Scan to know paper details and author's profile

Results on Complex Valued Complete Fuzzy Metric Spaces

Praveen Kumar Sharma & Shivram Sharma

ABSTRACT

This work demonstrates certain standard fixed point theorems on complex-valued fuzzy metric spaces. We show certain fixed point findings in the situation of complex-valued fuzzy metric spaces, inspired by Singh et al. [25].

To begin, we extend some well-known existing conclusions from metric spaces to complex-valued fuzzy metric spaces and then prove them in the complex-valued complete fuzzy metric space context. We provide an example that supports our main result and supports our hypotheses.

Keywords: common fixed point; metric spaces; complex-valued fuzzy metric spaces (CVFMS).

Classification: DDC Code: 515.42 LCC Code: QA312

Language: English

Great Britain Journals Press

LJP Copyright ID: 925614
Print ISSN: 2631-8490
Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 23 | Issue 2 | Compilation 1.0

Results on Complex Valued Complete Fuzzy Metric Spaces

Praveen Kumar Sharma[α] & Shivram Sharma[σ]

ABSTRACT

This work demonstrates certain standard fixed point theorems on complex-valued fuzzy metric spaces. We show certain fixed point findings in the situation of complex-valued fuzzy metric spaces, inspired by Singh et al. [25].

To begin, we extend some well-known existing conclusions from metric spaces to complex-valued fuzzy metric spaces and then prove them in the complex-valued complete fuzzy metric space context. We provide an example that supports our main result and supports our hypotheses.

Keywords: common fixed point; metric spaces; complex-valued fuzzy metric spaces (CVFMS).

Author α: Department of Mathematics, SVIS, Shri Vaishnav Vidyapeeth Vishwavidyalaya, Indore-453111 (M.P.), India.
σ: Department of Mathematics, Govt. P.G. College, Guna-473001 (M.P.), India.

I. INTRODUCTION

In 1965, Zadeh [3] coined the term "fuzzy set." Following that, a slew of authors worked on fuzzy sets, expanding the fuzzy set theory and its applications [4-6]. The idea of fuzzy metric spaces was given by Kramosil and Michalik [7]. After then, George and Veeramani [9] updated this idea. Grabiec [8] investigated fuzzy metric space fixed-point theory. The idea of complex-valued metric spaces was introduced by Azam et al. [21].

Verma et al. [23] recently established 'Max' functions and the partial order relation'for complex numbers, and used properties (E-A) and CLRg to prove fixed point theorems in complex valued metric space. Singh et al. [25] were the first to present the concept of complex-valued fuzzy metric spaces and to create the complex-valued fuzzy version of some metric space results.

The goal of this study is to expand well-known metric-space results to complex-valued fuzzy metric spaces and then prove them in complex-valued complete fuzzy metric spaces.

II. PRELIMINARIES

Def.2.1.[21]. Let $\mathbb{C}$ be the set of complex numbers and $\eta_1, \eta_2 \in \mathbb{C}$, where $\eta = \mu + i\nu$. Then a partial order relation '$\precsim$' on $\mathbb{C}$ is defined as follows:

$\eta_1 \precsim \eta_2 \Leftrightarrow Re(\eta_1) \leq Re(\eta_2)$ and $Im(\eta_1) \leq Im(\eta_2)$

Hence $\eta_1 \precsim \eta_2$ if one of the following satisfies;

London Journal of Research in Science: Natural and Formal

(PO1) $Re(\eta_1) = Re(\eta_2)$ and $Im(\eta_1) = Im(\eta_2)$

(PO2) $Re(\eta_1) < Re(\eta_2)$ and $Im(\eta_1) = Im(\eta_2)$

(PO3) $Re(\eta_1) = Re(\eta_2)$ and $Im(\eta_1) < Im(\eta_2)$

(PO4) $Re(\eta_1) < Re(\eta_2)$ and $Im(\eta_1) < Im(\eta_2)$

In particular, $\eta_1 \lnsim \eta_2$ if $\eta_1 \neq \eta_2$ and one of (PO2), (PO3), and (PO4) is satisfied, and we write $\eta_1 \prec \eta_2$ if only (PO4) is satisfied.

It can be noted that;

$0 \lesssim \eta_1 \lnsim \eta_2 \Rightarrow |\eta_1| < |\eta_2|,\ \eta_1 \lesssim \eta_2, \eta_2 \prec \eta_3 \Rightarrow \eta_1 \prec \eta_3$.

Def.2.2.[21]. Complex-Valued Metric Space (CVMS)

Let X be a non-empty set. Assume that the mappings $d: X \times X \rightarrow \mathbb{C}$ satisfies:

(CV1) $0 \lesssim d(a, b)$, for all $a, b \in X$ and $d(a, b) = 0$ iff $a = b$;

(CV2) $d(a, b) = d(b, a)$, for all $a, b \in X$;

(CV3) $d(a, c) \lesssim d(a, b) + d(b, c)$, for all $a, b, c \in X$

Then d is called a complex-valued metric on X, and (X, d) is called a CVMS.

Def.2.3.[23]. The 'max' function with partial order relation '$\lesssim$' is defined as

(1) $max\,\{\eta_1, \eta_2\} = \eta_2 \Leftrightarrow \eta_1 \lesssim \eta_2$
(2) $\eta_1 \lesssim max\,\{\eta_2, \eta_3\} \Rightarrow \eta_1 \lesssim \eta_2$ or $\eta_1 \lesssim \eta_3$

And the 'min' functions can be defined as

(1) $min\,\{\eta_1, \eta_2\} = \eta_1 \Leftrightarrow \eta_1 \lesssim \eta_2$
(2) $min\,\{\eta_1, \eta_2\} \lesssim \eta_3 \Rightarrow \eta_1 \lesssim \eta_3$ or $\eta_2 \lesssim \eta_3$.

Following Zadeh's [3] contribution to fuzzy set theory, a number of scholars [4-6] contributed to the field's basics and core theories.

Buckley [10] was the first to present the concept of fuzzy complex numbers. Other authors were inspired by Buckley's work and continued their research on fuzzy complex numbers. Ramot et al. [1] expanded fuzzy sets to complex fuzzy sets in this chain.

Singh et al. [25], inspired by Ramot et al. [1,] constructed complex-valued fuzzy metric spaces using continuous t - norms, defined a Hausdorff topology on complex - valued fuzzy metric space, and gave the concept of Cauchy sequences in CVFMS.

We establish certain fixed-point conclusions in the situation of complex -valued fuzzy metric spaces, inspired by Singh et al. [25]. We begin by extending several well-known metric-space results to complex-valued fuzzy metric spaces, and then we prove those results in the setting of CVFMS.

Def.2.4.[1]. The complex fuzzy set S is given by $S = \{(x, \mu_s(x))/x \in U\}$.

Where U is a universe of discourse, $\mu_s(x)$ is a membership function and defined as $\mu_s(x) = r_s(x).e^{iw_s(x)}, (i = \sqrt{-1}\,)$ where $r_s(x)$ and $w_s(x)$ both real-valued, with $r_s(x) \in [0,1]$.

Def.2.5. [25]. Complex Valued Continuous t-norm

A binary operation $*: \ r_s e^{i\theta} \times r_s e^{i\theta} \rightarrow r_s e^{i\theta}$, wherein $r_s \in [0,1]$ and a fix $\theta \in [0, \frac{\pi}{2}]$, is called complex valued continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a * e^{i\theta} = a, \forall a \in r_s e^{i\theta}$, where $r_s \in [0,1]$,
(4) $a * b \lesssim c * d$ whenever $a \lesssim c$ and $b \lesssim d$, for all $a, b, c, d \in r_s e^{i\theta}$, where $r_s \in [0,1]$.

Ex.2.5. [25]. The following binary operations defined in (i), (ii) and (iii) are complex valued continuous t-norm

(i) $a * b = min\,(a, b)$.
(ii) $a * b = max\,(a + b - e^{i\theta}, 0)$, for a fix $\theta \in [0, \frac{\pi}{2}]$.
(iii) $a * b = \begin{cases} \min\{a, b\}, if \max\{a, b\} = e^{i\theta}; \\ 0, otherwise, \end{cases}$ for a fix $\theta \in [0, \frac{\pi}{2}]$.

Def.2.6. [25]. Complex Valued Fuzzy Metric Spaces (CVFMS)

The triplet $(X, M, *)$ is said to be CVFMS if a complex valued fuzzy set $M : X \times X \times (0, \infty) \rightarrow r_s e^{i\theta}$ (where $X \neq \emptyset$, $*$ is a complex valued continuous t-norm) fulfil the following criteria:

(CF1) $M(a, b, t) > 0$,

(CF2) $M(a, b, t) = e^{i\theta}$ for all $t > 0 \Leftrightarrow a = b$,

(CF3) $M(a, b, t) = M(b, a, t)$,

(CF4) $M(a, b, t) * M(b, c, s) \gtrsim M(a, c, t + s)$,

(CF5) $M(a, b, .) : (0, \infty) \rightarrow r_s e^{i\theta}$ is continuous,

for all $a, b, c \in X, s, t > 0, r_s \in [0,1]$ and $\theta \in [0, \frac{\pi}{2}]$.

Note- Wherever appropriate to our study, we refer to [25] and the references mentioned in [25] for further basic definitions, examples, and fundamental features of CVMS.

Singh et al. [25] demonstrated the following lemmas in CVFMS before establishing the result on complex-valued fuzzy metric space, i.e. Theorem 2.7.

Lemma 2.7 [25]. Let $(X, M,*)$ be a CVFMS such that $\lim_{t\to\infty} M(a, b, t) = e^{i\theta}$, for all $a, b \in X$, if $M(a, b, \mathcal{K}t) \gtrsim M(a, b, t)$, for all $a, b \in X, 0 < \mathcal{K} < 1$, $t \in (0, \infty)$ then $a = b$.

Lemma 2.8 [25]. Let $\{b_n\}$ be a sequence in a CVFMS $(X, M,*)$ with $\lim_{t\to\infty} M(a, b, t) = e^{i\theta}$, for all $a, b \in X$. If there exists a number $\mathcal{K}$ which lies on $(0, 1)$such that

$M(b_{n+1}, b_{n+2}, \mathcal{K}t) \gtrsim M(b_n, b_{n+1}, t)$, $\forall\, t > 0$, $n = 0, 1, 2, \ldots$ Then $\{b_n\}$ is a Cauchy sequence in X.

The following theorem was established by Singh et al. [25], which is the resetting of the Banach contraction principle in CVFMS.

Theorem 2.7 [25]. Let $(X, M,*)$ be a CVFMS such that $\lim_{t\to\infty} M(a, b, t) = e^{i\theta}$, $\forall\, a, b \in X$, and $t > 0$. Let $T: X \to X$ be a mapping that satisfies $M(Ta, Tb, \mathcal{K}t) \gtrsim M(a, b, t)$, $\forall\, \mathcal{K} \in (0, 1)$. Then T has a fixed point that is unique.

III. MAIN RESULTS

Fisher [24] established the following theorem in complete metric space for three mappings.

Theorem A [24]. Let S and T be continuous mappings of a complete metric space (X, d) into themselves. Then S and T have a common fixed point in X iff a continuous mapping A of X into S(X) ∩T(X) exists, which commutes with S and T and satisfies;

$d(Ax, Ay) \leq \alpha\, d(Sx, Ty)$ for all $x, y \in X$ and $0 < \alpha < 1$. Indeed S, T and A have a unique common fixed point.

We can now extend the preceding theorem/result to complex-valued complete fuzzy metric space as follows:

Theorem -3.1. Let $(X, M,*)$ be a complex-valued complete fuzzy metric space (CVCFMS). S and T are continuous mappings from X to X. If A is a continuous mapping from X to $S(X) \cap T(X)$, it commutes with S and T, and if detailed maps satisfy the following contractive condition.

$M(Ax, Ay, kt) \gtrsim Min\{ M(Ty, Ay, t), M(Sx, Ax, t), M(Sx, Ty, t)\}$ for all $x, y \in X, t \in (0, \infty)$ and $0 < k < 1$

... (3.11)

Additionally, $\lim_{t\to\infty} M(x,y,t) = e^{i\theta}$, for all $x,y \in X$ and $\theta \in [0,\frac{\pi}{2}]$
... (3.12)

Then S,T, and A have a unique common fixed point.

Proof: Ax_n is a Cauchy sequence?

Since A is a continuous mapping from X to $S(X) \cap T(X)$ so for $x_1 \in X$, there exists any $x_0 \in X$ such that $Ax_0 = Sx_1$ and $Ax_0 = Tx_1$

On keep repeating this process for different x_1 and x_0, we get a sequence $\{x_n\}$ such that

$Ax_n = Sx_{n+1}$ and $Ax_n = Tx_{n+1}$

Or $Ax_{2n} = Sx_{2n+1}$ and $Ax_{2n} = Tx_{2n+1}$, $n = 1,2,3,\ldots$

On setting $x = x_{2r}$ and $y = x_{2r+1}$ in (3.11), we get for $r = 1,2,3,\ldots$

$$M(Ax_{2r}, Ax_{2r+1}, kt) \gtrsim Min\{ M(Tx_{2r+1}, Ax_{2r+1}, t), M(Sx_{2r}, Ax_{2r}, t), M(Sx_{2r}, Tx_{2r+1}, t)\}$$

$$M(Ax_{2r}, Ax_{2r+1}, kt) \gtrsim Min\{ M(Ax_{2r}, Ax_{2r+1}, t), M(Ax_{2r-1}, Ax_{2r}, t), M(Ax_{2r-1}, Ax_{2r}, t)\}$$

$$M(Ax_{2r}, Ax_{2r+1}, kt) \gtrsim Min\{ M(Ax_{2r}, Ax_{2r+1}, t), M(Ax_{2r-1}, Ax_{2r}, t)\} \ldots (I)$$

Now suppose $Min\{ M(Ax_{2r}, Ax_{2r+1}, t), M(Ax_{2r-1}, Ax_{2r}, t)\} = M(Ax_{2r}, Ax_{2r+1}, t)$

Then by (I), we have $M(Ax_{2r}, Ax_{2r+1}, kt) \gtrsim M(Ax_{2r}, Ax_{2r+1}, t)$

By lemma (4.1) or (5.1), we have $Ax_{2r} = Ax_{2r+1}$

Which is not possible

Hence by (I), we must have $M(Ax_{2r}, Ax_{2r+1}, kt) \gtrsim M(Ax_{2r-1}, Ax_{2r}, t), \forall\, t > 0 \ldots (II)$

In general, we get $M(Ax_{r+1}, Ax_{r+2}, kt) \gtrsim M(Ax_r, Ax_{r+1}, t), \forall\, t > 0 \ldots (III)$

Hence by lemma (4.2), $\{Ax_n\}$ is a Cauchy sequence in X.

Since the space X is complete, so there exists some $p \in X$ such that $\lim_{n\to\infty} Ax_n = p$

and $p = \lim_{n\to\infty} Sx_{n+1} = \lim_{n\to\infty} Tx_{n+1}$

It follows that $Ap = Sp = Tp$, and

$$M(Ap, A^2p, kt) \gtrsim Min\{ M(TAp, AAp, t), M(Sp, Ap, t), M(Sp, TAp, t)\}$$

$$M(Ap, A^2p, kt) \gtrsim M(Sp, ATp, t)$$

$$M(Ap, A^2p, kt) \gtrsim M(Ap, A^2p, t)$$

$$\ldots$$

$$M(Ap, A^2p, kt) \gtrsim M\left(Ap, A^2p, \frac{t}{k^n}\right) \ldots (IV)$$

On taking $n \to \infty$, then by lemma (4.1), we have; $Ap = A^2p$

This implies that $Ap = p$

Thus p is a common fixed point of A, S, and T.

Uniqueness: - let $q(\neq p)$ be another fixed point of A, S, and T. Then, by (3.11), we have

$$M(Ap, Aq, kt) \gtrsim Min\,\{\,M(Tq, Aq, t), M(Sp, Ap, t), M(Sp, Tq, t)\}$$

Which implies that

$$M(p, q, kt) \gtrsim Min\,\{\,e^{i\theta}, e^{i\theta}, M(p, q, t)\}$$

As $M(p, q, t) \in\ r_s e^{i\theta},\ r_s\ \in\ [0,1]$ and $\theta \in [0, \frac{\pi}{2}]$, also $M(p, q, t)\ \lesssim\ e^{i\theta}$

Then certainly we get, $Min\left\{\,e^{i\theta}, e^{i\theta}, M(p, q, t)\right\} = M(p, q, t)$

$$M(p, q, kt) \gtrsim\ M(p, q, t)$$

Which implies that $p = q$.

As a result, p is unique.

Ex. 3.1. Let $X = [3,21]$ with the metric d defined by $d(x, y) = |x - y|, \forall x, y \in X$.

For all $x, y\ \in X$ and $t \in (0, \infty)$, we define $M(x, y, t) = e^{i\theta}[\frac{t}{t+d(x,y)}]$ or $M(x, y, t) = e^{i\theta}\left[\frac{t}{kt+d(x,y)}\right]$, $k = \frac{1}{2}$, and t -norm $'*'$ is defined as $a * b = min\,\{a, b\}$ where $a, b \in r_s e^{i\theta}$, for $r_s\ \in\ [0,1]$ and $\theta \in [0, \frac{\pi}{2}]$. Here, $\lim_{t\to\infty} M(x, y, t)\ =\ e^{i\theta}$, for all $x, y\ \in\ X$.

$(X, M,*)$ is a CVCFMS with a given t-norm $*$.

$S, T\ :\ X\ \rightarrow\ X$ are defined as:

$$S(X) = \begin{cases} 3;\, at\ \ x = 3 \\ \frac{x}{3} + 2;\ \ 3 < x \leq 21 \end{cases},\text{ and } T(X) = \begin{cases} 3;\, at\ \ x = 3 \\ \frac{2x}{3} + 1;\ \ 3 < x \leq 21 \end{cases}$$

And A: $X\ \rightarrow S(X) \cap T(X)$ as:

$$A(X) = \begin{cases} 3;\, at\ \ x = 3 \\ \dfrac{3x + 21}{10};\ \ 3 < x \leq 21 \end{cases}$$

The mappings S and T are continuous. A is continuous from X to $S(X) \cap T(X)$.

Clearly, $A(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$

This implies that $A(X) \subseteq S(X) \cap T(X)$.

IV. CONCLUSION

Existing results from complete metric space have been extended to complex-valued complete fuzzy metric spaces in this study. We tested the extended version of the result using a new form of weaker contractive condition. We've offered an example that backup our major finding and proves our hypotheses. In this line, various complete metric space results can be extended and demonstrated in the context of complex-valued complete fuzzy metric spaces.

REFERENCES

1. D. Ramot, R. Milo, M. Friedman, and A. Kandel, IEEE Transactions of Fuzzy Systems 10(2) (2002), 171–186.
2. G. Jungck, Commuting maps and fixed points, Amer Math Monthly 83 (1976), 261– 263.
3. L.A. Zadeh, Fuzzy sets, Inform Control 8 (1965), 338–353.
4. A. Kandel, Fuzzy Mathematical Techniques and Applications. Reading, MA: Addison- Wesley, 1986.
5. G.J. Klir and B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, NJ: Prentice-Hall, 1995. [6] J.M. Mendel, Fuzzy logic systems for engineering: A tutorial, Proc IEEE 83 (1995), 345–377.
6. I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975), 336–344.
7. M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and System 27 (1988), 385–399.
8. A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets System 64 (1994), 395–399.
9. J.J. Buckley, Fuzzy complex numbers, in Proc ISFK, Guangzhou, China, 1987, 597aˆASˇ ¸700
10. J.J. Buckley, Fuzzy complex numbers, Fuzzy Sets System 33 (1989), 333aˆASˇ ¸345.
11. J.J. Buckley, Fuzzy complex analysis I: Differentiation, Fuzzy Sets System 41 (1991), 269aˆASˇ ¸284. [13]
12. J.J. Buckley, Fuzzy complex analysis II: Integration, Fuzzy Sets System 49 (1992), 171aˆASˇ ¸179.
13. M. Ousmane and C. Wu, Semi-continuity of complex fuzzy function, Tsinghua Science and Technology 8 (2003), 65–70.
14. N. Sethi, S.K. Das and D.C. Panda, Probabilistic interpretation of complex fuzzy set, IJCSEIT 2(2) (2012), 31–44.
15. D. Qiu and L. Shu, Notes on aˆAIJon the restudy of fuzzy ˘ complex analysis: Part I and part IIaˆAˇ ˙I, Fuzzy Sets System 159 (2008), 2185–2189.
16. D. Qiu, L. Shu and Z. Wen Mo, Notes on fuzzy complex analysis, Fuzzy Sets System 160 (2009), 1578–1589.
17. J. Qiu, C. Wu and F. Li, On the restudy of fuzzy complex analysis: Part I. The sequence and series of fuzzy complex numbers and their convergences, Fuzzy Sets System 115 (2000), 445–450.
18. J. Qiu, C. Wu and F. Li, On the restudy of fuzzy complex analysis: Part II. The continuity and differentiation of fuzzy complex functions, Fuzzy Sets System 120 (2001), 517–521.
19. C. Wu and J. Qiu, Some remarks on fuzzy complex analysis, Fuzzy Sets System 106 (1999),231-238
20. A. Azam, B. Fisher, and M. Khan, Common fixed point theorems in complex valued metric spaces, Numerical Functional Analysis and Optimization 32(3) (2011), 243–253.

21. F. Rouz ar an M. Im a , Some common ixe point t eorems on comp ex-va ue metric spaces, Computer and Mathematics with Applications 64 (1012), 1866–1874.
22. R.K. Verma and H.K. Pathak, Common fixed point theorems using property (E.A) in complex-valued metric spaces, Thai Journal of Mathematics 11(2) (2013), 347–355.
23. B. Fisher; Mapping with a common fixed point, Math. Sem. Notes Kobe Univ., Vol.7,81-84 (1979).
24. D. Singh, V. Joshi, M. Imdad, and P. Kumam; A novel framework of complex-valued fuzzy metric spaces and fixed-point theorems, Journal of Intelligent and fuzzy system, Vol.30, 3227-3238,2016.