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An Investigation into the Order of Integral Powers of Consecutive Elements of Set of Even and Odd Numbers

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This article analysed the order of difference of integral perfect powers of the set of even and odd numbers. The analysis was proof by the use of combinatorial terminologies, established property of the difference operator and the principle of mathematical induction. The results proved conclusively that "if any number of consecutive odd or even integers are raised to a positive power k, then the kth difference is equal to $2^k k!$

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I. INTRODUCTION

A perfect power is a number which has a rational root. Chase [1]. An integral perfect power is the irrational root that is an integer. A finite difference is a mathematical expression of the form, $f(x + b) - f(x + a)$ and a forward difference is of the form $\Delta h\,[f]\,(x) = f(x + h) - f(x)$[2].

In this article, an examination on the set of even numbers: {0, 2, 4, 6, 8, 10, 12, 14, 16, 18} and the set of odd numbers: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} reveals that the first difference of the square of consecutive elements of both the set of even and odd numbers is even. The second difference of the two distinct sets is {8, 8, 8, 8, 8, 8, 8, 8, 8} = {8}, a singleton set.

Clearly, the third difference is {0}. The emerging pattern motivates the investigation of whether or not this pattern persists for all perfect squares of elements of even and odd sets, resulting from integers. In their contribution in Exponential Diophantine Equations, Shorey and Tijdeman[3] investigated perfect powers at integral values of a polynomial with rational integer coefficients and obtain in particular the following.

Let f(x) be a polynomial with rational coefficients and with at least two simple rational zeros.

Suppose $b \neq 0$, $m \geq 0$, x and y with $|y| > 1$ are rational integers. Then the equation $f(x) = by^m$ implies that m is bounded by a computable number depending only on b and f. Also, in their contribution in the difference of perfect powers of integers, Ladan, Ukwu and Apine[4] investigated order of integral perfect powers and proved that "if any number of consecutive integers are raised to a positive integral power k, then the k^{th} difference is equal to k!

More generally, the question at the heart of the matter is the following. What is the computational disposition of the orders of difference of integral perfect powers of successive elements of set of even and odd numbers? The Review of Literature shows that no such investigation has been undertaken. Thus, this article adds to the existing body of knowledge, by providing answers to the above question.

II. METHODS

2.1 Preliminary Definitions

In what follows, the difference of finite order will be defined.

2.1.1 Difference of Order One (1)

Given a sequence $\{U_j\}_{j=0}^{\infty}$, define the difference of order one at j with respect to the sequence by:

$$\Delta(U_j) = U_{j+1} - U_j, \text{ for every integral } j.$$

2.1.2 Higher Order Difference

Higher difference can be defined recursively by:

$$\Delta^k(U_j) = \Delta \ \ \Delta^{k-1}(U_j) \ \ = \Delta^{k-1} \ \ \Delta(U_j) \ \ = \Delta^{k-1}[U_{j+1} - U_j] \text{ for } k \geq 2.$$

III. RESULTS AND DISCUSSION

3.1 Preliminary Theorem

Suppose that $U_j = j$ and $U_j = \hat{j}$, for integral j.

Let $\propto_j U_j = \propto_j j$ be the set of even numbers and $\propto_j \hat{U}_j + \beta_j = \propto_j j + \beta_j$ be the set of odd numbers.

Where $\propto_j = 2$ fixed, $j \geq 0$, $\beta = 1$, fixed.

Then,

$(i)_1$ $\quad \Delta(\propto_j U_j)^2 = \propto_j^2(\text{odd})$ for even case

$(i)_2$ $\quad \Delta(\propto_j \hat{U}_j + \beta)^2 = \propto_j^2(\text{even})$ for odd case

(ii) $\quad \Delta^2(\propto_j U_j)^2 = \Delta^2(\propto_j \hat{U}_j + \beta)^2 = 8 = 2^2(2!)$

(iii) $\quad \Delta^k(\propto_j U_j)^2 = \Delta^k(\propto_j \hat{U}_j + \beta_j)^2 = 0, k \in \{3, 4, \ldots\}$

Proof:

Case 1. (even)

Let $\langle \propto_j U_j \rangle_{j=0}^{9} = \langle \propto_j j \rangle_{j=0}^{9} = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

Then, the table below display the values of $\Delta^k(\propto_j U)^p$ for selected values of k and P in the set {0, 1, 2, 3} and {1, 2} respectively when $\Delta^0(\propto_j U_j)^p = (\propto_j U_j)^p$.

Table I: Difference Order Table for Selected Consecutive Positive Integers.

$\propto_j j$	$(\propto_j j)^2$	$\Delta(\propto_j j)^2$	$\Delta^2(\propto_j j)^2$	$\Delta^3(\propto_j j)^2$
0	0	4	8	0
2	4	12	8	0
4	16	20	8	0
6	36	28	8	0
8	64	36	8	0
10	100	44	8	0
12	144	52	8	0
14	196	60	8	0
16	256	68	8	
18	324			

It is clear that $\Delta^k(\propto_j j)^2 = 0$, for all $k \geq 3$, $\propto_j j = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$. Obviously the theorem is valid for $j \geq 0$ as observed from columns 4 and 5.

Case II. (odd)

Let $\langle \propto_j + \beta \rangle_{j=0}^{9} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$.

Then the table below displayed the values of $\Delta^k(\propto_j + \beta_j)^p$ for selected values of k and p in the set {0, 1, 2, 3} and {1, 2} respectively when $\Delta^0(\propto_j j + \beta_j)^p = (\propto_j j + \beta_j)^p$.

Table II: Difference Order table for selected Consecutive Positive Integers

$\propto_j + \beta$	$(\propto_j + \beta)^2$	$\Delta(\propto_j + \beta)^2$	$\Delta^2(\propto_j + \beta)^2$	$\Delta^3(\propto_j + \beta)^2$
1	1	8	8	0
3	9	16	8	0
5	25	24	8	0
7	49	32	8	0
9	81	40	8	0
11	121	48	8	0
13	169	56	8	0
15	225	64	8	
17	289	72		
19	361			

Similarly, it is clear that $\Delta^k(\propto_j \hat{U}_j + \beta_j) = 0$, for all $k \geq 3$, $\propto_j + \beta = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$. The theorem is valid as observed from column 4 and 5.

The proof of (i), (ii) and (iii) are direct.

(i1) $U_j = j \Rightarrow \propto U_j = \propto_j \Rightarrow (\propto U_j)^2 = \propto^2 j^2$

$$\Rightarrow \quad (\propto^2 j^2) = \propto^2 \Delta j^2 = \propto^2 [(j+1)^2 - j^2] = \propto^2 [j^2 + 2j + 1 - j^2]$$

$$= \propto^2 (2j+1) = \propto^2 (\text{odd}) = \text{even}.$$

(i2) $\hat{U}_j = \hat{j} \Rightarrow \propto \hat{U}_j + \beta = \propto \hat{j} + \beta$

$$(\hat{U}_j + \beta)^2 = (U_{\hat{j}} + \beta)^2 = (\propto_{\hat{j}})^2 + 2\propto \beta_j + \beta^2$$

$$\Delta(\propto \hat{U}_j + \beta_j)^2 = \Delta(\propto^2 j^2 + 2 \propto \beta_j + \beta^2)$$

$$= \propto^2 \Delta j^2 + 2 \propto \beta \Delta_j + \beta^2$$

$$= \propto^2 [(j+1)^2 - j^2] + 2\propto \beta[(j+1) - j] + 0$$

$$= \propto^2 (j^2 + 2j + 1 - j^2) + 2\propto \beta \quad (1)$$

$$= \propto^2 (2j+1) + 2\propto \beta$$

$$= 2\propto^2 j + \propto^2 + 2\propto \beta$$

$$= \propto^2 (2j+1) + 2\propto \beta$$

$$\Rightarrow \quad \Delta(\propto \hat{U}_j + \beta_j)^2 = \propto^2 (2j+1) + 2\propto \beta = \text{even}$$

For every $j \geq 0, \propto = 2, \beta = 1$.

(ii) $\Delta^2(\propto U_j)^2 = \Delta(\Delta(\propto j))^2 = \Delta[\propto^2 (2j+1)]$

$$= \Delta(2 \propto^2 j + \propto^2) = 2 \propto^2 \Delta j + \Delta \propto^2$$

$$= 2 \propto^2 [(j+1) - j] + 0$$

$$= 2 \propto^2 (1) = 2 \propto^2$$

$$\Rightarrow \Delta^2(\propto U_j)^2 = 2 \propto^2$$

Similarly:

$$\Delta^2(\propto \hat{U}_j + \beta)^2 = \Delta[\Delta(\propto j + \beta)^2]$$

$$= \Delta[\propto^2 (2j+1) + 2 \propto \beta]$$

$$= \Delta[2 \propto^2 j + \propto^2 + 2 \propto \beta]$$

$$= 2 \propto^2 \Delta j + \Delta\propto^2 + 2\Delta\propto \beta$$

$$= 2 \propto^2 [(j+1) - j] + 0 + 0$$

$$= 2 \propto^2(1) = 2 \propto^2$$

$$\Rightarrow \Delta^2\left(\propto \hat{U}_j + \beta\right) = 2 \propto^2$$

$$\Rightarrow \Delta^2\left(\propto U_j\right)^2 = \Delta^2\left(\propto \hat{U}_j + \beta\right) = 2 \propto^2. \text{ proving (ii)}$$

The principle of mathematical induction is needed in the proof of (iii).

For k = 3, $\Delta^3\left(\propto_j U_j\right)^2 = \Delta^3(\propto j)^2 = \Delta[\Delta^2(\propto j)^2] = \Delta(2 \propto^2) = 0$

Similarly, $\Delta^3\left(\propto \hat{U}_j + \beta\right)^2 = \Delta^3(\propto j + \beta)^2 = \Delta[\Delta^2(\propto j + \beta)^2] = \Delta(2 \propto^2) = 0$

Consequently, $\Delta^k\left(\propto \hat{U}_j\right)^2 = \Delta^{k-3}[\Delta^3(\propto j)^2] = \Delta^{k-3}(0) = 0$

and $\Delta^k\left(\propto \hat{U}_j + \beta\right) = \Delta^{k-3}[\Delta^3(\propto j + \beta)^2] = \Delta^{k-3}(0) = 0$

proving $\Delta^k\left(\propto U_j\right)^2 = \Delta^k\left(\propto \hat{U}_j + \beta\right)^2 = 0$, for all positive integer $k \geq 3$. This established the proof of (iii).

Thus, we have seen clearly that:

Theorem 3.1 ⇒ $\Delta\left(\propto U_j\right)^2$ and $\Delta\left(\propto \hat{U}_j + \beta\right)^2$ are even .... (i)

$\Delta^2\left(\propto U_j\right)^2$ and $\Delta^2\left(\propto \hat{U}_j + \beta\right)^2 = \propto^2 (2!), \propto = 2$ .... (ii)

$\Delta^3\left(\propto U_j\right)^2$ and $\Delta^3\left(\propto \hat{U}_j + \beta\right)^2 = 0 \not\forall\, k \geq 3$ .... (iii)

In the sequel, we examine the computational disposition of $\Delta^k\left(\propto U_j\right)^p$ and $\Delta^k\left(\propto \hat{U}_j + \beta\right)^p$

for every integral j and positive integral k and p. The results are summarized as in the following theorem.

3.2 Main Theorem

Let $\propto U_j = \propto j$ and $\propto \hat{U}_j + \beta = \propto j + \beta$, where j is any integer. Then for arbitrary positive integer k and p, $\Delta^k\left(\propto U_j\right)^p$ and $\Delta^k\left(\propto \hat{U}_j + \beta\right)^p$ is given by:

$$\Delta^k(\propto U_j)^p = \Delta^k(\propto \widehat{U}_j + \beta)^p \begin{cases} 0, & \text{if } k > p \quad (a) \\ \sum_{i=1}^{p} \binom{p}{i} j^i, & \text{for } k = 1 \quad (b) \\ \propto^k k!, & \text{if } k = p \quad (c) \\ \text{even}, & \text{for } p \geq 1 \quad (d) \\ \text{even}, & \text{for } 2 \leq k < p \quad (e) \end{cases}$$

3.2.1 Proof of (a)

$$\Delta^2(\propto U_j) = \Delta(\Delta\propto j) = \Delta[\propto \Delta(j)] = \Delta[((j+1) - j)]$$

$$= \Delta[\propto (1)] = \Delta(\propto) = 0$$

Similarly,

$$\Delta^2(\propto U_j + \beta) = \Delta[\Delta(\propto j + \beta)] = \Delta[\propto \Delta j + \Delta\beta]$$

$$= \Delta[\propto ((j+1) - j)] = \Delta[\propto (1)]$$

$$= \Delta(\propto) = 0$$

$$\Rightarrow \quad \Delta^2(\propto U_j) = \Delta^2(\propto U_j + \beta) = 0.$$

So (a) is valid for k = 2 and p = 1, which are the least values for the respective exponents. Assume that (a) is valid for all pairs of integers $\hat{k}$, $\hat{p}$ for which $\hat{k} + \hat{p} \leq k + p$ for some positive integers $\hat{k}$ and $\hat{p}$ such that $p \geq 1$, $k \geq 2$, $p < k$.

Then $\Delta^k(\propto U_j)^p = \Delta[\Delta^k(\propto U_j + \beta)^p] = \Delta(0) = 0$

and $\Delta^{k+1}(\propto U_j + \beta) = \Delta[\Delta^k(\propto U_j + \beta)] = \Delta(0) = 0$

(by induction hypothesis hypothesis) = 0. Therefore, the validity of (a) is established.

3.2.2 Proof of (b)

$$(j+1)^p = \sum_{i=1}^{p} \binom{p}{i} j^i = \sum_{i=1}^{p-1} \binom{p}{i} j^i + \binom{p}{p} j^p$$

$$\Rightarrow \quad (j+1)^p - j^p = \sum_{i=1}^{p-1} \binom{p}{i} j^i \quad \text{proving (b)}$$

$$\Rightarrow \quad \Delta^1(\propto U_j)^p = \propto^p \left(\Delta U_j^p\right) = \propto^p \sum_{i=1}^{p-1} \binom{p}{i} j^i$$

$$\text{and} \quad \Delta^1(\propto U_j + \beta)^p = \sum_{i=1}^{p-1} \binom{p}{i} (\propto j)^{p-i} = \propto^p \sum_{i=1}^{p-1} \binom{p}{i} j^i$$

Observe that, since β = 1, implies that:

$$\propto \widehat{U}_j + \beta = \propto j + 1$$

$$\therefore \quad (\propto j + \beta)^p = \sum_{i=1}^{p-1} \binom{p}{i} (\propto j)^{p-i} \beta^i = \sum_{i=1}^{p-1} \binom{p}{i} (\propto j)^{p-i}$$

For simplicity of complexity of the odd form, we can logically express it as a single form with respect to even form, since they have same characteristic structures.

Claim that proof of even case is necessary and sufficient for the proof of the odd case.

Considering the even form for the remaining part of the proof ∵ proof even ⇔ proof of odd.

We have that:

$$\Delta(\propto U_j)^p = \Delta\left(\propto^p U_j^p\right) = \propto^p \left(\Delta U_j^p\right) = \propto^p (\Delta j^p)$$

$$= \propto^p [(j+1)^p - j^p] = \propto^p \sum_{i=1}^{p-1} \binom{p}{i} j^i$$

3.2.3 Proof of (c)

We examine $\Delta^k(\propto U_j)^p$: k =1

$\Rightarrow (\propto U_j) = \propto (\Delta j) = \propto (j + 1 - j) = \propto (1) = \propto = 2!$

$k = 2, \Delta^2(\propto U_j)^2 \quad = \propto^2 \Delta^2 U_j^2 = \Delta\propto^2 (\Delta j^2) = \propto^2 \Delta[(j+1)^2 - j^2]$

$= \propto^2 \Delta(j^2 + 2j + 1 - j^2) = \propto^2 \Delta(2j + 1)$

$= \propto^2 \Delta(2j + 1)$

$= 2 \propto^2 \Delta j + \Delta\propto^2$

$= 2 \propto^2 (j + 1 - j) = 0$

$= 2 \propto^2 = \propto^2 2! = 2^2.2!$

$\Rightarrow \Delta^2(\propto U_j)^2 = \Delta^2(\propto \widehat{U}_j + \beta) = \propto^2 2! = 2^2.2! = 8.$

by (ii) of theorem 3.1 ⇒ the theorem is valid for k ϵ {1, 2}. Assume the validity of the theorem for k ϵ {3,...,q} for some integer q ≥ 4. Then $\Delta^{q}(\propto^{q}\, Uj^{q}) = \propto^{q} q!$ ⇒ $\Delta^{q}(\propto^{q}\, Uj^{q}) = \propto^{q}(\propto^{q}\, Uj^{q}) = \propto^{q} q!$

by induction hypothesis. Finally, we need to prove that:

$\Delta^{q+1}(\propto^{q+1}\, Uj^{q+1}) = \propto^{q+1}(q+1)!$

Claim Δ^{r} is reproductive. Reproductive is understood to mean the following:

$$\Delta^{r}\sum_{j=1}^{n}\widehat{\propto}_{j}\, g_{j}(x) = \sum_{j=1}^{n}\widehat{\propto}_{j}\, \Delta^{r}\big(g_{j}(x)\big)$$

Where $\propto_{j}$ are arbitrary constants.

3.2.4 Proof of Claim

Consider $\Delta^{r}\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big), r = \{1, 2 \ldots\ldots\}$.

for r = 1, $\Delta\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big) = \Delta\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big)$

$$= \widehat{\propto}_{1}\,(j+k)^{k_1} + \widehat{\propto}_{2}\,(j+k)^{k_2} - \widehat{\propto}_{1}\, U_{j_1}^{k_1} - \widehat{\propto}_{2}\, U_{j_2}^{k_2}$$

$$= \widehat{\propto}_{1}\left[(j_1+1)^{k_1} - j_{i}^{k_1}\right] + \widehat{\propto}_{2}\left[(j_2+1)^{k_2} - j_{2}^{k_2}\right]$$

$$= \widehat{\propto}_{1}\, \Delta\big(j_{i}^{k_1}\big) + \widehat{\propto}_{2}\, \Delta\big(j_{2}^{k_2}\big)$$

$$= \widehat{\propto}_{1}\, \Delta\left(U_{j_i}^{k_1}\right) + \widehat{\propto}_{2}\, \Delta\left(U_{j_2}^{k_2}\right)$$

⇒ Δ is a reproductive ⇒ the claim is valid for 2 ≤ r ≤ t, for some integer t ≥ 3.

Then $\Delta^{t}\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big) = \widehat{\propto}_{1}\, \Delta^{t}\big(U_{j_1}^{k_1}\big) + \widehat{\propto}_{2}\, \Delta^{t}\big(U_{j_2}^{k_2}\big)$

Finally,

$$\Delta^{t+1}\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big) = \Delta^{t}\big[\Delta\big(\widehat{\propto}_{1}\, U_{j_1}^{k_1} + \widehat{\propto}_{2}\, U_{j_2}^{k_2}\big)\big]$$

$$= \Delta^{t}\big(\widehat{\propto}_{1}\, \Delta U_{j_1}^{k_1} + \widehat{\propto}_{2}\, \Delta U_{j_2}^{k_2}\big) = \widehat{\propto}_{1}\, \Delta^{t}\big(\Delta U_{j_1}^{k_1}\big) + \widehat{\propto}_{2}\, \Delta^{t}\big(\Delta U_{j_2}^{k_2}\big)$$

(by induction hypothesis)

$$= \widehat{\propto}_{1}\, \Delta^{t+1}\big(U_{j_1}^{k_1}\big) + \widehat{\propto}_{2}\, \Delta^{t+1}\big(U_{j_2}^{k_2}\big)$$

So the theorem is valid for r = t+1 and hence valid for all positive integer r.

Now, $\Delta^{q+1}(\propto^{q+1}\, Uj^{q+1}) = \Delta^{q}[\propto^{q+1}\, \Delta(Uj^{q+1})]$

$$= \Delta^{q}[\propto^{q+1}\,((j+1)^{q+1} - j^{q+1})]$$

$$= \Delta^q \left[\propto^{q+1} \sum_{i=1}^{q} \binom{q+1}{i} j^i \right]$$

$$= \Delta^q \left[\propto^{q+1} \sum_{i=1}^{q} \binom{q+1}{i} Uj^i \right]$$

$$= \propto^{q+1} \sum_{i=1}^{q} \binom{q+1}{i} \Delta^q (Uj^q) \quad \text{(by reproductive property)}$$

$$= \propto^{q+1} \binom{q+1}{i} \Delta^q (Uj^q) \quad \text{(by (a) of theorem 3.2)}$$

$$= \propto^{q+1} \binom{q+1}{i} q! \quad \text{(by the induction hypothesis)}$$

$$= \propto^{q+1} \binom{q+1}{i} !$$

Therefore, $\Delta^k(\propto^k U_j{}^k) = \propto^k k!$

For every positive integer k, proving (c).

3.2.5 Proof of (d)

$$\Delta(\propto U_j)^p = \propto^p [(j+1)^p - j]$$

$$\Rightarrow \qquad \propto^p [(j+1)^p - j] = \begin{cases} \text{even} - \text{even} = \text{even for even case} \\ \text{odd} - \text{odd} = \text{odd for odd case} \end{cases}$$

So in all cases $\Delta(\propto U_j)^p$ and $\Delta(\propto \widehat{U}_j + \beta)^p$ are even, proving (d).

3.2.6 Proof of (e)

Consider $\Delta^k(\propto U_j)^p$ for k ≥ 2, k < p. If k = 2, then the condition k < p ⇒ p ≥ 3.

$\Delta^2(\propto U_j)^p = \Delta[\Delta(\propto U_j)^p]$

$$= \propto^p \Delta \left[\sum_{i=1}^{p-1} \binom{p}{i} j^i \right] = \Delta \text{ (an even)from (a)and (b)}$$

$$= \propto^p \Delta[(j+1)^p - j^p] = [\Delta[(j+1)^p] - \Delta[j^p]] \propto^p$$

(by the reproductive property.

$$= \propto^p [(j+2)^p - (j+1) - [(j+1)^p - j^p]]$$

$$= \propto^p [(j+2)^p - 2(j+1)^p - j^p] = \begin{cases} \text{even} - \text{even} + \text{even} = \text{even (for even case)} \\ \text{even} - \text{even} + \text{even} = \text{even (for odd case)} \end{cases}$$

So in all cases, $\Delta^2(\propto U_j)^p$ and $\Delta^2(\propto \widehat{U}_j + \beta)^p$ is even for p ≥ 2.

Assume the validity of (e) for all positive integer k and p such that k + p ≤ m, for some positive integer m.

Then

$$\Delta^{k+1}(\propto U_j)^p = \Delta^k(\Delta(\propto U_j)^p)$$

$$= \Delta^k \propto^p [[(j+2)^p - 2(j+1)^p + j^p]]$$

$$= \Delta^k \propto^p \left[[(j+2)^p] - 2\Delta^k[(j+1)^p] + \Delta^k[j^p]\right]$$

(by the reproductive property of Δ^k)

$$\left(\Delta^k\left(U_{j+2}^p\right) - 2\Delta^k\left(U_{j+1}^p\right) + \Delta^k\left(U_j^p\right)\right) \propto^p$$

= even – even + even = even.

$$= \propto^p \Delta^k \left[U_{j+2}^p\right] - 2\propto^p \Delta^k \left[U_{j+1}^p\right] + \propto^p \Delta^k \left[U_j^p\right]$$

Finally, we examine:

$$\Delta^k(\propto U_j)^{p+1}.$$

$$\Delta^k(\propto U_j)^{p+1} = \Delta^k(\propto j)^{p+1} = \Delta^{k-1}[\Delta(\propto j)^{p+1}]$$

$$= \Delta^{k-1}[\propto^{p+1} (j+1)^{p+1} - 2(j+1)^{p+1} + j^{p+1}]$$

$$= \Delta^{k-1}\left[\propto^{p+1}\left[U_{j+2}^p\right] - 2 \propto^{p+1} \Delta^{k-1}\left[U_{j+1}^{p+1}\right] + \propto^{p+1} \Delta^{k-1}\left[U_j^{p+1}\right]\right]$$

= even – even + even = even.

(by the induction hypothesis). Since k – 1 + p + 1 = k + p ≤ m.

This completes the proof, that is the relation (e) hold for all +ve integers k and p for which k < p, k ≥ 2. Thus, the theorem is established.

3.3 Corollary

$$\Delta^k(\propto j)^k = \Delta^k\left(\propto^k j^k\right) = \propto^k \Delta^k j^k = \propto^k k!$$

For any integer j and for any positive integer k.

In other words, for any set of even and odd integers ($\propto U_j$ and $\propto \widehat{U}_j + \beta$), the k^{th} order difference of the k^{th} power of any integers is equal to $2^k k!$

The implication of (c) in theorem 3.2 is the following: "if any number of consecutive even or odd integers are raised to a positive integral power k, then the k^{th} order difference is equal to $2^k k!$

3.4 Remarks

The existence of the triddle between the difference operator Δ and the D operator (differential operator).

$$D^k(x^p) = 0 \text{ if } k > p$$

$$D^k(x^k) = k!$$

$$D^k(\propto^k x^k) = \propto^k k!$$

The coefficient of x^{p-k} in $D^k(x^p)$:

$$D^k(x^p) = \frac{p!}{(p-k)!} x^{p-k} \text{ for } k \geq 2, k < p \quad \text{is even.}$$

$$\Rightarrow \quad \propto^p D^k(x^p) = \frac{\propto^p p!}{(p-k)!} x^{p-k}, 2 \leq k < p \quad \text{is even}$$

IV. CONCLUSION

This article established the structures of finite orders with respect to powers of consecutive elements of even and odd sets. Specifically, the results reveal a similarity between the difference orders and the D operator powers of monomials with positive integral powers as reflected in (a) and (c) of theorem 3.2 and (I) and (II) of remark 3.4.

REFERENCES

1. Chase, P. E. The elements of arithmetic in which decimal and integral arithmetic are combined and taught inductively. U. Hunt and Son 1844.
2. Finite difference: 2015. Retrieved October, 2015. Available at: http://www.en.wikipedia.org/wiki/finite-diffference
3. McGraw Hill Dictionary of Scientific and technical terms. 6. E. Perfect power. The McGraw Hill Companies Inc, 2003. Retrieved November 23, 2015. Available at: www.encyclopedia2.the-free-dictionary.com/perfect+power
4. Ladan, Umaru Ibrahim Ukwu Chukwunenye and Apine Elijah. An investigation into the order of integral perfect powers. Ibrahim et al: AJOMCOR 10(1):12:18:2016.
5. Cambridge tracts in Mathematics 87. Exponential Diophantine equations, Cambridge University Press 1988: 169-183.
6. Andreescu, T. and Adrica D. Number theory. Birkhauser Boston, a part of springer science + business, Media, LLCl 2009. DOI: 10.1007/611856.2
7. Perfect Powers: 2015. Retrieved November 23, 2015. Available at: http://en.wikipedia.org/wiki/perfect-power

8. Perfect Powers of Successive Naturals: Can you always reach a constant difference? 2015 Retrieved November 23, 2015. Available at: http://mathstachexchange.com/questions/1345205/perfect-powers-of-successive-natural-can-you-always-reach-a-constant-difference
9. Catalan's Conjecture; 2002, Retrieved November 23, 2015. Available at: http://en.wikipedia.org/wiki/catalan%27sconjecture