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Abstract

This work demonstrates certain standard fixed point theorems on complex-valued fuzzy metric spaces. We show certain fixed point findings in the situation of complex-valued fuzzy metric spaces, inspired by Singh et al. [25].To begin, we extend some well-known existing conclusions from metric spaces to complex-valued fuzzy metric spaces and then prove them in the complex-valued complete fuzzy metric space context. We provide an example that supports our main result and supports our hypotheses.

Index terms—

1 I. INTRODUCTION

In 1965, Zadeh [3] coined the term "fuzzy set." Following that, a slew of authors worked on fuzzy sets, expanding the fuzzy set theory and its applications [4][5][6]. The idea of fuzzy metric spaces was given by Kramosil and Michalik [7]. After then, George and Veeramani [9] updated this idea. Grabiec [8] investigated fuzzy metric space fixed-point theory. The idea of complex-valued metric spaces was introduced by Azam et al. [21].

Verma et al. [23] recently established 'Max' functions and the partial order relation'for complex numbers, and used properties (E-A) and CLRg to prove fixed point theorems in complex valued metric space. **??**ingh et al. [25] were the first to present the concept of complex-valued fuzzy metric spaces and to create the complex-valued fuzzy version of some metric space results.

The goal of this study is to expand well-known metric-space results to complex-valued fuzzy metric spaces and then prove them in complex-valued complete fuzzy metric spaces.

2 II. PRELIMINARIES

Def.2.1. [21]. Let ? be the set of complex numbers and ? 1 , ? 2 ? ?, where ? = ? + ??. Then a partial order relation '? ' on ? is defined as follows:? 1 ? ? 2 â??" ??(? 1) ? ??(? 2) and ??(? 1) ? ??(? 2)

Hence ? 1 ? ? 2 if one of the following satisfies;

London Journal of Research in Science: Natural and Formal (PO1) ??(? 1) = ??(? 2) and ??(? 1) = ??(? 2) (PO2) ??(? 1) < ??(? 2) and ??(? 1) = ??(? 2) (PO3) ??(? 1) = ??(? 2) and ??(? 1) < ??(? 2) (PO4) ??(? 1) < ??(? 2) and ??(? 1) < ??(? 2)

In particular, ? 1 ? ? 2 if ? 1 ? ? 2 and one of (PO2), (PO3), and (PO4) is satisfied, and we write ? 1 ? ? 2 if only (PO4) is satisfied.

It can be noted that;0 ? ? 1 ? ? 2 ? |? 1 | < |? 2 |, ? 1 ? ? 2 , ? 2 ? ? 3 ? ? 1 ? ? 3 . Def.2.2.[21]. Complex-Valued Metric Space (CVMS)

Let ? be a non-empty set. Assume that the mappings ?: ? × ? ? ? satisfies: (CV1) 0 ? ?(?, ?), for all ?, ? ? ? and ?(?, ?) = 0 iff ? = ? ;

(CV2) ?(?, ?) = ?(?, ?), for all ?, ? ? ? ;

(CV3) ?(?, ?) ? ?(?, ?) + ?(?, ?), for all ?, ?, ? ? ? Then ? is called a complex-valued metric on ?, and (?, ?) is called a CVMS. Def.2.3. [23]. The 'max' function with partial order relation '?' is defined as(1) ??? {? 1 , ? 2 } = ? 2 â??" ? 1 ? ? 2 (2) ? 1 ? ??? {? 2 , ? 3 } ? ? 1 ? ? 2 or ? 1 ? ? 3

And the 'min' functions can be defined as(1) ??? {? 1 , ? 2 } = ? 1 â??" ? 1 ? ? 2 (2) ??? {? 1 , ? 2 } ? ? 3 ? ? 1 ? ? 3 or ? 2 ? ? 3 .

Following Zadeh's [3] contribution to fuzzy set theory, a number of scholars [4][5][6] contributed to the field's basics and core theories.

Buckley [10] was the first to present the concept of fuzzy complex numbers. Other authors were inspired by Buckley's work and continued their research on fuzzy complex numbers. Ramot et al. [1] expanded fuzzy sets to complex fuzzy sets in this chain.

3 Singh et al. [25], inspired by Ramot et al. [1,

] constructed complex-valued fuzzy metric spaces using continuous t -norms, defined a Hausdorff topology on complex -valued fuzzy metric space, and gave the concept of Cauchy sequences in CVFMS.

We establish certain fixed-point conclusions in the situation of complex -valued fuzzy metric spaces, inspired by **??**ingh et al. [25]. We begin by extending several well-known metric-space results to complex-valued fuzzy metric spaces, and then we prove those results in the setting of CVFMS. Def.2.4. [1]. The complex fuzzy set ? is given by ? = {(?, ? ? (?)) ? ? ? ?}.

Where ? is a universe of discourse, ? ? (?) is a membership function and defined as ? ? (?) = ? ? (?). ? ?? ? (?) The triplet (?, ?, *) is said to be CVFMS if a complex valued fuzzy set ? ? ? × ? × (0, ?) ? ? ? ? ?? (where ? ? ?, * is a complex valued continuous t-norm) fulfil the following criteria: and ? > 0. Let ?: ? ? ? be a mapping that satisfies ?(??, ??, ??) ? ?(?, ?, ?), ? ? ? (0, 1). Then ? has a fixed point that is unique.

Fisher [24] established the following theorem in complete metric space for three mappings.

Theorem A [24]. Let S and T be continuous mappings of a complete metric space (X, d) into themselves. Then S and T have a common fixed point in X iff a continuous mapping A of X into S(X) ?T(X) exists, which commutes with S and T and satisfies;

?(??, ??) ? ? ?(??, ??) for all ?, ? ? ? and 0 < ? < 1. Indeed ?, ? and ? have a unique common fixed point.

We can now extend the preceding theorem/result to complex-valued complete fuzzy metric space as follows:

Theorem -3.1. Let (?, ?, *) be a complex-valued complete fuzzy metric space (CVCFMS). ? and ? are continuous mappings from ? to ?. If ? is a continuous mapping from ? to ?(?) ? ?(?), it commutes with ? and ?, and if detailed maps satisfy the following contractive condition.

?(??, ??, ??) ? ???{ ?(??, ??, ?), ?(??, ??, ?), ?(??, ??, ?)} for all ?, ? ? ?, ? ? (0, ?) and 0 < ? < 1 ? (3.11)

4 III. MAIN RESULTS

Additionally, lim ??? ?(?, ?, ?) = ? ?? , for all ?, ? ? ? and ? ? [0, Then ?, ?, and ? have a unique common fixed point.

Proof: ?? ? is a Cauchy sequence?

Since ? is a continuous mapping from ? to ?(?) ? ?(?) so for ? 1 ? ?, there exists any ? 0 ? ? such that ?? 0 = ?? 1 and ?? 0 = ?? 1

On keep repeating this process for different ? 1 and ? 0 , we get a sequence {? ? } such that In general, we get ?(?? ?+1 , ?? ?+2 , ??) ? ?(?? ? , ?? ?+1 , ?), ? ? > 0 ? (???)??

Hence by lemma (4.2), {?? ? } is a Cauchy sequence in ?.

Since the space ? is complete, so there exists some ? ? ? such that lim The mappings ? and ? are continuous. ? is continuous from ? to ?(?) ? ?(?).

Clearly, ?(?) ? ?(?) and ?(?) ? ?(?)

This implies that ?(?) ? ?(?) ? ?(?). [1]

Figure 1: (

[1] Volume 23 | Issue 2 | Compilation 1.0 © 2023 Great Britain Journal Press

Figure 2:

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Figure 3: 2] 1 2 2]. 3 ? 3 + 2 ; 3 < 3 2? 3 + 1 ; 3 < ? ? 21 And

Def.2.5. [25]. Complex Valued Continuous t-norm
A binary operation * ? ? 2], is
called complex valued continuous t-norm if it satisfies the following conditions:
(1) * is associative and commutative,
(2) * is continuous,
(3) ? 2].
(iii) ? * ? = { min{?, ?} , ?ð ??” max{?, ?} = ? ?? ; 0, ?????????, for a fix ? ? [0, ? 2].

? ? [0, 1]. Ex.2.5. [25]. The following binary operations defined in (i), (ii) and (iii) are complex valued continuous t-norm (i) ? * ? = ??? (?, ?). (ii) ? * ? = ??? (? + ? -? ?? , 0), for a fix ? ? [0, Def.2.6. [25]. Complex Valued Fuzzy Metric Spaces (CVFMS)

Figure 4:

Lemma 2.7 [25]. Let (?, ?, *) be a CVFMS such that lim ??? ?(?, ?, ?) = ? ?? , for all ?, ? ? ?, if ?(?, ?, ??) ? ?(?, ?, ?), for all ?, ? ? ?, 0 < ? < 1, ? ? (0, ?) then ? = ?.
Lemma 2.8 [25]. Let {? ? } be a sequence in a CVFMS (?, ?, *) with lim ??? ?(?, ?, ?) = ? ?? , for all ?, ? ? ?. If there exists a number ? which lies on (0, 1)such that ?(? ?+1 , ? ?+2 , ??) ? ?(? ? , ? ?+1 , ?), ? ? > 0, ? = 0, 1, 2, . .. Then {? ? } is a Cauchy sequence in ?.

The following theorem was established bySingh et al. [25], which is the resetting of the Banach contraction principle in CVFMS. Theorem 2.7 [25]. Let (?, ?, *) be a CVFMS such that lim ??? ?(?, ?, ?) = ? ?? , ? ?, ? ? ?,

Figure 5:

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