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In this paper, the condition under which composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0 .

Keywords: composite multiplication operator, conditional expectation, Quasi-p-normal, multiplication operator, class Q operator.

I. INTRODUCTION

Let (X,Σ,μ) be a σ-finite measure space. Then a mapping T from X into X is said to be a measurable transformation if $T^{-1}(E)\in\Sigma$ for every $E\in\Sigma$.A measurable transformation T is said to be non-singular if $\mu(T^{-1}(E))=0$ whenever $\mu(E)=0$. If T is non-singular then the measure μT^{-1} defined as $\mu T^{-1}(E)=\mu(T^{-1}(E))$ for every E in Σ , is an absolutely continuous measure on Σ with respect to μ .Since μ is a σ-finite measure, then by the Radon-Nikodym theorem, there exists a non-negative function f_0 in $L^1(\mu)$ such that $\mu T^{-1}(E)=\int_E f_0\, d\mu$ for every $E\in\Sigma$. The function f_0 is called the Radon-Nikodym derivative of μT^{-1} with respect to μ.

Every non- singular measurable transformation T from X into itself induces a linear transformation C_T on $L^p(\mu)$ defined as $C_T f = f\circ T$ for every f in $L^p(\mu)$. In case C_T is continuous from $L^p(\mu)$ into itself, then it is called a composition operator on $L^p(\mu)$ induced by T. We restrict our study of the composition operators on $L^2(\mu)$ which has Hilbert space structure. If u is an essentially bounded complex-valued measurable function on X, then the mapping M_u on $L^2(\mu)$ defined by $M_u f = u\cdot f$, is a continuous operator with range in $L^2(\mu)$. The operator M_u is known as the multiplication operator induced by u.
A composite multiplication operator is linear transformation acting on a set of complex valued Σ measurable functions f of the form

$$M_{u,T}(f) = C_T M_u(f) = u\circ T \ \ f\circ T$$

Where u is a complex valued, Σ measurable function. In case $u = 1$ almost everywhere, $M_{u,T}$ becomes a composition operator, denoted by C_T.

In the study considered is the using conditional expectation of composite multiplication operator on L^2 -spaces. For each $f\in L^p(X,\Sigma,\mu)$, $1\le p\le\infty$, there exists an unique $T^{-1}(\Sigma)$-measurable function $E(f)$ such that

$$\int_A g f\, d\mu = \int_A g E(f)\, d\mu$$

for every $T^{-1}(\Sigma)$-measurable function g, for which the left integral exists. The function $E(f)$ is called the conditional expectation of f with respect to the subalgebra $T^{-1}(\Sigma)$. As an operator of $L^p(\mu)$, E is the projection onto the closure of range of T and E is the identity on $L^p(\mu)$, $p \geq 1$ if and only if $T^{-1}(\Sigma) = \Sigma$. Detailed discussion of E is found in [1-4].

1.1 Normal operator

Let H be a Complex Hilbert Space. An operator T on H is called normal operator if $T^*T = TT^*$

1.2 Quasi-normal operator

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T\,T^*T = T^*T\,T$ ie, T^*T commute with T

1.3 Quasi p-normal operator [13]

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^*T\left(T + T^*\right) = \left(T + T^*\right)T^*T$

1.4 Power -normal operator

Let H be a Complex Hilbert Space. An operator T on H is called 2 power-normal operator if $T^2\,T^* = T^*T^2$

1..5 Class Q-operator [14]

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^{*2}\,T^2 = \left(T^*\,T\right)^2$

II. RELATED WORK IN THE FIELD

The study of weighted composition operators on L^2 spaces was initiated by R.K. Singh and D.C. Kumar [5]. During the last thirty years, several authors have studied the properties of various classes of weighted composition operator. Boundedness of the composition operators in $L^p(\Sigma), (1 \leq p < \infty)$ spaces, where the measure spaces are σ-finite, appeared already in [6]. Also boundedness of weighted operators on $C(X,E)$ has been studied in [7]. Recently S. Senthil, P. Thangaraju, Nithya M, Surya devi B and D.C. Kumar, have proved several theorems on n-normal, n-quasi-normal, k-paranormal, and (n,k) paranormal of composite multiplication operators on L^2 spaces [8-12]. In this paper we investigate composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0.

III. CHARACTERIZATION ON COMPOSITE MULTIPLICATION OF QUASI P NORMAL OPERATORS ON L^2 SPACE

3.1 Proposition

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$ .Then for $u \geq 0$

(i) $M^*_{u,T}\,M_{u,T}\,f = u^2\,f_0\,f$

(ii) $M_{u,T}\,M^*_{u,T}\,f = u^2 \circ T \cdot f_0 \circ T \cdot E(f)$

(iii) $M^n{}_{u,T}(f) = (C_T M_u)^n(f) = u_n\ (f \circ T^n)$, $u_n = u \circ T . u \circ T^2 . u \circ T^3 .... u \circ T^n$

(iv) $M^*{}_{u,T}\, f = u f_0 \cdot E(f) \circ T^{-1}$

(v) $M^{*^n}{}_{u,T}\, f = u f_0 \cdot E(u f_0) \circ T^{-(n-1)} \cdot E(f) \circ T^{-n}$

where $E(u f_0) \circ T^{-(n-1)} = E(u f_0) \circ T^{-1} \cdot E(u f_0) \circ T^{-2} .... E(u f_0) \circ T^{-(n-1)}$

Theorem 3.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M_{u,T}$ is Quasi p-normal operator

(ii) $u \circ T\ u^2 \circ T\ h \circ T\ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1} = h u^2\ u \circ T\ f \circ T + h^2\ u^3\ E(f)$

Proof:

For $f \in L^2(\mu)$, $M_{u,T}$ is Quasi P-normal operator if

$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right) f = \left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right) f$ and we have,

$$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right) f = M_{u,T}\left(M^*{}_{u,T} M_{u,T}\right) f + M^*{}_{u,T}\left(M^*{}_{u,T} M_{u,T}\right) f$$

$$= M_{u,T}\, M^*{}_{u,T}\,(u \circ T\ f \circ T) + M^*{}_{u,T}\, M^*{}_{u,T}\,(u \circ T\ f \circ T)$$

$$= M_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right] + M^*{}_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right]$$

$$= M_{u,T}\left[h u^2 f\right] + M^*{}_{u,T}\left[h u^2 f\right]$$

$$= u \circ T\left(h u^2 f\right) \circ T + h u E\left(h u^2 f\right) \circ T^{-1}$$

$$= u \circ T\ u^2 \circ T\ h \circ T\ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1}$$

Consider

$$\left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right) f = \left(M^*{}_{u,T} M_{u,T}\right) M_{u,T}\, f + \left(M^*{}_{u,T} M_{u,T}\right) M^*{}_{u,T}\, f$$

$$= \left(M^*{}_{u,T} M_{u,T}\right)(u \circ T\ f \circ T) + \left(M^*{}_{u,T} M_{u,T}\right)\left(h u E(f) \circ T^{-1}\right)$$

$$= M^*{}_{u,T}\ u \circ T\ (u \circ T\ f \circ T) \circ T + M^*{}_{u,T}\ u \circ T\ \left(h u E(f) \circ T^{-1}\right) \circ T$$

$$= h u E\left[u \circ T\ u \circ T^2\ f \circ T^2\right] \circ T^{-1} + h u E\left[u \circ T\ h \circ T\ u \circ T\ E(f)\right] \circ T^{-1}$$

$$= h u^2\ u \circ T\ f \circ T + h^2\ u^3\ E(f)$$

Suppose, $M_{u,T}$ is Quasi P-normal operator. Then

$$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right) f = \left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right) f$$

$$\Leftrightarrow u \circ T\ u^2 \circ T\ h \circ T\ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1} = h u^2\ u \circ T\ f \circ T + h^2\ u^3\ E(f)$$

almost everywhere.

Corollary 3.2

The composition operator C_T on $B(L^2(\mu))$ is Quasi p-normal operator if and only if

$h \circ T\ f \circ T + h\,u E\left(h\ f\right) \circ T^{-1} = h\,f \circ T + h^2\ E(f)$ almost everywhere.

Proof:

The proof is obtained from Theorem 3.1 by putting $u = 1$.

Theorem 3.3

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M^*_{u,T}$ is Quasi p-normal operator

(ii)

$$h^2 u^3 E(f) \circ T^{-1} + h \circ T^2\, u \circ T\, u^2 \circ T^2\, E(f) \circ T$$
$$= h \circ T\, u^2 \circ T\, E(h)\, E(u) E(f) \circ T^{-1} + h \circ T\, u \circ T\, u^2 \circ T\, f \circ T$$

almost everywhere.

Proof:

For $f \in L^2(\mu)$, $M^*_{u,T}$ is Quasi P-normal operator if

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\, M^*_{u,T}\right) f = \left(M_{u,T}\, M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f$$

and then we have

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\, M^*_{u,T}\right) f = M^*_{u,T}\left(M_{u,T}\, M^*_{u,T}\right) f + M_{u,T}\left(M_{u,T}\, M^*_{u,T}\right) f$$
$$= M^*_{u,T} M_{u,T}\left[h\, u\, E(f) \circ T^{-1}\right] + M_{u,T}\, M_{u,T}\left[h\, u\, E(f) \circ T^{-1}\right]$$
$$= M^*_{u,T}\ u \circ T\left[h\, u\, E(f) \circ T^{-1}\right] \circ T + M_{u,T}\ u \circ T\left[h\, u\, E(f) \circ T^{-1}\right] \circ T$$
$$= M^*_{u,T}\left[u \circ T\ h \circ T u \circ T\ E(f)\right] + M_{u,T}\left[u \circ T\ h \circ T u \circ T\ E(f)\right]$$
$$= h\, u\, E\left(u \circ T\ h \circ T u \circ T\ E(f)\right) \circ T^{-1} + u \circ T\left[u \circ T\ h \circ T u \circ T\ E(f)\right] \circ T$$
$$= h^2\, u^3\, E(f) \circ T^{-1} + h \circ T^2\, u \circ T\, u^2 \circ T^2\, E(f) \circ T$$

Consider

$$\left(M_{u,T}\, M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f = \left(M_{u,T}\, M^*_{u,T}\right) M^*_{u,T}\ f + \left(M_{u,T}\, M^*_{u,T}\right) M_{u,T}\ f$$
$$= \left(M_{u,T}\, M^*_{u,T}\right) h\, u\, E(f) \circ T^{-1} + \left(M_{u,T}\, M^*_{u,T}\right)\left(u \circ T\ f \circ T\right)$$
$$= M_{u,T}\ h\, u\, E\left(h\, u\, E(f) \circ T^{-1}\right) \circ T^{-1} + M_{u,T}\ h\, u\, E\left(u \circ T\ f \circ T\right) \circ T^{-1}$$
$$= M_{u,T}\ h\, u\, E(h) \circ T^{-1} E(u) \circ T^{-1} E(f) \circ T^{-2} + M_{u,T}\ h\, u^2\, f$$
$$= u \circ T\left(h\, u\, E(h) \circ T^{-1}\, E(u) \circ T^{-1}\, E(f) \circ T^{-2}\right) \circ T + u \circ T\left(h\, u^2 f\right) \circ T$$
$$= h \circ T\, u^2 \circ T\, E(h)\, E(u) E(f) \circ T^{-1} + h \circ T\, u \circ T\, u^2 \circ T\, f \circ T$$

Suppose $M^*_{u,T}$ is Quasi p-normal operator. Then

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\, M^*_{u,T}\right) f = \left(M_{u,T}\, M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f$$
$$\Leftrightarrow h^2\, u^3\, E(f) \circ T^{-1} + h \circ T^2\, u \circ T\, u^2 \circ T^2\, E(f) \circ T$$
$$= h \circ T\, u^2 \circ T\, E(h)\, E(u) E(f) \circ T^{-1} + h \circ T\, u \circ T\, u^2 \circ T\, f \circ T$$

almost everywhere.

Corollary 3.4

The composition operator C^*_T on $B(L^2(\mu))$ is Quasi P-normal operator if and only if

$$h^2\, E(f)\circ T^{-1} + h\circ T^2\, E(f)\circ T = h\circ T\;E(h)\;E(f)\circ T^{-1} + h\circ T\;\; f\circ T$$

almost everywhere.

Proof:

The proof is obtained from Theorem 3.3 by putting $u=1$.

III. CHARACTERIZATIONS ON N POWER CLASS Q COMPOSITE MULTIPLICATION OPERATOS ON L²-SPACE

Theorem 4.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then $M_{u,T}$ is n power class Q composite multiplication operator if and only if

$$h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2}$$
$$= h\,u\,E(u_n)\circ T^{-1}\,h\circ T^{n-1}\,u\circ T^{n-1}\,E(u_n)\circ T^{n-2}\,f\circ T^{2n-2}$$

Proof:

Now Consider,

$$M^{*2}{}_{u,T}\,M^{2n}{}_{u,T}\;f = M^{*2}{}_{u,T}\left[u_{2n} f\circ T^{2n}\right]$$

where $u_{2n} = u\circ T^2\;\; u\circ T^4 \ldots\ldots\ldots\ldots\ldots\; u\circ T^{2n}$

$$= M^*{}_{u,T}\left(\,h\,u\,E\left(u_{2n}\,f\circ T^{2n}\right)\circ T^{-1}\right)$$
$$= M^*{}_{u,T}\;h\,u\,E\,(u_{2n})\circ T^{-1}\,f\circ T^{2n-1}$$
$$= h\,u\,E\left(h\,u\,E\,(u_{2n})\circ T^{-1}\,f\circ T^{2n-1}\right)\circ T^{-1}$$
$$= h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2}$$

Next we consider,

$$\left(M^*{}_{u,T}M^n{}_{u,T}\right)^2 f = \left(M^*{}_{u,T}M^n{}_{u,T}\right)\left(M^*{}_{u,T}M^n{}_{u,T}\right)f$$
$$= \left(M^*{}_{u,T}M^n{}_{u,T}\right)M^*{}_{u,T}\left(u_n\;f\circ T^n\right)$$

where $u_n = u\circ T\;\; u\circ T^2 \ldots\ldots\ldots\ldots\ldots\; u\circ T^n$

$$= \left(M^*{}_{u,T}M^n{}_{u,T}\right)h\,u\,E\left(u_n\;f\circ T^n\right)\circ T^{-1}$$
$$= \left(M^*{}_{u,T}M^n{}_{u,T}\right)h\,u\,E(u_n)\circ T^{-1}\,f\circ T^{n-1}$$
$$= M^*{}_{u,T}\;\;u_n\left(\,h\,u\,E(u_n)\circ T^{-1}\,f\circ T^{n-1}\right)\circ T^n$$
$$= M^*{}_{u,T}\;\;u_n\;h\circ T^n\;u\circ T^n\;E(u_n)\circ T^{n-1}\;f\circ T^{2n-1}$$
$$= h\,u\,E\left(u_n\;h\circ T^n\;u\circ T^n\;E(u_n)\circ T^{n-1}\;f\circ T^{2n-1}\right)\circ T^{-1}$$
$$= h\,u\,E(u_n)\circ T^{-1}\;h\circ T^{n-1}\;u\circ T^{n-1}\;E(u_n)\circ T^{n-2}\;f\circ T^{2n-2}$$

Given $M_{u,T}$ is n power class Q composite multiplication operator

$$\Leftrightarrow\; M^{*2}{}_{u,T}\,M^{2n}{}_{u,T}\;f = \left(M^*{}_{u,T}M^n{}_{u,T}\right)^2 f$$

$$\Leftrightarrow h\, u\, E(h)\circ T^{-1}\, E(u)\circ T^{-1}\, E(u_{2n})\circ T^{-2}\, f\circ T^{2n-2}$$
$$= h\, u\, E(u_n)\circ T^{-1}\, h\circ T^{n-1}\, u\circ T^{n-1}\, E(u_n)\circ T^{n-2}\, f\circ T^{2n-2} \quad \text{almost everywhere.}$$

Corollary 4.2

The composition operator C_T on $B(L^2(\mu))$ is n power class Q if and only if

$$h\; E(h)\circ T^{-1}\, f\circ T^{2n-2} = h\;\, h\circ T^{n-1}\, f\circ T^{2n-2}$$

almost everywhere.

Proof:

The proof is obtained from Theorem 4.1 by putting $u = 1$.

Theorem 4.3

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then $M^*_{u,T}$ is n power class Q composite multiplication operator if and only if

$$u\circ T\; u^2\circ T^2\, h\circ T^2\, E(u\,h)\circ T^{-(2n-3)}\, E(f)\circ T^{-(2n-2)}$$
$$= u^2\circ T\;\, h\circ T\; E(u\,h)\circ T^{-(n-2)}\, h\circ T^{-(n-2)}\, E(u\,h)\circ T^{-(2n-3)}\, E(f)\circ T^{-(2n-2)}$$

Proof:

Now if we consider

$$M^2{}_{u,T}\, M^{*2n}{}_{u,T}\; f = M^2{}_{u,T}\left(\, h\, u\, E(h\, u)\circ T^{-(2n-1)}\;\, E(f)\circ T^{-2n}\right)$$
$$= M_{u,T}\left(u\circ T\left(\, h\, u\, E(h\, u)\circ T^{-(2n-1)}\;\, E(f)\circ T^{-2n}\right)\circ T\right)$$
$$= M_{u,T}\left(h\circ T\;\; u^2\circ T\;\, E(h\, u)\circ T^{-(2n-2)}\, E(f)\circ T^{-(2n-1)}\right)$$
$$= u\circ T\;\left(h\circ T\;\; u^2\circ T\;\, E(h\, u)\circ T^{-(2n-2)}\, E(f)\circ T^{-(2n-1)}\right)\circ T$$
$$= u\circ T\;\, u^2\circ T^2\;\, h\circ T^2\;\, E(u\, h)\circ T^{-(2n-3)}\;\, E(f)\circ T^{-(2n-2)}$$

and we consider

$$\left(M_{u,T}\, M^{*n}{}_{u,T}\right)^2 f = \left(M_{u,T}\, M^{*n}{}_{u,T}\right)\left(M_{u,T}\, M^{*n}{}_{u,T}\right) f$$
$$= \left(M_{u,T}\, M^{*n}{}_{u,T}\right) M_{u,T}\;\, u\, h\, E(u\, h)\circ T^{-(n-1)}\; E(f)\circ T^{-n}$$
$$= \left(M_{u,T}\, M^{*n}{}_{u,T}\right) u\circ T\left(u\, h\, E(u\, h)\circ T^{-(n-1)}\; E(f)\circ T^{-n}\right)\circ T$$
$$= M_{u,T}\, M^{*n}{}_{u,T}\left(u^2\circ T\;\, h\circ T\;\, E(u\, h)\circ T^{-(n-2)}\;\;\, E(f)\circ T^{-(n-1)}\right)$$
$$= M_{u,T}\; u\, h\, E(u\, h)\circ T^{-(n-1)}\, E\left(u^2\circ T\;\, h\circ T\;\, E(u\, h)\circ T^{-(n-2)}\;\;\, E(f)\circ T^{-(n-1)}\right)\circ T^{-n}$$
$$= M_{u,T}\;\left(u\, h\, E(u\, h)\circ T^{-(n-1)}\;\, u^2\circ T^{-(n-1)}\;\, h\circ T^{-(n-1)}\, E(u\, h)\circ T^{-(2n-2)}\;\, E(f)\circ T^{-(2n-1)}\right)$$
$$= u\circ T\left(\, u\, h\, E(u\, h)\circ T^{-(n-1)}\;\, u^2\circ T^{-(n-1)}\;\, h\circ T^{-(n-1)}\, E(u\, h)\circ T^{-(2n-2)}\;\, E(f)\circ T^{-(2n-1)}\right)\circ T$$
$$= u^2\circ T\;\; h\circ T\;\; E(u\, h)\circ T^{-(n-2)}\;\, h\circ T^{-(n-2)}\;\, E(u\, h)\circ T^{-(2n-3)}\;\, E(f)\circ T^{-(2n-2)}$$

Since $M_{u,T}$ is a Composite multiplication operator, by definition

$$\Leftrightarrow\;\; M^2{}_{u,T}\, M^{*2n}{}_{u,T}\;\; f = \left(M_{u,T}\, M^{*n}{}_{u,T}\right)^2\, f$$

$$\Leftrightarrow\ u\circ T\ u^2\circ T^2\ h\circ T^2\ E(uh)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$
$$= u^2\circ T\ h\circ T\ E(uh)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(uh)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

almost everywhere.

Corollary 4.4

The composition operator $C^*{}_T$ on $B(L^2(\mu))$ is n power class Q if and only if

$$h\circ T^2\ E(h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$
$$= h\circ T\ E(h)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

almost everywhere.

Proof:

The proof is obtained from Theorem 4.3 by putting $u = 1$.

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