nonnegative u, v, are nonnegative and therefore the inequality

$$\lambda_1(y,t)\, \lambda_2(z,t) \leq [\lambda_1(y,t) + \alpha_y(y,z,t)] \times [\lambda_2(z,t) + \alpha_z(y,z,t)]$$

always holds.

The latter together with (19), provides solutions of (19) in the form:

$$\Psi(y,z,t) = \omega(t)\, \lambda_1(y,t)\, \lambda_2(z,t) ,$$

where the coefficient function $\omega(t)$ satisfy $0 \leq \omega(t) \leq 1$, and, in particular, $\omega(t)$ may be considered a constant in t , especially one may set $\omega(t) = 1$ for each t.

The model (or rather a candidate for a model in a practical application) we have obtained, has the form of the time dependent joint survival function of the random vectors (Y_t,Z_t) for $t \geq 0$. It has the following form:

$$S_t(y,z) = P(Y_t \geq y)\, P(Z_t \geq z) \exp[- \omega(t) \int_{-\infty}^{y} \int_{-\infty}^{z} \lambda_1(u,t)\, \lambda_2(v,t)\, du\, dv]$$
$$= \exp[- \int_{-\infty}^{y} \lambda_1(u\ t)du - \omega(t) \int_{-\infty}^{y} \int_{-\infty}^{z} \lambda_1(u,t)\, \lambda_2(v,t)\, du\, dv - \int_{-\infty}^{z} \lambda_2(v,t)dv] .$$

For the above considered '*bivariate Wiener stochastic process*' the marginals $P(Y_t \geq y)$ and $P(Z_t \geq z)$ present in the last formula, are given by (17) and (17*).

Also, in the above case, $\lambda_1(u,t)$, $\lambda_2(v,t)$ are the corresponding hazard rates associated with the considered normal distributions. It is clear, however, that the obtained class of models is much wider than the class of the bivariate Wiener. Write inequality (19) in the form:

$$\Psi(y,z,t) \leq [\lambda_1(y,t) + \int_{-\infty}^{z} \Psi(y,v,t)dv] \times [\lambda_2(z,t) + \int_{-\infty}^{y} \Psi(u,z,t)du] . \quad (19^*)$$

Another candidate for a set of the models (i.e., set of "bivariate Wiener stochastic processes") will be given by the set of the functions $\Psi(y,z,t)$ that are solutions of the following integral equation directly derived from inequality (19*):

$$\Psi(y,z,t) = [\lambda_1(y,t) + \omega_1(t) \int_{-\infty}^{z} \Psi(y,v,t)dv] \cdot [\lambda_2(z,t) + \omega_2(t) \int_{-\infty}^{y} \Psi(u,z,t)du] , \quad (20)$$

Where $0 \leq \omega_i(t) \leq 1$, for i = 1, 2 .

Realize that the right-hand side of (20) is always less than or equal than the right hand side of (19*). For equation (20) to hold the condition $\Psi(y,z,t) \geq 0$ is , in general, essential.

The integral equation (20) is nonlinear. However, it can be simplified to the following:

$$\Psi(y,z,t) = g(y,z,t) + \omega_1(t)\, \lambda_2(z,t) \int_{-\infty}^{z} \Psi(y,v,t)dv + \omega_2(t)\, \lambda_1(y,t) \int_{-\infty}^{y} \Psi(u,z\, t)du\,, \qquad (20^*)$$

where $0 \leq g(y,z,t) \leq \lambda_1(y,t)\, \lambda_2(z,t)$ is any continuous function.

Equation (20*) is linear (nonhomogeneous), but the (in general, known) coefficients are still variable.

The set of solutions of (20*) is contained in the set of solutions of equation (19*) , so that any solution of (20*) is a solution of (19*).

Equation (20*) can be simplified more. This will yield a smaller set of solutions being still solutions of (20*). Namely, setting $g(y,z,t) = 0$, we obtain the (purely) linear integral equation

$$\omega_1(t)\, \lambda_2(z,t) \int_{-\infty}^{z} \Psi(y,v,t)\, dv + \omega_2(t)\, \lambda_1(y,t) \int_{-\infty}^{y} \Psi(u,z,t)\, du = \Psi(y,z,t)\,. \qquad (20^{**})$$

So the set of solutions of (20**) forms a vector space over the field of real numbers. In the narrowest version of the problem the coefficients can be made constants. Especially the assumptions $\lambda_1(y,t) = \lambda_1$ and $\lambda_2(z,t) = \lambda_2$ comprise the exponential case. But to stick with the "Wiener model" one must keep the hazard rates (coefficients) $\lambda_1(y,t)$, $\lambda_2(z,t)$ in (20*) and in (20**) as corresponding to the distributions given by (17) and (17*).

Now, having any solution $\Psi(y,z,t)$ of any integral equation above, we find the joint survival function of the corresponding bivariate stochastic process as expressed by formula (18), where $\alpha(y,z,t\,) = \int_{-\infty}^{y} \int_{-\infty}^{z} \Psi(s,r,t)\, dsdr$. The so obtained joiner $\alpha(y,z\,t)$ (or $\alpha(y,z)$ in the case of random vectors only) would be a candidate for the bivariate stochastic model.

Next steps are statistical verifications of an eventual fit of the obtained models to a given data that might yield the eventual choice of the best solution among all the obtained. This subject is, however, out of scope of this paper.

Notice that the above presented method for construction of bivariate Wiener processes is applicable to any two (marginal) stochastic processes $\{Y_t\}$, $\{Z_t\}$ such that each random variable Y_t and Z_t possesses a hazard rate. Thus, one obtains more models. The method also includes discrete time cases like the case described below.

Example B. In this example we seek an additional method for constructions. From now on we slightly change the notation by replacing the symbols (Y, Z) for the random vectors by the symbols (X,Y) . Consider the following first Pareto survival functions for two random variables X , Y :

$$S(x) = P(X \geq x) = (x_0 / x)^{\alpha}\,,$$

$$R(y) = P(Y \geq y) = (y_0 / y)^{\beta}\,,$$

$$\text{where} \quad x \geq x_0 > 0\,,\ y \geq y_0 > 0\,,\ \alpha > 0\,,\ \beta > 0\,.$$

In this (Pareto) case the variables X , Y usually (but not always) describe income or wealth redistributed within a society. We consider the case where the same society is subdivided in some manner into two groups that differ by professions or some other social indicator (ethnicity, race, gender, age etc ...). In the models the differences between the two groups income redistributions are reflected by the parameters x_o , y_o (minimal incomes), and by the shape parameters α , β . It is reasonable to assume that the probability distributions of the incomes together with the incomes themselves will evolve over time. That is why we consider stochastic (Pareto) processes $\{X_t\}$, $\{Y_t\}$ description, where the discrete time t is defined as multiplicities $t = r, 2r, 3r, \ldots$ of some time period r such as a year, a quarter, or a month. In our notation we adopt $r = 1$, and therefore $t = 1, 2, 3 \ldots$. We now assume that, for each t, the random variables X_t , Y_t are distributed according to the rules:

$$S_t(x) = P(X_t \geq x) = (x_o(t) / x)^{\alpha(t)} ,$$

$$R_t(y) = P(Y_t \geq y) = (y_o(t) / y)^{\beta(t)} .$$

For simplicity we will assume that $x_o(t) = x_o$ = constant, and $y_o(t) = y_o$ = constant.

The above univariate "Pareto stochastic processes" are determined by the constants x_o , y_o , and the way we define the functions $\alpha(t)$, $\beta(t)$ for t = 1, 2, 3, … . Our (simplest) proposition is to define

$$\alpha(t) = A + (t-1)r , \quad \beta(t) = B + (t-1)s ,$$

where (as a particular example) $1 \leq A \leq 2$, $1 \leq B \leq 2$, $r = 0.10$, $s = 0.15$.

Now, for each t, we find the joint survival functions $U_t(x,y)$ of the random vector (X_t,Y_t) . Recall the general form of the joint survival functions:

$U_t(x,y) = S_t(x)\, R_t(y) \exp[- \alpha^*(x,y,t)]$, (here, $\alpha^* \neq \alpha$ and the meaning of α^* is different than that of α).

Since we already have both $S_t(x)$ and $R_t(y)$ as given above, we need to find a class of proper joiners $\alpha^*(x,y,t)$ to make, for each t, the joint survival function $U_t(x\ y)$ of (X_t,Y_t) properly defined. Set $\alpha^*(x,y,t) = \alpha^*_t(x,y)$.

Recall that we have:

$$\alpha^*_t(x,y) = \int_{xo}^{x} \int_{yo}^{y} \Psi_t(u,v)\, du dv,$$

where, for every t, the function $\alpha_t^*(x,y)$ must satisfy conditions (1) - (3) from section 3.1. Since the domain of this function is reduced to $[x_o,+\infty) \times [y_o,+\infty)$,

with positive x_o and y_o , condition (3) (section 3.1) now takes the form:

$$\alpha_t^*(x_o,y) = \alpha_t^*(x,y_o) = 0 , \text{ for each } t .$$

Also, for each t, we have $S_t(x_o) = 1$ and $R_t(y_o) = 1$.

Therefore, as pointed out in Property 1 (section 3.1), it follows that, for each t, the functions $S_t(x)$, $R_t(y)$ are the marginal survival functions for the joint $U_t(x,y)$. Moreover, we restrict ourselves to positive stochastic dependences only, so we have, for every t , $\Psi_t(u,v) \geq 0$. [As already mentioned, the last assumption is not necessary (and not necessarily true in this case), but makes the *initial* considerations clearer.] In order to determine a proper class of the (not necessarily all) joiners $\alpha^*_t(x,y)$ [which are equivalent to the corresponding functions $\Psi_t(u,v)$], we need to examine (for each t) inequality (9*) which is

$$\Psi_t(u,v) \leq \lambda_1(u,t)\, \lambda_2(v,t) , \qquad (21)$$

where, in this 'Pareto case', we have $\lambda_1(u,t) = \alpha(t) / u$ and $\lambda_2(v,t) = \beta(t) / v$.

Thus inequality (21) takes the form

$$\Psi_t(u,v) \le \alpha(t)\,\beta(t) / uv \text{, for every } t \text{.}$$

Like in Example A, the first easiest solution to our problem is

$$\Psi_t(u,v) = \omega(t)\,\alpha(t)\,\beta(t) / uv \text{,}$$

where, for every t, $0 \le \omega(t) \le 1$.

Recall at this point that like the variables x, y also the variables u, v satisfy $u \ge x_o$, $v \ge y_o$.

The natural assumption in this Pareto case is that both $x_o \ge 1$, $y_o \ge 1$ (i.e., the minimal incomes in both social groups are at least 1 ”unit”). Taking the above under the consideration we may choose for $\Psi_t(u,v)$ (the determinant of the stochastic dependence between X_t and Y_t) the following:

$$\Psi_t(u,v) = \alpha(t)\,\beta(t) / u^{\gamma_1} v^{\gamma_2}, \quad \text{for } \gamma_1 \ge 1,\ \gamma_2 \ge 1 \text{.}$$

This $\Psi_t(u,v)$ satisfies (21).

Finally, the full version of the so derived joint survival function of the Pareto random vectors (X_t, Y_t), for every t, is:

$$U_t(x,y) = S_t(x)\, R_t(y) \exp[- \alpha(t)\,\beta(t) \int_{xo}^{x} \int_{yo}^{y} u^{-\gamma_1} v^{-\gamma_2}\, dudv],\ \gamma_1 \ge 1,\ \gamma_2 \ge 1 \text{.}$$

Annotation: Just in recent days, when this paper was already finalized, we learned that similar results as for the bivariate distributions (and not bivariate stochastic processes) were considered by Finkelstein [7] in 2003. The author also considered the “Aalen factor” (see formulas (4) and (5) in [7]) but under different names and with no reference to Aalen or Cox. Our impression, however, is that the generality of Finkelstein’s approach is somewhat limited. Also, in the references he provides, the results are rather specific and the generality or even universality of this bivariate distributions’ representation was probably overlooked. But, anyway, the novum of our results became somehow (but not drastically) limited.

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Research Progress on the Schrödinger Equation that Can Describe the Earth's Revolution and its Applications

Runsheng Tu

Huanggang Normal University

ABSTRACT

In the scientific community, the traditional concept has been formed that 'quantum mechanics is only applicable to microscopic systems, classical mechanics is only applicable to macroscopic systems, and the Schrödinger equation cannot be used to describe macroscopic objects'. Under the constraints of this traditional concept, there has been no attempt for a long time to establish and apply the Schrödinger equation of gravitational potential energy to describe the motion of the Earth. By replacing the potential energy in the Hamiltonian operator from electromagnetic interaction potential energy to gravitational interaction potential energy, the Schrödinger equation that can describe planetary motion was successfully obtained. Many examples provided can prove that classical mechanics and quantummechanicsarecompatible. We can combine classical mechanics and quantum mechanics to describe the same system. For describing objects, mass size is no longer an important determining factor in choosing between classical mechanics and quantum methods. Establishing the Schrödinger equation for gravitational potential energy can prompt us to change our mindset and liberate our minds.

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In the scientific community, the traditional concept has been formed that 'quantum mechanics is only applicable to microscopic systems, classical mechanics is only applicable to macroscopic systems, and the Schrödinger equation cannot be used to describe macroscopic objects'. Under the constraints of this traditional concept, there has been no attempt for a long time to establish and apply the Schrödinger equation of gravitational potential energy to describe the motion of the Earth. By replacing the potential energy in the Hamiltonian operator from electromagnetic interaction potential energy to gravitational interaction potential energy, the Schrödinger equation that can describe planetary motion was successfully obtained. Many examples provided can prove that classical mechanics and quantum mechanics are compatible. We can combine classical mechanics and quantum mechanics to describe the same system. For describing an object, mass size is no longer an important determining factor in choosing between classical mechanics and quantum mechanics methods. Establishing the Schrödinger equation for gravitational potential energy can prompt us to change our mindset and liberate our minds. The idea that classical mechanics and quantum mechanics are opposed to each other and should be mutually exclusive can be transformed into the idea that they coexist due to complementarity.

Keywords: planetary model, schrödinger equation, quantum mechanics, classical mechanics, compatibility, wave function, ravitational potential, macroscopic systems, fluctuation mechanics, quantum-classical relationship.

Author: Huanggang Normal University, China.

I. INTRODUCTION

Can quantum mechanics and classical mechanics be compatible? It's a fascinating question. We haven't found anyone else exploring, but we can be sure that there are many people who hope the answer is yes and have a desire to explore this question.We believe that this compatibility has a certain theoretical foundation and strive to find or establish such a foundation. Ehrenfest's law proves that "taking the classical limit, the laws of quantum mechanics will be reduced to the laws of classical mechanics" [1,2]. In principle, this allows wave me mechanics to be used to describe macroscopic objects under certain conditions. When the scale of quantum systems becomes very large or quantum effects become less apparent, the description of quantum mechanics (QM) approaches the results of classical mechanics. This is the characteristic of quantum decoherence. Although there is no macroscopic object in the object it describes, there are manifestations (features) that contain macroscopic objects. If the two different behaviors of particles before and after quantum decoherence are determined by their composition, structure, and external conditions, then the two extreme cases of Ehrenfest's theorem (microscopic particles and macroscopic matter) and the starting and ending points of the quantum decoherence process are interconnected. This connection is also the connection between classical states and quantum states. The compatibility between quantum mechanics and classical mechanics has a structural foundation. Previously, people used two methods (& the ideas and concepts determined by them) to sever this connection: the changes in microscopic particles did not conform to the law of causality; The process of quantum decoherence and other microscopic changes is an instantaneous process, and there is no causal relationship in between (non local realism changes, without real signals and matter maintaining causal relationships or connections). It is not difficult to see that this "disconnection operation" is largely artificial or subjective. The deeper you delve, the more you feel that it is unreliable. The Ehrenfest's law only indicates that the described object is not limited by mass when using de Broglie waves. This happens to be useful for the theme of this article.

I hope the above analysis can lead people to explore the possibility of establishing localized realism QM) and weaken the dominant position of existing quantum mechanics, especially the explanatory system of QM. This anticipated new theory cannot completely exclude existing quantum theories, and can only seek compatibility between classical and quantum theories. introduced my own established Schrödinger equation (SE) for gravitational potential energy in references [3-5]. This equation is obviously not unique to quantum theory, nor is it unique to classical theory. It can be said that local reality quantum mechanics requires such equations. This equation also need the support of the application results of local reality quantum mechanics. In references [3-5], the introduction of this equation is relatively rough. This article will review the establishment process of this equation and provide a detailed introduction to the application scope, theoretical basis, application examples, and practical significance of this equation. The SE for the Earth's revolution also refers to it. Sometimes it is also called the 'SE for macroscopic objects'. The classical system SE (the SE of local realism). For convenience, we refer to this type of equation as the "Schrödinger-Tu equation". We will also discuss how to handle the contradictions between existing quantum mechanics and classical mechanics.

I have made significant efforts in establishing local realism quantum mechanics [6-9]. I found that this research work lacks something that is currently unclear to me in theory. After the SE that can describe macroscopic objects has been established, I knew that this equation was one of the things I lacked.

Preliminary ideas on localized realism has been gradually proposed through the development of material structure theory. Its conceptual system and mathematical logic system are both obtained by transforming the existing theoretical system. The biggest feature is the ability to simultaneously use fluctuation mechanics and classical mechanics to describe microsystems [4-9], which recognizes and applies the compatibility of fluctuation mechanics and classical mechanics in terms of ideological concepts and logic. The newly established SE for gravitational potential energy and the original SE may form a system of SEs. Under appropriate conditions, entities as small as elementary particles and as large as planets can be described using this equation, and both the objects of local realism and non local realism can be described using this set of SEs. For convenience, we refer to the group SE as the Schrödinger-Tu equation. The theoretical system of local realism (including conceptual system, explanatory system, and mathematical formal system) has been relatively complete. However, some details still need to be continuously supplemented and improved. One of the imperfections is that the SE cannot be used to describe macroscopic systems, but the Schrödinger–Tu equation can describe the Earth's revolution in references. Thus, the theoretical system of local realism QM has been further perfected. That is to say, the macroscopic SE is an important component of the theoretical system of local realism QM (after establishing the macroscopic SE, we can see this intuitively). This article has introduced the theoretical basis (background), application achievements, and significance of the macroscopic SE.

This work demonstrates the compatibility between classical mechanics and fluctuation mechanics, suggesting that there is no insurmountable gap between them, and briefly explains how classical theory and quantum theory are combined in the context of this study briefly explains how classical theory and quantum theory are combined in the context of this study.

Define or derive the relationship between various physical quantities of macroscopic objects (such as momentum, energy, wavelength, frequency, velocity, wave function) and de Broglie waves. Replace the electromagnetic potential energy function in SE with the gravitational potential energy function, and then derive the SE for the background of gravitational potential energy. We will clarify that the equation can simultaneously or separately describe macroscopic objects and microscopic objects. As long as classical theory and quantum theory are compatible, we can combine them for use. We can also avoid considering the quality of the described object as an important factor in choosing which method to use.If the application effect can be relatively ideal, it indicates that the "compatibility" and "combination" mentioned above have been basically successful in theory and practice. Readers can make judgments after reading the following chapters.

II. THE THEORETICAL BASIS OF THE SECHRÖDINGER EQUATION THAT CAN BE USED TO DESCRIBE THE EARTH'S REVOLUTION

The existing theories of quantum mechanics and material structure cannot explain the source of electron spin magnetic moment. What kind of intrinsic composition structure and motion lead to the wave particle duality of microscopic particles? What are the wave functions widely used in quantum mechanics and the essence of de Broglie waves? Is the frequency of de Broglie waves on Earth determined by its actual vibrations? What is the relationship between the kinetic energy of physical particles and the energy E in the wave function? Is a moving elementary particle also a wave packet composed of multiple waves of different wavelengths? Will the wave packet collapse when it encounters the instrument? If the answer is yes, why can diffraction still occur after the wave packet collapses when the elementary particle flow passes through the slit? For the sake of convenience, we refer to these questions as the first set of unsolved mysteries concerning the structure, properties, and waves of matter. Once any of these questions are clarified, it will definitely lead to significant progress in theoretical research in physics. The thinking method of local realism is obviously the theoretical basis for describing macroscopic and microscopic objects using quantum mechanics and classical mechanics methods. The theory of wave element material structure and the macroscopic object SE are mutually necessary and mutually confirmed.

Reference [6] proposes a wave element material structure theory (whose core argument is that particles are composed of waves. For convenience, we refer to it as Hypothesis 1). That is, assuming that electrons are localized entities composed of waves. The formation method is that the wave propagates along a very small circle, and the wave can propagate forward, but the center of gravity of the circular solid formed by the "head tail connected wave" can be stationary. Assumption 1 provides a good explanation for the source of electron spin magnetic moment. In the context of assumption 1, an electron is a small charged wave ring that can expand according to Huygens' principle and form a hydrogen atom with a proton. Such hydrogen atoms are particles with planetary structures. This also smoothly explains why the calculation results of Bohr hydrogen atoms (planetary structure type hydrogen atoms) are very close to the facts. Under the same material structure framework, this calculation method can be extended to diatomic small molecules such as hydrogen molecular ions and hydrogen molecules. The calculation method is simple, but the results are close to the experimental values [7-10]. The approach of local realism is clearly the theoretical basis for directly describing macroscopic objects using quantum mechanics methods. The theory of wave element material structure and the macroscopic object SE are mutually necessary and mutually confirmed. Because the macroscopic object SE is an equation that can describe the "real object in the local domain".

Since there are the unsolved mysteries above, they leave a huge imagination space for explorers. We can boldly choose the relationship between the physical quantities of macroscopic and microscopic entities (such as kinetic energy, velocity, etc.) and the energy, velocity, wavelength, frequency, and momentum of de Broglie waves based on some clues. We define the displacement velocity of the center of mass of a macroscopic object in motion as equal to the phase velocity of its de Broglie waves, and the kinetic energy as equal to the wave energy of its de Broglie waves. The wave energy corresponding to the wave function used is currently not very clear, but can be selected based on some facts. We define the displacement velocity of the center of mass of a macroscopic object in motion as equal to the phase velocity of its de Broglie waves, and the kinetic energy as equal to the "wave energy" of its de Broglie waves:

$$\upsilon_{centroid} = \upsilon_{phase} = \upsilon_d = \lambda_d \nu_d. \quad (1)$$

$$2E_k = m\upsilon^2_{centroid} = h\nu_d. \quad (2)$$

In the equation, ν_d is the frequency of the de Broglie wave of a moving object.Equation (2) represents that the de Broglie wave of a moving object with non-zero stationary mass is a "kinetic wave", and the intrinsic energy (internal energy, which corresponds to the stationary mass of the object) of the object does not belong to the wave energy component of its de Broglie wave. That is to say, when an object with non-zero stationary mass is in motion, its de Broglie waves resemble a moving spring harmonic oscillator (or vibration propagating on a string or water waves). These are examples where the energy converted from static mass does not belong to wave energy. By doing so, we can solve the problem of de Broglie's phase velocity exceeding the speed of light without using the concept of group velocity. Establishing two Eqs. (1) and (2) is a key step in deriving the SE for gravitational potential energy. It is currently unclear whether the essence of de Broglie waves used for counting is the same type of wave. It is not clear whether the three different energies of E contained in the SE are the same. Therefore, Eqs. (1) and (2) are defined as tools. If readers want to investigate the falsity of equations (1) and (2), please include this project in the research plan for further exploration. The first equation of Eq. (2) is the relationship between wave energy and kinetic energy, as well as the relationship between macroscopic material entities and de Broglie waves. The physical quantity of classical motion represented on its right side, and the wave energy of the corresponding de Broglie wave (or wave energy in the wave function) on its left side. In this way, we can say that the de Broglie wave of a moving object is not a real wave. It is highly likely to be a virtual wave, or more accurately, a tool for computation. All the energy of fluctuations is provided by their motion, and static energy does not contribute. It is obvious that Eqs. (1) and (2) are all components of the theoretical basis of the macroscopic object SE. The successful application of the calculation method for microscopic particles established based on hypothesis 1 is the practical basis for the macroscopic object SE.

If you do not believe that the wave energy of de Broglie waves is only the kinetic energy of moving particles and does not include the static energy of moving particles, or if you do not believe in Eqs. (1) and (2), please take a look at the logical proof below. Assuming the potential energy of the system is zero, the kinetic energy of the moving object is calculated according to the SE. This can be seen from any SE. After the potential energy increases and becomes a bound system, the energy attribute classification previously obtained by logical methods remains unchanged.

One of the SE for the Earth's revolution established by references [3-5] is:

$$-\frac{i\hbar}{2}\frac{\partial}{\partial t}\left|\psi\right\rangle = \hat{H}\left|\psi\right\rangle. \quad (3)$$

In Eq. (1), there is an additional – (1/2) coefficient compared to the original SE. From now on, we will carefully explain how it was obtained. How was this coefficient obtained?

III. VERIFICATION OF THE SCHRÖDINGER EQUATION OF THE EARTH'S REVOLUTION

In mechanics, the potential energy of phase interactions takes negative values, while the kinetic energy takes positive values. Therefore, the algebraic symbol V is used to represent the interaction potential energy, and there is no negative sign before V. If a specific potential energy calculation formula is used instead of V, a negative sign needs to be added before the formula. The universal Hamiltonian operator is $\hat{H} = -\left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V\right]$. Considering gravitational potential energy is a negative value, the Hamiltonian operator for gravitational potential energy is:

$$\hat{H} = -\left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{GMm}{R}\right] \tag{4}$$

Equation (3) uses the Hamiltonian operator shown in Eq. (4). One of the widely used forms of wave functions is:

$$\psi(x,t) = Ae^{-i2\pi(\nu t - x/\lambda)}. \tag{5}$$

Where, ψ is often called wave function. This is a frequently used function in quantum mechanics. We also use it in the SE of gravitational potential energy. The reason is that moving macroscopic objects can also be seen as de Broglie waves.

The SE for time-dependent gravitational potential energy obtained by combining Eqs. (3) and (4) is:

$$-\frac{i\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{GMm}{R}\psi = E\psi. \tag{6}$$

The standard orbital motion of the Earth conforms to this equation.

To verify equation (6), Eqs. (1) and (2) are required for each calculation. In planetary systems, Viry's theorem holds. This classical mechanics theorem ensures that the absolute value of the third term from the left in Eq. (6) is twice that of the second term. By solving the equation by taking partial derivatives (also using Viry's theorem), the result obtained by solving Eq. (6) is: the first term from the left is $-(1/2)m\upsilon^2$ (the total energy of the system, excluding m_0c^2), the second term is $(1/2)m\upsilon^2$ (kinetic energy), the third term is $V = -$ (GMm)/R$= -m\upsilon^2$ (potential energy), and the fourth term is $-(1/2)m\upsilon^2$ (the energy eigenvalue of the system, excluding m_0c^2). Let $f(x)\frac{\partial}{\partial t}\psi = -(1/2)m\upsilon^2\psi$, The solution is $f(x) = -\frac{i\hbar}{2}$. We have already explained the source of Eq. (3) or Eq. (6) clearly. In fact, Eq. (6) can be verified based on Eqs. (1) and (2).

IV. TYPICAL APPLICATION EXAMPLES OF THE SCHRÖDINGER EQUATION FOR GRAVITATIONAAL POTENTIAL ENERGY

It is worth noting that the de Broglie wave is not the same type of wave as the eigenwave of the wave function (The essence of the wave function is not matter waves). The relationship between some physical quantities is inconsistent, therefore, the relationship between hν and E or E_k. The reasons behind this are not very clear and are worth exploring in depth. Reference [8] discusses this situation from the perspective of material structure and proposes a solution: choose one of the two sets of relationship equations based on different situations.

Bohr had already done the work of calculating hydrogen atoms using planetary models (*i.e.* classical mechanics methods). Now, let's first establish a planetary model of hydrogen atoms like Bohr did, and then use the SE to calculate the energy of hydrogen atoms. In the context of planetary models, the radius of the ground state hydrogen atom is the planetary orbit of an electron, with the the alue is a_0, and is a constant value, the solution of the SE for this hydrogen atom is easy to obtain, which is E= $\left(-Ze^2/a_0\right)$. For hydrogen atoms, the combination of classical mechanics and quantum mechanics has both advantages and disadvantages. The advantage is that the solving process is simple and fast. The stronghold cannot obtain an ideal solution. The combination of classical mechanics and quantum mechanics has other advantages. That is, after hydrogen atoms combine to form hydrogen molecules, the combination of quantum mechanics and classical mechanics can generate new quantum chemistry calculation methods (to be introduced later, readers can refer to references [7,10] for details).

From a purely mathematical perspective, the use of the SE is not limited by the mass of the object being described (the value of mass has no discontinuous points). There are two lampshade balls with opposite charges, one of which rotates around the other. If someone believes that the original SE for electromagnetic potential energy cannot describe this bound system, what is the reason? Is it non locality and uncertainty? From a purely mathematical perspective, that's not the reason! The use of the SE is limited by the mass of the described object "is a subjective conclusion. The conclusion (If available) that non local realism and non deterministic objects cannot be described by the SE is also subjectively obtained by some people. Let's discuss the situation where the constraint system is scaled. Continuously shrinking the solar system, it eventually became a microscopic system with a mass similar to that of fluorine atoms. Logically speaking, Eq. (6) can still be used to describe the miniature planets involved. Amplify a hydrogen atom (by increasing its mass and charge in the same proportion) to a mass close to the Earth, and the applicability of Eq. 6 should change in the same way as in the case of the bulb above. Can the SE be used to describe these two hypothetical systems, and which SE to use? The choice method and limitations cannot be seen from the SE itself. This is a logical method for explaining the applicability of the gravitational potential SE and the original SE. The macroscopic system cannot be described using the SE, which is determined based on ideological concepts rather than mathematical logic. Some people may believe that it was determined based on experimental facts. However, we have not seen any reports on determining the applicability of the SE through experimental methods.

Can the explanatory system of quantum mechanics be used as a determining factor in selecting the applicable scope of the SE? Worth further discussion. It is not difficult to see that when deriving the SE (including the later gravitational potential SE), "non locality" and "uncertainty" were not used. Only when using the SE did quantum mechanics scientists consider the characteristics of microscopic particles (for example, when solving the equation, they consider the radius r that determines potential energy to be uncertain, that is, the motion state and mode of particles are uncertain).

The framework structure of localized realistic hydrogen molecular ions established using classical electrodynamics and planetary models is similar to the wheels of a unicycle [7-10]. This skeletal structure can achieve classical electrodynamic equilibrium. Merge the similar terms of the potential energy function and establish a SE using the merged potential energy function (merging the potential energy of multiple particles into similar terms yields a merged potential energy, equivalent to the potential energy of a virtual object). Solving this equation can yield bond length and dissociation energy data [7-10]. Multiple examples of simultaneously using classical mechanics and quantum mechanics to calculate a microscopic object are presented in references [7,8,10]. A common feature of the calculation methods in these examples is that they first provide the composition and structural framework of the system based on the idea of local realism, then find the potential energy function, establish the Schrödinger method, and finally solve the established SE. In the process of calculating the same molecule, classical mechanics and quantum mechanics methods are used successively (this is a combination of quantum mechanics and classical mechanics methods). For hydrogen atoms, their skeletal structure is a planetary model. The calculation of hydrogen atoms based on planetary models and classical mechanics methods has long been done by Bohr. Later on, people felt that it was not very useful and ultimately had to use quantum mechanics methods for precise calculations. We found in references [7-10] that although the planetary skeleton of hydrogen atoms is not very useful for calculating hydrogen atoms, it is very helpful for calculating hydrogen molecular ions and hydrogen molecules (which can greatly simplify the calculation process and make it clearer). The planetary model SE (classical system SE) is the theoretical basis of this method.

Both the SE and the planetary model can be used simultaneously or separately to describe macroscopic objects such as The Earth. There is no reason to restrict the simultaneous use of the SE and classical mechanical models". We have no reason to restrict the simultaneous and separate use of the SE and classical mechanical models. When we calculating hydrogen molecular ions hydrogen molecules and hydrogen molecules, the combination of classical mechanics and quantum mechanics methods is more evident and successful [7,8].

V. ANALY AND EXPLAIN

The second section points out that there is a set of unsolved mysteries in physics. Equation (2) serves as a bridge between classical mechanics and quantum mechanics (or micro and macro). There are two such bridges: Equation 2 and the de Broglie relationship. Like the de Broglie relationship, Eq. (2) can also strengthen the connection between waves and particles (or macroscopic and microscopic). This also reduces obstacles for the hybrid use of classical mechanics and quantum mechanics.

The establishment and application of this equation [Eq. (2)] further confirms that the de Broglie waves of physical particles are not real waves but tools. The wave of wave function (the noumenon of wave function) is more like a computational tool of quantum mechanics (or a tool that can be more widely applied). Because the objects of local realism and determinism can also be described using the methods of quantum mechanics. If the wave of the wave function is not a real wave but a tool, then in the context of the widespread application of de Broglie waves, we cannot say that de Broglie waves and Schrödinger equations cannot be used to describe macroscopic objects. When an electron is stationary, its fluctuation energy is zero (Due to $\upsilon = \lambda\nu$, therefore, when υ equals zero, the frequency is zero, and the fluctuation energy is zero). But the internal energy of electrons is still a non-zero constant value m_0c^2. This fully demonstrates that the energy of the de Broglie waves of moving electrons is not the total energy including internal energy of the electrons.

What is the relationship between the wave of wave function and de Broglie wave? No one dares to confidently say that the de Broglie wave of the nuclear tooth electron of a hydrogen atom is the wave of wave function used to describe electrons in the SE. After completing the solution process of the SE for the hydrogen atom, the wave function was set aside. Because none of the values obtained for some physical quantities belong to the ontological wave of the wavefunction. Because none of the values obtained for some physical quantities belong to the wave of wave function. This makes people believe that the wave function in the SE is just a computational tool. It is not unique to microsystems. As long as it can be demonstrated that wave functions can also be confidently used by macroscopic objects, the compatibility-remixed use of classical mechanics and quantum mechanics eliminates an important subjective barrier.

Microscopic particles are either "completely waves" or "completely particles", which are subjective choices made by people in order to choose the "appropriate method".In fact, the state of objectively existing particles is not subject to people's subjective consciousness. Under constant external conditions, particles will not frequently jump between pure waves and pure particles. Logically speaking, the "duality (Double representation)" in wave particle duality should have existed from the beginning. Just like intersex individuals, different sexual organs are displayed and used depending on the gender of the sexual partner. Particles, like intersex individuals, manifest and utilize different sexual organs depending on the sexual object.

Let's analyze the properties of the energy of each term in equation (6). We will not discuss items that include time for now [That is, let's first discuss the latter equation in Eq.(6)]. In the case of V=0, whether based on the original SE or the Schrödinger-Tu equation, the energy of the term $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$ can be calculated as $E_k=(1/2)m\upsilon^2$. In classical mechanics, this is the kinetic energy of a rotating object in a bound state system. Because in a bound state system, internal energy is excluded, and when the potential energy is zero, there is only kinetic energy in the system.In this way, according to the Viry theorem, the last term of the SE is the total energy of the system (excluding internal energy), which is the energy eigenvalue of the system.It is the sum of kinetic energy and potential energy. Now, let's discuss the first equation in Eq. (6). .

According to the Viry theorem, the absolute value V of potential energy in a bound system is twice the kinetic energy. In Eq. (6), V is $-mv^2$. The same result can also be calculated directly based on $V=-(GMm)/R$. The properties of these physical quantities calculated according to the SE will not change regardless of whether the object being calculated is macroscopic or microscopic. From this perspective, it cannot be said that the SE can only be enjoyed by the microscopic world. It is natural to conclude that classical mechanics and quantum mechanics are compatible.

The above discussion can experience the simultaneous or combined use of quantum mechanics and classical mechanics, either separately or separately. The SE has a Hamiltonian operator. The applicability of the SE is strongly correlated with the applicability of the Hamiltonian. And the Hamiltonian operator is not entirely within the scope of quantum mechanics (on the contrary, it is closer to classical mechanics). The Hamiltonian operator, potential energy function, Viry's theorem, etc. are closely related to classical theory, while the core content of quantum mechanics — the Schrödinger equation — is closely related to the Hamiltonian operator, potential energy function, Viry's theorem, etc. The problem explained by the combination of multiple factors is also that the SE is not unique to quantum mechanics.

The sharp contradiction between classical mechanics and quantum mechanics is not a dialectical conclusion, but a metaphysical conclusion. Dialectically speaking, the conflicting parties are interdependent. Even if we do not use dialectics, the two sides of the so-called contradiction are highly likely to be complementary. From the perspectives of philosophy, logic, and ideological concepts, and from the perspective of establishing Schrödinger's method, the "scope of application of the SE" and the "relationship between quantum mechanics and classical mechanics" have little to do with the explanatory system of quantum mechanics. The randomness of microscopic particles is highly likely the result of neglecting the detailed description of causal transmission due to the difficulty of calculating causal relationships step by step. If we don't care about details, we can also say that the weather in a place is random. We are not sure if anyone regards the randomness exhibited by particularly complex things as the absolute randomness of that thing. Similarly, it cannot be assumed that the characteristics of photons in a beam can represent all situations. This passage illustrates that the randomness of microscopic objects can sometimes also be related to subjective selection.

Pour a lot of sesame seeds into a bucket, and the direction of the sesame seeds' extension is random. If you believe that the randomness of sesame's stretching direction is an inherent characteristic of sesame, then the laws followed by sesame contradict those followed by macroscopic objects (sesame is definitely a macroscopic object). This is an example where randomness is sometimes related to subjective selection. Whether the nature of tiny particles (even the microscopic world) contradicts the inherent nature of common objects also depends on subjective opinions. Microscopic particles are too small and have many interfering factors, making spontaneous ordering difficult. It is extremely difficult to clarify the details of various small causal relationships. So that it can give others the illusion that the randomness of sesame is an inherent characteristic of sesame. The randomness of sesame extension direction is an inherent characteristic of sesame, which does not conform to determinism and causality. Although this is a rough analogy, we cannot deny that the situation with microscopic particles is not like this.

In the process of deriving and applying the SE, both quantum mechanics methods (such as using wave functions and de Broglie relations, and solving equations using the uncertainty properties of electrons) and classical mechanics methods (such as using potential energy functions, Viry's theorem, classical interactions, mathematical logic, etc.) are used. In addition, there is a phenomenon of 'you have me, I have you' in terms of methods, features, and attributes. These phenomena can be considered as complementary or coexisting relationships, rather than a life and death contradictory relationship. For a particle to have multiple manifestations (such as wave and particle characteristics, coherent and decoherent states), it can be considered that "external conditions can determine which state the described object presents". It can also be considered that the phenomenon just mentioned is similar to the situation of intersex people, that is, similar to the "binary existence and single manifestation" of "male female duality". Existence does not equal expression. It is entirely possible that the (objectively existing) characteristics of microscopic particles are manifested, while another objectively existing characteristic is not manifested.

For the SE of gravitational potential energy, there are both classical mechanics and quantum mechanics methods for its derivation and application in quantum mechanics. The methods of quantum mechanics and classical mechanics coexist or complement each other. As mentioned earlier, there are some unsolved mysteries in quantum mechanics and material structure theory. Let's change our mindset and perhaps explain some of the unsolved mysteries. Simultaneously eliminating some contradictions and giving birth to new interpretations. The establishment of the SE for gravitational potential energy can promote work in this area.

We are not currently pursuing more accurate results calculated directly using the SE of gravitational potential energy, but rather using it to change some concepts (For example, changing the concept of "classical mechanics and quantum mechanics being mutually opposed and contradictory"), liberate people's minds, and develop physics theories and establish new methods. Furthermore, by utilizing its compatibility and complementarity with quantum mechanics, classical mechanics and quantum mechanics can be used simultaneously in quantum chemistry calculations, simplifying the calculation process and even improving the accuracy of calculations. The coexistence referred to in this article refers to the complementarity between the quantum mechanical properties and classical mechanical properties of particles or other physical objects, and/or the complementarity between classical mechanical theories and methods and quantum mechanical theories and methods. These calculation examples would help to explain in greater detail why and how these two can be used together without causing contradictions. "both the SE and the planetary model can be used simultaneously or separately to describe macroscopic objects such as Earth" and "there is no reason to restrict the simultaneous use of the SE and classical mechanical models".

VI. CONCLUSION

As mentioned above, our conclusion is that quantum mechanics and classical mechanics are compatible, and the contradiction between quantum theory and classical theory may be a misunderstanding that "these two theories are complementary and can coexist". Some contradictions can disappear or transform into complementary or coexisting relationships after changing one's mindset. By simultaneously utilizing quantum mechanics and classical mechanics to calculate microscopic systems such as hydrogen molecules, ideal results can be obtained, and the SE that can describe macroscopic systems has been established. This situation forces us to search for the essence of de Broglie waves and wave function ontology. From a theoretical (or logical) and practical perspective, the essence of the wave function and the de Broglie wave are not the same type of wave. This indicates that at least one of the ontologies of de Broglie waves and wave functions is imaginary. The difficulty in observing the wave characteristics of large objects suggests that moving large objects are unlikely to have clear and useful de Broglie waves. Most people believe that the volatility of large objects is extremely weak. Even if one believes in the existence of de Broglie waves, people do not know how the de Broglie waves of matter and the essence of wave functions fluctuate. However, the SE established using imaginary waves can yield accurate calculation results. The SE is like a calculation program distributed by God, which can calculate various physical quantities of microscopic systems without knowing the specific motion of microscopic particles (now it can also calculate some physical quantities of macroscopic systems). This leads one to speculate that it may not be the peculiar properties of microscopic particles, but rather the mystique of the SE as a tool and method, rather than the particles themselves. Perhaps both have a certain degree of mystery and wonder. As long as the tools of quantum mechanics have mystery (or wonderful properties), the mystery of microscopic particles is not as important as before. Cutting off the connection between quantum mechanics and classical mechanics, even putting them in opposition, is no longer as important as before. It is natural to use the appropriate form of the SE in both the micro and macro worlds.

The important significance of establishing the SE for a macroscopic system is to change ideas, liberate thoughts, create more new methods and establish new theories, thereby promoting the development and progress of theory and methods. The existing quantum mechanics is not without any problems [7,12]. This clearly leaves us with space for research and exploration. It also indicates that we cannot use the existing quantum mechanics theory as an absolute criterion for judgment. Let non local realism and local realism coexist and complement each other. For problems that cannot be perfectly solved by the ideas and methods of local realism, use the original Schr ö dinger equation and the ideas and methods of non local realism to solve them. For example, the hydrogen atom can solve quantization and electron spin problems, as well as fine structure problems, using planetary models, the original SE and non local realism methods. Another excellent example is the calculation of helium atoms. Assuming that the two s-electrons of the atom are completely overlapping 'double electrons'. Calculate the energy of this pair of electrons using quantum mechanics methods, and then use a prepared regression equation to determine the interaction energy between the electrons. Finally seeking peace. You can obtain the energy and bond length of helium molecules.

Another question that needs to be explored is whether the SE, which is used to describe macroscopic systems, can be extended to three-dimensional space like the original SE to perform unique functions, thereby calculating quantized orbits and specific orbital radii? If possible, the significance of establishing the SE for gravitational potential energy would rapidly increase. What new ideas, methods, and theories will emerge when ideas change and thoughts are liberated?

VII. DISCUSSION

Comment from group member "Little Stone": The description is not quite accurate, you can go and learn about quantum decoherence. When the scale of quantum systems becomes very large or quantum effects become less apparent, the description of quantum mechanics approaches the results of classical mechanics. Teacher Chen said: After looking at it, I feel that it is of the same type as Gan Yongchao's innovative ideas (Note: Gan Yongchao's behavior is to combine the de Broglie relationship and uncertainty relationship into a matrix, and then say that this action has great significance). My group friend Yue Shandong said to me: Personally, I think this is a very creative and inspiring path. The opinion of an anonymous reviewer is as follows: This manuscript is important to the scientific community. The paper explains The direct significance of establishing and applying SEs and the use of classical mechanics and fluctuation mechanics to describe all objects (no longer limited by the mass of the objects), simplifying the calculation process of quantum mechanics.

In today's world, scholars have two drastically different ways of evaluating a newly emerging theory or idea. One approach is to use current mainstream theories and concepts to measure newly emerging things (evaluating new theories based on old theories). Anything that does not conform to existing old theories and ideas is considered wrong and unacceptable. It seems that innovation has original sin. Another approach is not to pay much attention to whether newly emerging theories and ideas are completely consistent with old theories and ideas (whether they conflict with each other), but to focus on whether there are differences between newly emerging theories and ideas and methods and existing ones. Next, pay attention to whether the new theory can achieve logical consistency. In other words, the latter type of scholars are concerned about whether new theories, ideas, concepts, and methods can be interesting and arouse their interest (in physics, they are concerned about whether what they see can cause their body to secrete more dopamine, increase their excitability points, and enhance their excitement level). The third step is to care about the usefulness of new things. Including whether it has enlightening effects, development prospects, characteristics that can change human concepts, and storage value. The first type of commentator belongs to the static conservative category, while the second type belongs to the dynamic reformist category.

Use the commenting style of dynamic reformist commentators to comment. The research work introduced in this article has two innovative points: the first innovative point (highlight) is the incorporation of gravitational potential energy into the SE. The second innovation point (highlight) is the first use of the SE to describe the ideal orbital motion of the Earth, calculating the energy eigenvalues of the Earth's ideal orbital motion, and changing the old concept that humans have persisted in for many years that the SE is limited to describing microscopic objects. The significance or value of this work is to improve the theoretical system of local realism QM.

Once the Schrödinger-Tu equation is established, it has nothing to do with quantum decoherence. As long as the method of establishing the SE conforms to rigorous logic, the existence of quantum decoherence phenomena cannot interfere with the use of the SE. Because macroscopic matter is not generated by quantum decoherence. They can, in fact, make up a macro world. Schrödinger-Tu equation is used in this macroscopic world without being affected by quantum coherence.

VIII. DISCLAIMER

This paper is an extended version of a preprint document of the same author. The preprint document is available in this link: https://vixra.org/abs/2411.0133 [As per journal policy, preprint /repository article can be published as a journal article, provided it is not published in any other journal].

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Peter Mbile, Fadcre, Lylliane Elomo, Fadcre & Yaya Fodoue

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Much of Africa's forest ecosystems heritage can be found in Cameroon. The country has since ratified numerous Conventions and enacted laws to protect and valorize these connecting ecosystems benefiting local people and global climate. These forest and non-forest ecosystems constitute Cameroon's green infrastructure today. However, due to anthropogenic and natural processes, these ecosystems face degradation, thereby weakening their superstructure, diminishing their services value; threatening livelihoods, and contributing to climate change.

In this paper, we draw parallels between green infrastructure and traditional (hard) infrastructure, in order to bring to ecosystem restoration,comparable maintenance mindset, historically reserved for hard infrastructure. We use the prisms of three ecosystems assessments for restoration, as case studies. These are; (i) the northern savannah, (ii) Sanaga-Kadey watershed and (iii) the forest transition zones of Cameroon. By analyzing some common parameters across these ecosystems, including (i) land tenure, (ii) multifunctionality, (iii) climate resilience, (iv) critical resource use efficiency, (v) carbon neutrality, (vi) connectivity, (vii) stakeholder engagement, (viii) social inclusivity and (ix) maintenance-friendliness, we simultaneously make a case for adopting analogous maintenance mindsets towards securing and re-building Cameroon's threatened green infrastructure

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I. INTRODUCTION

1.1. *Traditional versus green infrastructure*

Traditional infrastructure[1] refers to the fundamental physical and organizational structures and facilities needed for the operation of a society, enterprise, or system. It includes essential services such as transportation systems (roads, bridges, airports), utilities (water supply, electricity, telecommunications), buildings (schools, hospitals, offices), and other key facilities necessary for the functioning of a community or business. Such infrastructure plays a critical role in supporting economic development, social well-being, and overall quality of life. Such traditional infrastructure can also have negative environmental impacts, such as increased pollution, habitat destruction, and water runoff[2].

While traditional infrastructure is primarily concerned with meeting human needs through built structures, green infrastructure emphasizes the integration of natural systems to provide multiple

[1] . "Green Infrastructure." Environmental Protection Agency, www.epa.gov/green-infrastructure/what-green-infrastructure

[2] United States Department of Agriculture. "Green Infrastructure." Natural Resources Conservation Service, www.nrcs.usda.gov/wps/portal/nrcs/main/national/plantsanimals

benefits for both people and the environment[3]. Principally, green infrastructure focuses on incorporating natural elements and processes to provide a range of ecosystem services while also promoting sustainability and resilience. Green infrastructure typically uses nature-based solutions [1,2,3] to mimic natural processes, enhance biodiversity, improve air and water quality, reduce heat and island effect in urban areas, to create more livable and healthy built-up communities.

At the scale of a country, green infrastructure refers to the network of natural and semi-natural areas that have evolved (modified and managed) to deliver multiple ecological, social, and economic benefits. Ideally, large-scale green infrastructure comprises interconnected green systems, such as national parks, wetlands and hydrological systems, high value forests, watersheds, agricultural and rangelands, etc. They provide ecosystem services, improve air and water quality, protect wildlife habitats and enhance biodiversity. Green infrastructure facilitates seed dissemination, agricultural crop pollination, soil fertility and regeneration; promoting human health, well-being and economic development.

Similar to the rehabilitation and maintenance needs, historically restricted to the built, urban and or traditional infrastructure, concerns about the state of green infrastructure are rising, and restoring degrading, green interconnected ecosystems [4] and landscapes, is today of major national and global concern, yet nowhere near the maintenance mindset reserved for hard infrastructure.

1.2 Cameroon's green infrastructure endowments

Cameroon is located in Central Africa and shares borders with Nigeria, the Central African Republic, Chad, the Republic of the Congo, Gabon and Equatorial Guinea. With a population of approximately 25.7 million people as of 2020, Cameroon has a mixed economy featuring state-owned and private enterprises. Food and export crop agriculture drives the economy, accounting for 22% of GDP with export crops like cocoa, coffee, banana, and palm oil. The country also has oil and gas reserves and manufacturing sectors for textiles, food processing, and construction materials. Services contribute 50% to GDP, with a per capita income of $2,300. Challenges include corruption, infrastructure limitations, and security concerns [5]

Cameroon's green infrastructure is represented by its forests repartitioned into permanent and non-permanent estates under the Cameroon Forestry Law of 1994. The country's permanent forest estate is estimated at 15,7 million hectares, and the non-permanent estates at 6.9 million hectares. This is out of a total forest area estimated at 22.5 million hectares [6].

By 2013 Cameroon[4] had officially partitioned the national territory into five (5) Agroecological zones; two forest zones; (i) a coastal with mono-modal rainforest regime (ii), bimodal hinterland forests; (iii) a high plateau, (iv) high savannah and (v) sudano-sahel zones. Each zone possesses slightly peculiar, dominant vegetation (main component of green infrastructure) with elevation (e.g., montane forests), nearness to sea (mangrove systems), low humidity and or low soil organic matter (sudano-sahel ecosystems) being additional factors influencing the classification and functioning of ecosystems or biomes. By 2020 there were nineteen protected areas in Cameroon with at least one in each of the five agro-ecological zones. The 1994 Forestry Law and one of its texts of Application – the 1996 National Environmental Management Plan (NEMP), stipulates a network of protected areas, to ensure that a viable sample of each type of ecosystem or biome is represented/protected, continues to provide its unique ecosystems functions, and to the extent possible, is connected to other natural systems. The presence of five agro-ecological zones, including sub biomes hasn't only earned Cameroon the title of

[3] Benedict, Mark A., and Edward T. McMahon. Green Infrastructure: Linking Landscapes and Communities. Island Press, 2006
[4] A central African

"Africa in miniature", but Cameroon is also famously, a meeting point of two major continental biomes – the west African upper Guinean and the central African-Congolian forest systems.

1.3 Cameroon's commitments to secure her green infrastructure

To solidify her commitment to secure her green infrastructure heritage, Cameroon has signed and ratified several major multilateral environmental agreements related to forests, biodiversity, and climate change. Some of the key agreements and strategies include:

- Convention on International Trade in Endangered Species of Wild Fauna and Flora (CITES): signed since 1983 and implemented to regulate the trade in endangered species.
- Convention on Biological Diversity (CBD) signed in 1992 ratified in 1994, committing to conserve and sustainably use her biodiversity.
- As required by the CBD Cameroon developed a National Biodiversity Strategy and Action Plan (NBSAP) in 2001 and updated, in 2015.
- The Kunming – Montreal Global Biodiversity Framework, being an offshoot of the CBD was adopted by Cameroon in 2022, guiding the revision of her NBSAP in 2023.
- Cameroon signed the United Nations Framework Convention on Climate Change (UNFCCC) in 1992 and ratified it in 1994.
- Cameroon adopted the Declaration of the Summit of Heads of State of Central African Countries on the Conservation and Sustainable Management of Tropical Forests of 1999.
- Cameroon has been a party to the United Nations Forests for People Programme (UNFP) since its inception in 2000.
- Cameroon ratified the Central African Forest Commission (COMIFAC) Convergence Plan in February 2005 on harmonized, sustainable management of Congo Basin forests.
- Cameroon is a signatory to the African Forest Landscape Restoration Initiative (AFR100): a continental initiative launched in 2015 to restore 100 million hectares of degraded landscapes across Africa by 2030.
- Cameroon has developed a National Development Strategy with a 2030 horizon (SND30) that includes sustainable management of the country's forest and other green infrastructure.

Despite these commitments to address environmental issues such as climate change, biodiversity loss, and deforestation, and demonstrated willingness to work with global actors to achieve common goals, Cameroon's green infrastructure heritage is still confronted with degradation.

For instance, from 2001 to 2023, Cameroon lost 2.05 million ha of tree cover, equivalent to a 6.5% decrease in tree cover since 2000, and equivalent to 1.23 Gt of CO_2eq of emissions (GFW, 2023). Amongst Cameroon's commitments, her pledge through the AFR100 and Bonn Challenge to bring 12, 062,768 hectares of degraded landscapes under restoration by 2030 stands out as particularly relevant towards rebuilding and maintaining the country's green infrastructure.

To update the country's environmental management commitments, a new forest and wildlife law no. 2024 008 of 24 July 2024 has been adopted. It builds on the 1994 framework laws and focuses on biodiversity conservation, climate change and the sustainable development goals; community rights; a landscape approach to environmental management, landscape restoration, use of advanced monitoring systems, stiffer law enforcement regulations for critically endangered species, greater transparency and much stronger alignment with international conventions.

1.4 Cross-cutting issues on the nexus of hard and green infrastructure

Like traditional (hard) infrastructure, development of green infrastructure (tree-based ecosystems), has some important prerequisites or requirements such as; land tenure, stakeholder engagement, social inclusivity and maintenance-friendliness. Any infrastructure is also expected to possess certain characteristics enabling them to deliver services in a particular manner, including; multifunctionality, climate resilience, critical resource use efficiency, carbon neutrality and connectivity, among others. Through these commonalities, green and traditional infrastructure appear to seek similar goals – that of achieving resilience, by reducing the risk of failure, improving system performance, and ensuring that infrastructure can adapt to changing conditions over time.

From a socio-cultural perspective, one overriding requirement for green infrastructure in Cameroon (equally important to traditional, hard infrastructure) – is tenure, and this deserves some emphasis.

The tenure status across all the areas of opportunities for green infrastructure development are also a reflection of Cameroon's 1974 land tenure laws. A number of exceptions to this Law can exist under specific conditions. Although rarely invoked, such exceptions in Cameroon may include land title deeds listed under the pre-independence *Grundbuch*[5] in former west Cameroon. Even with such lands, either retroceded to traditional authorities or held in trust by customary entities such as lands within Lamidats[6] in northern Cameroon, only a land title transfers full rights of ownership from one moral entity to another; the rest being considered as lands in the *National Domain*[7]. The tenure status of such lands must often be ascertained prior to developing either green or hard infrastructure.

Furthermore, in 1994, forested lands (as defined by Law n°94/01 of January 20, 1994) came under the jurisdiction of the forest and wildlife laws, and were demarcated as Permanent (PFE) and Non-Permanent Forest Estates (NPFE). Whereas PFE are "the private property" of the State (comprising protected areas, timber concessions, council forests, etc.), NPFE are defined to include agroforestry zones, comprising *inter alia,* community forests, and private forests, where individuals and communities can exercise ownership and control rights under certain agreements with the supervising authorities (e.g. in the case of Community forests). Given that forests have been defined to cover the national territory, such estates can occur everywhere, including in the northern regions, not traditionally considered to be "a forest zone". Tenure statuses therefore, have implications for all types of green infrastructure development to be implemented, and who the stakeholders can be.

II. CASE STUDIES OF GREEN INFRASTRUCTURE OPPORTUNITIES ASSESSMENTS

To illustrate issues pertaining to green infrastructure development in Cameroon, three case studies are used to articulate prerequisites and requirements of successful green infrastructure development. These case studies illustrate salient issues to consider in the event of a shift in mindset towards a stronger maintenance culture for green infrastructure, similar to that hitherto reserved for hard, traditional infrastructure.

For a consistent ecological cognition rather than a hard infrastructural one, the expression "ecosystem restoration" will be used to mean "rebuilding green infrastructure", meanwhile, the principal micro ecosystems examined in this paper are tree-based systems.

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[5] Refers to lands held/contested as being under customary trust (especially by the Bakweri ethnic group) and believed to have been excluded from the 1974 Land Tenure Ordinance of Cameroon that stipulates that, all land is the property of the State.

[6] A Lamidat is a traditional Muslim chiefdom in northern Cameroon (currently the Far North, North and Adamaoua regions).

[7] An attribution meaning Sovereign lands or "property of the State" of Cameroon.

The three ecosystem restoration case studies are therefore, (i) tree savannah, northern region; (ii) the Sanaga – Kadey watershed, in the eastern region, and (iii) a forest-savannah transition zone, Centre region, all in Cameroon.

2.1 Northern tree savannah ecosystem restoration (ER) assessment – overview

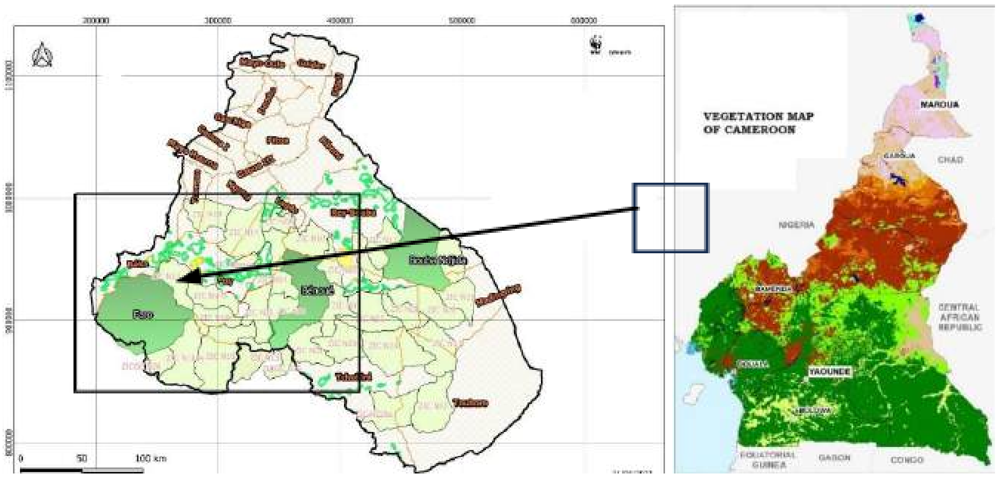

Figure 2: Northern savannah assessment and opportunities for ER

2.1.1 Survey of restoration opportunities in the northern savannah

In June of 2022 a sub national Restoration Opportunities Assessment Methodology [4], was completed here by WWF Cameroon [7]. The assessment identified 26,029 ha to be brought under ecosystem restoration. 89.59 % of this area are agricultural lands, 10.11 % agroforestry and 0.3% appropriate for woodlands. The area is densely populated, water-stressed with river banks suffering strong erosion; low soil organic matter and water retention potential; and characterized by impoverished and degraded agricultural soils (leached and low in soil nutrients). Indigenous tree species; *Pterocarpus erinaceus, Afzelia Africana* and *Kiguelia Africana,* are selectively overexploited here for fuel, timber, furniture and fodder.

The ecological needs highlighted by this case study pertain to; multifunctionality, climate resilience and critical resource use efficiency aspects of ecosystems restoration (or green infrastructure development).

2.1.2 The Sanaga – Kadey watershed assessment – overview

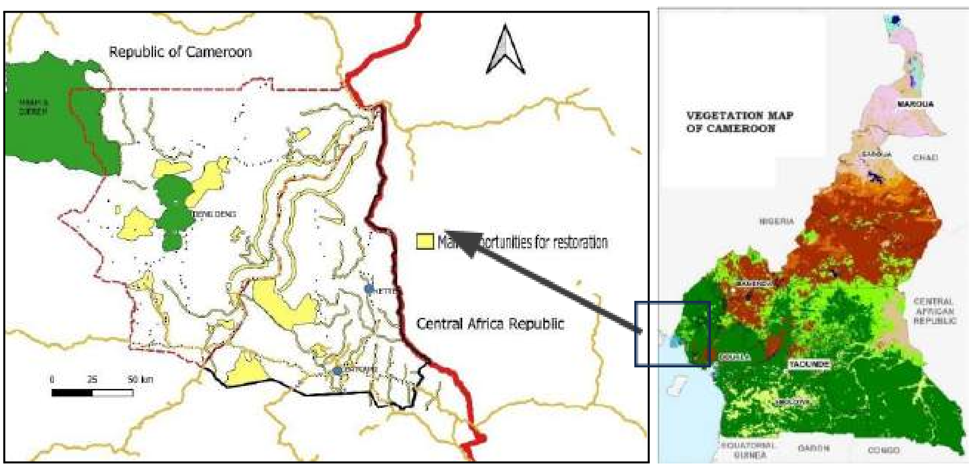

Figure 3: Sanaga - Kadey watershed and opportunities for restoration

2.1.3 Survey of restoration opportunities in the Sanaga – Kadey watershed

In June of 2023 a second sub national Restoration Opportunities Assessment Methodology [4], was completed by WWF Cameroon [8] in the Sanaga-Kadey watershed and identified 426 904 ha of ecosystems restoration opportunity. 98 942 ha of this is in communal forests (state forests); 10 070 ha of abandoned mines within the national domain; 65 062 ha of micro zoned areas also in the national domain; 107 001 of gallery (riparian) forests within the national domain and 145 829 ha of degraded agroforestry areas within the national domain and within community areas. With non-uniform population distribution, community areas including riparian forests are densely settled, whereas the national domains and abandoned mines are sparsely populated. Agricultural areas are dominated by annual crops (cereals and tubers); and agroforests of indigenous multipurpose trees species; *Irvingia gabonensis, Ricinodendron heudelotii* and *Trichoscypha acuminata;* medicinal species; *Baillonella toxisperma, Enantia chlorantha* and *Garcinia cola;* and by timber species, *Diospyros spp. Triplochiton scleroxylon, Afzelia bipidensis* and *Entandophragma cylindricum.* Most sensitive degradation drivers; small scale agriculture and fuelwood extraction impacts riparian forests, while artisanal mining impacts land cover and water quality.

The ecological needs highlighted by this case study pertain to; multifunctionality, critical resource use efficiency and connectivity aspects of green infrastructure development.

2.2 The forest-savannah transition zone agroforestry assessment: overview

Figure 4: Forest transition agroforestry landscape opportunities for restoration

2.2.3 Restoration practice in the "Ndong community" (see Mbile & Elomo, 2024)

Between 2003 and 2004, the World Agroforestry Centre (ICRAF) transferred 28,000 trees of *Irvingia wombolu*, an indigenous fruit tree, into the NDONG community, Centre region, under the tree domestication programme that began in the 90s [9]. This fragmented forest transition zone just outside Yaoundé (Cameroon's capital City) is a forest degradation hotspot [10] and community livelihoods here have partly depended, and continue to do so, on consumption and sales of products from indigenous tree species including *Ricinodendron heudelotii*, *Monodora myristica*, as well as exotic economic trees like Avocado, Citruses and Oil palm. The participants of the programme included men, youth and a women's group – "MERUNGA". This restoration programme has been monitored for over 20 years and the results published [11].

The ecological lessons learned in this case study involve indigenous tree domestication through the displacement of viable planting materials for ecosystem restoration, and are particularly relevant to;

multifunctionality, social inclusiveness and maintenance-friendly aspects of green ecosystems restoration (or green infrastructure development).

III. DISCUSSION OF THE CASE STUDIES IN THE CONTEXT OF GREEN INFRASTRUCTURE DEVELOPMENT

3.1 Building resilience

To the same extent that the problems of ecosystem degradation are numerous and interconnected, so too should the interventions to resolve them, are of a multifunctional nature.

The northern regions of Cameroon are amongst the most climate vulnerable with longer hours of insolation, high seasonal temperatures and short-lived, but torrential rains. This facilitates soil crusting, and formation of hardpans leading to low water infiltration and potentially higher volumes of surface water. Low soil organic matter, low soil carbon holding capacity and high runoffs exacerbate erosion and soil loss (Figure 5). The low overall forest cover, high exposure enhances vulnerability to strong winds and dust storms causing agricultural land degradation (Figure 6).

Figure 5: Dry river beds, eroded river banks and degraded gallery forests in the Faro National Park, landscape, North region

Figure 6: Agricultural fields in the north region suffer from low organic matter, tree cover, high insolation, desertification and high exposure

Moving on to the Sanaga-Kadey watershed on the eastern forest transition zones of Cameroon, the population is more dispersed compared to the north region. However, this ecosystem faces similar pressures on riparian forests (Figure 7) that protect rivers and streams.

Whereas in the drier northern savannah, there is strong preference for arable agricultural land (89.59 % of assessed opportunities) with fewer trees, rampant wildfires used to prepare agricultural and pastoral lands, including unsustainable extraction of fuelwood drives tree-loss. This form of degradation is also common here in the Sanaga-Kadey watershed (Figure 3 and Figure 7).

In both cases, tree loss on these vulnerable ecosystems generally increases the likelihood of hazards from extreme weather events, like storms, flooding, causing loss of top soil, siltation of waterways, while extreme droughts, exposure and strong winds drive uncontrolled wildfires, decimating what little vegetation is left, thereby reenforcing the degradation cycle.

Figure 7: Riparian forest degradation on the banks of the Bindiki watercourse (Garoua - Boulai) Sanaga – Kadey watershed, east region

As human populations grow, there is increasing need for more, often low-productivity farmlands and so more forests are converted to farmlands.

The recommended restoration opportunities for both the northern savannah and the Sanaga-Kadey watershed thus emphasize how to enhance the multifunctionality of the ecosystems by sustainably managing and conserving critical resources like water, and tree systems diversity (abundance and evenness) for increased resilience. Such ecosystems restoration opportunities were therefore most strongly recommended for sensitive, high value and easy-to-manage areas of both northern savannah and Sanaga-Kadey watersheds, such as managed farmlands, community home gardens, and relatively smaller areas such as river banks (or gallery forests), community forests and sacred forests (making 11% of total assessed area).

Development of agroforestry systems in both the northern savannah region and the Sanaga-Kadey watershed by planting a range of available, multipurpose trees species (fruit, timber, fuel wood, soil improvers) is one solution. For agricultural land, application of bio regenerating products like Biochar (a carbon-rich material produced by partial oxidation- pyrolysis, at ≤700 °C in the absence or limited supply of oxygen, using organic materials such as forestry and agricultural wastes as substrate (32, 33) to improve soil water holding capacity, cation exchange capacity and overall fertility are also recommended.

3.2 Enhancing critical resources use efficiencies

Critical resource use efficiency is an important ecosystem restoration strategy in multifunctional landscapes. Such critical resources include soil nutrients holding capacity and species (e.g. trees) biodiversity with a direct correlation with system resilience [12, 13, 14]. Due to relatively low growth rates of especially indigenous species, and the harsh soil biophysical factors of low organic matter and hard-pans, the recommended technology to rehabilitate such soils in the north savannah region is the relatively costly Revitech[8] [15, 16] estimated to reach $US 1,500 per hectare. This technology, although tested and viable in the targeted context, is costly, and well beyond the financial wherewithal of the relatively poor communities. Its low adoption rates to-date, despite it proven effectiveness is largely due to cost.

The preference for slow-growing indigenous species, for biodiversity and resilience, can actually add to the ecosystem restoration costs per hectare, as it takes longer for indigenous trees to grow, and establish, if at all. As a consequence, and based on lessons from the application of vegetation successions in restoration [16,17, 18,] the application of Revitech on hardpans is still recommended despite costs. The strategy involves the use of fast-growing exotic species (e.g. Acasia) as part of a vegetation succession, creating enabling micro-conditions, which then allows slow growing indigenous species to establish as part of assisted natural regeneration.

Another recommended strategy to restore hard-pans (including abandoned mines where topsoil has been removed), support soil fertility (a critical resource), assist natural regeneration and grow biodiversity, is through the use of biochar through private sector participation. In the northern savannah, biochar production is envisaged through pyrolysis of rice husks, a waste product of the rice-producing company SEMRY (*YAGOUA Rice Expansion and Modernization Company*).

Similarly, restoration interventions to enhance soil carbon and organic matter in the Sanaga – Kadey watershed are expected to source forestry waste produced from lumbering in communal forests or supplied under contract from private timber companies like PALISCO LLC (a private forestry company in the southeastern forest) operating just south of the watershed. The coordinated role of the private sector is further discussed below under private sector engagement.

Furthermore, the northern savannah degraded areas are in a transhumance zone, prone to farmer-grazer conflicts and frequent bush fires. An interesting (bitter-sweet) relationship exists here between farmers and graziers. The sweet part involves agricultural land fertilization via cow dung. Meanwhile, the bitter parts involve deliberate fires set by pastoralists to stimulate re-sprouting of fresh fodder, crop and saplings raiding by livestock. Managing this bitter-sweet scenario, prevalent in both the northern savannah and Sanaga-Kadey watershed requires (costly) effective and reliable monitoring missions (passive restoration) to manage conflicts and ensure survival of species undergoing natural regeneration.

The restoration assessments [4] for both northern savannah and Sanaga-Kadey watershed, thus recommended species with multiple uses (for soil fertility, medicinal, food values, fodder value, fuel-wood value, etc.), and with strong economic potential (e.g., Cashew nut) to serve as incentive for engaging local communities [20].

3.3 Carbon capture and climate action

A significant amount of biochar use is envisaged in these recommended ecosystems restoration processes. Biochar is a negative emissions technology, identified by the Intergovernmental Panel on Climate Change (IPCC) as effective in mitigating climate change and achieving net-zero emissions [21]. Biochar increases the soil's capacity to retain, absorb carbon and support natural carbon sinks. It improves soil structure and health (soil organic matter, soil carbon content, microbial and fungal

[8] Revitech increases soil organic matter and thereby nutrient and moisture holding capacity of soils.

activity, cation exchange and pH), the ability of the soil to retain and absorb carbon and water; and reduces the fluxes of NO_2, CH_4 and CO_2 in the soil.

The quantities to be applied per hectare for either mosaic (e.g., farmer-managed agricultural systems with community tenure) or wide-scale (e.g., abandoned mines, plantations, state-owned forest systems) restoration approaches are yet to be determined for both the north region and Sanaga-Kadey watershed sites. However, there is a 1:3 ratio with respect to biochar use versus carbon removal and storage; i.e., 1 MT of biochar applied to agricultural plots can permanently sequesters and remove up to 3 MT/CO_2eq from the atmosphere [21, 22, 23, 24].

In terms of above ground capture and storage, the transition from a degraded cropping system to a tree-based system or agroforest can more than double surface carbon gains [25, 26]. If later converted to a purely tree-based system (e.g. a gallery forest) or through reforestation (e.g., in communal or community forests), this can more than triple above-ground carbon stock gains [27, 28].

Restoring the significant areas of gallery forests in the Sanaga – Kadey watersheds using mosaic (29 364 ha) and wide-scale (82 604 ha) approaches through protection, local by-laws, assisted natural regeneration and reforestation, to healthy states, can be comparable to restoring to primary or secondary mixed tropical forests. These will result in over 100,000 ha of gains in both above-ground carbon stocks being permanently stored including the associated biodiversity benefits [29, 30].

Bringing abandoned mines under restoration through a combination of ReviTech®, biochar application, reforestation and assisted natural regeneration is likely to be expensive in the short-term. However, through flexibility in managing costs (such as by using local labor and organic matter) and achieving permanence, through the development of locally useful, mixed tree systems there is good potential for below and above ground, long-term carbon capture and storage (using biochar). Through high integrity voluntary carbon capture and trading (e.g. Gold Standard or VCS) some of the financial investments can be recovered.

Finally, biochar use irrespective of the feedstock requires expensive specialized knowledge, processes and materials for chemical (elemental) analysis of the feedstock to match with the soil properties, and tailor the biochar production to the needs of soils to be brought under restoration. Dealing with assisted natural regeneration also requires selection of most appropriate tree species, determining their nutrient needs and tolerance to relevant environmental factors (local pests, drought conditions, etc.). These are areas that carbon trading can finance and create a circular, self-sustaining, win-win, process of revenue generation, environmental protection, and climate action.

3.4 Connectivity and biodiversity conservation

Across rivers and streams in the Sanaga-Kadey watershed, progressive eutrophication of streams like the Bindiki river (Fig. 6) results in the reduction of the volume of water and the deterioration of its physico-chemical properties to support fish for home use by local populations. The recommendations for ecosystem restoration through development of viable woodlots, agroforests and green belts by communities (including women) will reduce siltation and help recovery of the river and freshwater life.

This setting is an ideal case for the combined application of tailor-made biochar with heavy metal absorption properties and to support both active and passive restoration (e.g., natural regeneration) for increased hydrolytic retention as well as the accumulation of minerals necessary for the growth of biodiversity on the completely eroded land. Women's groups, through robust stakeholder engagement processes, are then an effective mechanism to ensure monitoring and compliance to ensure recovery of green banks.

3.5 Social inclusiveness and private sector engagement

Women are some of the victims of degrading multifunctional landscapes; such as loss of soil fertility, precarity in domestic energy resources and deterioration of water quality. Individual women have also been known to be an important part of the problem of forest landscape degradation as they fend for their families through smallholder agriculture, fuelwood harvesting, etc.

Well organized women's groups can however, play a significant role in accelerating landscape restoration. Securing the consent and participation of women's groups in envisaged ecosystem restoration programs in the Sanaga-Kadey watershed was assessed through the participator process of Free Prior and Informed Consent (FPIC). Getting their views required focused group discussions with the women, and accompanying them on field visits, to their different places of activities (Figure 8). FPIC is an immersion approach which does not only secure the consent of social groups during project implementation, but helps program proponents to develop and appreciate the motivation of social groups; understand their perception and actual states of wellbeing; the challenges they face, and their actual livelihoods strategies pertaining to how they interact with critical resources facing degradation.

Figure 8: Accompanying women to an artisanal gold mining site in Yassa village (Bindiba) to experience landscape degradation within the Sanaga – Kadey watershed.

Social inclusiveness in ecosystem restoration often targets the vulnerable. Therefore, one deliberate strategy to strengthen inclusiveness is to link it to engagement with the private sector, given the latter often tend s to be powerful and influential. The private sector can thus leverage social legitimacy, while the vulnerable groups can ride on the private sector power, resources and influence.

This restoration assessment thus specifically proposes an innovative human network built on a stakeholder infrastructure composed of three (03) main categories; (i) *Investors* – such as Biochar developers and carbon markets facilitators/brokers like FACET POWER[9]; (ii) *service providers* – nursery managers, trainers, seedlings distributors, researches and knowledge holders (ReviTech), and (iii) *active and passive restorers* - farmers, women's groups, planters, plantation developers.

FACET POWER seeks to produce biochar by engaging contractually in the north with SEMRI (rice company producing husks as feedstock), and with PALISCO - a timber processing company (producing wood waste as feedstock). Another potential private entity is *Terra Formations, a private* forestry regeneration and carbon markets facilitator who have expressed interest in the restoration assessment

[9] FACET power established a Memorandum of Understanding in 2022 with the WWF to take forward this initiative.

results in the Sanaga – Kadey watershed. Service providers abound and include ReviTech, small nursery managers and NGOS in Garaoua and Bertoua, or others even further afield in established institutions willing to invest resources if a business opportunity emerges, either directly or via local satellites. The third group in this innovative human network of delivery infrastructure are the active and passive restorers. Electronic (virtual) and physical market-place options which need investments and further development, have been proposed to link these three actors in real-time, dynamize their interaction and accelerate rebuilding of Cameroon's crumbling green infrastructure.

3.6 Developing maintenance-friendly restoration designs

The lesson of ecosystem restoration in multifunctional, farmer- management landscapes come from the restoration practice in the forest-savanna transition zone (Box 3) of central Cameroon. This case study is the subject of a recent publication [9] which makes a strong case for sustainability and a maintenance culture to be built into restoration processes. In the time left in this decade (2020-2030) of landscape restoration, where restoration involving vulnerable groups is expected to play a major role, it is critical to have a long-term view (beyond 2030). Practitioners of ecosystem restoration must be attentive to specificities - the long-term needs of farmers (in particular, vulnerable groups), and of sensitive ecosystems under restoration. Especially, restoration must be sensitive to the physiological transformations that beneficiaries (especially rural women) would have to go through, and how these changes would influence their ability to sustain multiple, often labour intensive, maintenance activities, critical to the sustainability of restoration investments. These include phytosanitary challenges, marketing challenges; renewal of tree-based systems and general support with new knowledge and technologies, to ensure these restored landscapes remain relevant to the livelihoods of their beneficiaries. If such precautions, safeguards are ignored, restoration investments, just like traditional infrastructure, would fall into neglect and be lost, including their benefits.

The spirit of the Rio 1992 declaration on Sustainable Development (Agenda 21), should remind us of a 'common future', between project developers, promoters, investors and beneficiaries. The earlier concept, that once the restoration intervention has ended, agencies and project leaders may turn their back on green infrastructure, needs to be revisited in the same way all traditional infrastructure are only as good as their maintenance plans.

IV. CONCLUSIONS

Green infrastructure and traditional hard infrastructure have strong parallels and understanding how the latter can be secured helps us deal better with the former. Just like traditional hard infrastructure, restoring ecosystems requires similar outcomes comprising *inter alia*; multifunctionality, climate resilience, critical resource use efficiency, carbon neutrality, connectivity, land tenure, stakeholder engagement, social inclusivity and maintenance-friendliness, as central notions.

The linkage to hard infrastructure should make ecosystem restoration less abstract to policy makers and society. It may be approached and valued at least in a similar way as society values and cares for infrastructure, not viewed only as a consumable, but as a necessary part of production and sustainability.

However, a challenge persists that, green infrastructure like biodiversity is utilitarian, is extracted and can be depleted, not a perception of traditional real-estate or hard infrastructure. However green infrastructure also regenerates and renews itself. So, one way to address this perceived deficit in green infrastructure is to use it in ways that enhance renewal, regeneration and to avoid permanent damage.

It is now urgent that restoring green infrastructure is accompanied by a look forward, a maintenance mentality, an "after-sales service". This should not be an afterthought that comes when ecosystems degrade. It should be built from the outset; in the same way maintenance is at the core of other types of infrastructure development.

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Conflict of Interest

The authors declare no conflict of interest.

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Imaginary Numbers: An Absurd Starting Point or a Mathematical Necessity?

Ulaanbaatar Tardad

Mongolian University of Science and Technology

ABSTRACT

Since the 19th century, complex analysis, including imaginary numbers, has become a major branch of mathematics. However, can a single algebraic operation not only reveal the infallibility of imaginary numbers (complex number theory) but also destroy it? This paper aims to challenge the mathematical basis of imaginary numbers and look at their historical development from a new perspective. In this article, we demonstrate the possibility of the non-existence of imaginary numbers based on examining imaginary numbers byusing the exponential function.

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Author: Department of Physics, School of Applied Sciences, Mongolian University of Science and Technology. email: tarzad@must.edu.mn

I. INTRODUCTION

If a negative number multiplied by itself equals a positive number, then it's hard to understand the square root of a negative number. [1]

I read somewhere but cannot find the source: "When we teach complex numbers, we usually start with an absurd assumption. We define i to be the square root of -1. Then, we construct an elegant theory. But since we start with an absurd assumption, many people have lingering doubts. We don't have to start from an absurd point." What is this starting point of date for imaginary and complex numbers?

The imaginary numbers may be that existence is hidden in little things we don't understand.

Negative numbers were not commonly accepted among mathematicians until the late 18th century. Think about it! The Egyptians could build the pyramids, the Romans could build and maintain their huge empire. Even Newton (and Kepler and others) could work out the laws of physics and predict planetary motions, WITHOUT the concept of negative numbers. [2].

Understanding the primordial date of the beginning of an imaginary number is also an important aspect of history. Accounting back from Heron of Alexandria [3], it is 1963 years, Bhaskara Acharya [4][5] 1549 years, Girolamo Cardano [6][7] 490 years, and René Descartes [8] 398 years passed. Since that early time, the imaginary number has been studied without interruption.

In mathematics, a complex number is an element of a number system that extends the real numbers with a specific element denoted i, called the imaginary unit, and satisfies the equation $i^2 = -1$; every complex number can be expressed in the form

$$a + bi \qquad [8][9] \qquad (1)$$

Where a and b are real numbers.

Because no real number satisfies the above equation, i was called an imaginary number by René Descartes.

We used the exponential functions for naked 1 to analyze the nature of imaginary numbers.

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II. THE IMAGINATION OF THE IMAGINARY NUMBER

The adjective imaginary was first used (as French imaginaire) by René Descartes in 1673, La Geometrie, referring to imaginary numbers in the broad sense, as non-real roots of polynomials. [8]

Euler only used the imaginary number but did not explain it. The word imaginary means that the real numbers may be slightly true in content. However, any imaginary number under the root is always different from real numbers and cannot be mixed with any other number or vanish (Identity (2) and Identity (3)).

$$\sqrt{-a} = \sqrt{a\cdot(-1)} = \sqrt{a}\cdot\sqrt{-1} \quad (2)$$

$$a + ib = a + \sqrt{-1}\cdot b \quad (3)$$

2.1 The Imagination of The Imaginary Number

Expression √-1 does not exist in nature. And there is no imaginary number that has nature's innermost secrets. Human abstractions such as imaginary birds, imaginary flowers, and imaginary melodies can exist only in painting, poetry, and music.

The adjective imaginary was first used (as French imaginary) by René Descartes in 1673, La Geometrie, referring to imaginary numbers in the broad sense, as non-real roots of polynomials. [7][8][9]

Euler only used the imaginary number but did not explain it. The word imaginary means that the real numbers may be true in content. However, any imaginary number under the root is always different from real numbers and cannot be mixed with any other number or vanish (Identity (2) and Identity (3)).

$$i = \sqrt{-1} \quad (4)$$

The most magic number in mathematics is the number ONE, which stands at the junction of the highest and lowest numbers on the number line.

Let's check the imaginary number using the following exponent equation (5) and the magic number 1.

$$a^{-x} = \frac{1}{a^{x}} \qquad [10] \quad (5)$$

And, if $a = 1;\ x = 1$ the intriguing problem comes in. In this case, we see the next naked identity (6):

$$1^{-1} = \frac{1}{1^{1}} = \frac{1}{1} = 1^{1} \quad (6)$$

And for 1, the next identities (7-8) are correct:

$$1^{-1} = 1^{1} \quad (7)$$

$$-1 = 1 \quad (8)$$

It is only valid when 1 is it. An imaginary number is inherently associated with the number 1.

Hence,

$$i = \sqrt{-1} = \sqrt{1} = 1 \quad (9)$$

This sounds like an incorrect solution but it is that at the end. What's true, it's true.

The very existence of imaginary numbers proves that humans create their problems! To err is human.

Great mathematician failing to come to a solution. [11].

So, I don't doubt that Identity (9) is correct. $\sqrt{-1}$ under the square root is a work of mathematicians in the dawn of mathematics. This was just such a thought experiment. From this viewpoint, simple imaginary numbers were combined with real numbers and moved into complex analysis.

In this case, Euler's Formula [12-16] is incorrect:

$$e^{x} \neq cosx + sinx$$

Either way, there is no imaginary number anywhere.

Thus, it is history that the number 1 has been able to give birth to an imaginary and complex number even for a while.

We need to know when and how to use Identities (5-9), or we get counterintuitive results for calculations of the negative number.

III. DISCUSSION

The first technique involves two functions with like bases. Recall that the one-to-one property of exponential functions tells us that, for any real numbers b, S, and T where $b > 0$, $b \neq 1$, $b^{S} = b^{T}$ if and only if $S = T$ [16][17].

In other words, when an exponential equation has the same base on each side, the exponents must be equal. This also applies when the exponents are algebraic expressions. Therefore, we can solve many exponential equations by using the rules of exponents to rewrite each side as a power with the same base. Then, we use that exponential functions are one-to-one to set the exponents equal and solve for the unknown [16][17].
My questions are

Why is it possible when b\neq1 in the case of exponential functions? Why is it not possible when $b = 1$?

1 is a real number. Why is it denied for 1?
According to one-to-one property

$$n^{S} = n^{T}, S = T$$

....

$$2^{S} = 2^{T}, S = T$$

$$1^{S} = 1^{T}, S = T$$

2. Is there a mathematical necessity to hide the imaginary number using 1?

IV. CONCLUSION

$1^{-1} = \frac{1}{1^{1}} = \frac{1}{1} = 1 = 1^{1}$, hence, $1^{-1} = 1^{1}$, then $\sqrt{-1} = 1$. So, we conclude that there is neither an imaginary nor a complex number.

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