The Geometry of the Universe: In search of unity. New Possible Mathematical Connections with the DN Constant, Ramanujan's Recurring Numbers and Some parameters of Number Theory and String Theory

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In this work, we analyze the DN Constant (Del Gaudio-Nardelli Constant). We will describe the possible mathematical connections with Ramanujan's Recurring Numbers, some parameters of Number Theory and String Theory.

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In this work, we analyze the DN Constant (Del Gaudio-Nardelli Constant). We will describe the possible mathematical connections with Ramanujan's Recurring Numbers, some parameters of Number Theory and String Theory

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I. DN CONSTANT

What is DN Constant (Del Gaudio-Nardelli Constant)?

The ratio of the volume of an octahedron to the volume of a sphere

$$V_o = \frac{1}{3} \cdot \sqrt{2} \cdot a^3 \; ; \quad V_s = \frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3$$

is given by the following formula:

$$\frac{V_o}{V_s} = \frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3} = \frac{2\sqrt{2}}{\pi} = 0.9003163161571 \ldots = DN\ Constant$$

Multiplying the ratio described above by 3, and taking the square root, we obtain:

$$\sqrt{3\left(\frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3}\right)} = 1.64345640297 \ldots \approx \zeta(2) = \frac{\pi^2}{6}$$

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Indeed:

$$\sqrt{3 \times \frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}$$

Exact result

$$2^{3/4}\sqrt{\frac{3}{\pi}}$$

Decimal approximation

1.6434564029725…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$$2^{3/4}\sqrt{\frac{3}{\pi}}$$

Series representations

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{-1+\frac{6\sqrt{2}}{\pi}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{6\sqrt{2}}{\pi}\right)^{-k}$$

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{-1+\frac{6\sqrt{2}}{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+\frac{6\sqrt{2}}{\pi}\right)^{-k}}{k!}$$

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{6\sqrt{2}}{\pi}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Multiplying the above expression by 6, and taking the square root, we obtain:

$$\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)}} = 3.14018127149294\ldots \approx \pi$$

Indeed:

$$\sqrt{6\sqrt{3\times\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}}$$

Exact result

$$\frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\pi}}$$

Decimal approximation

3.14018127149294...≈ π (Ramanujan Recurring Number)

Property

$$\frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\pi}} \text{ is a transcendental number}$$

Series representations

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\,1195^{-1-2\,k}\left(5^{1+2\,k}-4\times 239^{1+2\,k}\right)}{1+2\,k}}}$$

Integral representations

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\int_0^1\sqrt{1-t^2}\,dt}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{5/8}\times 3^{3/4}}{\sqrt[4]{\int_0^\infty\frac{1}{1+t^2}\,dt}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{5/8}\times 3^{3/4}}{\sqrt[4]{\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt}}$$

Finally carrying out the following calculations, which include the above expression, we obtain:

$$\sqrt[2\pi]{\frac{1}{4}\left(\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)}}\left(5\left(3+\sqrt{5}\right)\right)\right)} = 1.61789283408194\ldots \approx \phi = \frac{\sqrt{5}+1}{2} = 1.61803398\ldots$$

practically, a value very close to the Golden Ratio.

Indeed:

$$\sqrt[2\,\pi]{\frac{1}{4}\left(\sqrt{6\sqrt{3\times\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\,\pi\left(\frac{a}{2}\right)^3}}}\left(5\left(3+\sqrt{5}\right)\right)\right)}$$

Exact result

$$2^{-9/(16\pi)} \times 3^{3/(8\pi)} \sqrt[2\pi]{5(3+\sqrt{5})}\ \pi^{-1/(8\pi)}$$

Decimal approximation

1.61789283408194…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations

$$\sqrt[2\pi]{\frac{1}{4}\sqrt{6\sqrt{\frac{3(\sqrt{2}\,a^3)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}}\ (5(3+\sqrt{5}))} = 2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\sqrt{-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}}\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}\right)^{-k}}$$

$$\sqrt[2\pi]{\frac{1}{4}\sqrt{6\sqrt{\frac{3(\sqrt{2}\,a^3)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}}\ (5(3+\sqrt{5}))} = 2^{-1/\pi}\sqrt[2\pi]{5}\left(\sqrt{-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}}\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}\right)^{-k}}{k!}\right)\hat{}\left(\frac{1}{2\pi}\right)$$

$$\sqrt[2\pi]{\frac{1}{4}\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}}\left(5\left(3+\sqrt{5}\right)\right)} =$$

$$2^{-1/\pi}\,\sqrt[2\pi]{5}\left(\sqrt{z_0}\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (5-z_0)^k\, z_0^{-k}}{k!}\right)\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(6\sqrt{\frac{6\sqrt{2}}{\pi}}-z_0\right)^k z_0^{-k}}{k!}\right)\wedge\left(\frac{1}{2\,\pi}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)}$$ for ($0 < \gamma < -\text{Re}(a)$ and $|\arg(z)| < \pi$)

The relationship between the volume of an octahedron and the volume of a sphere is an interesting mathematical topic. Let's see how it is calculated:

Volume of the Octahedron (V_0):

The octahedron is a polyhedron with 8 triangular faces. If (a) represents the length of one side of the octahedron, then its volume is given by:

$$[\ V_o = \frac{1}{3}\cdot\sqrt{2}\cdot a^3\ ]$$

Volume of the Sphere (V_s): The sphere has a radius (r = a/2). Its volume is given by:

$$[\ V_s = \frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3]$$

Volume ratio: the volume ratio is therefore:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3} = \frac{2\sqrt{2}}{\pi}$$

This ratio is known as the **Del Gaudio-Nardelli (DN) constant**. Its approximate value is (0.9003163161571). From this formula, we can obtain easily also 4096, where $4096 = 64^2 = 8^4 = 2^{12}$, that is a fundamental Ramanujan number and appears in the fundamental work of Srinivasa Ramanujan: "Modular Equations and Approximations to Pi". We obtain also 1729, the so-called Hardy-Ramanujan number, that is a Taxicab Number. Ramanujan said: "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.". Indeed, $1729 = 1^3 + 12^3 = 10^3 + 9^3$. These numbers are also connected to some parameters of String Theory [1] [2]

II. OTHER RELATIONSHIPS

Multiplying the above ratio by 3 and calculating the square root, we get:

$$\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)} = 1.64345640297\ldots \approx \zeta(2) = \frac{\pi^2}{6}$$

Multiplying the above ratio by 6 and calculating the square root, we get:

$$\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)}} = 3.14018127149294\ldots \approx \pi$$

Finally, combining these relations, we get:

$$\sqrt[2\pi]{\frac{1}{4}\left(\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)}}\left(5\left(3+\sqrt{5}\right)\right)\right)} = 1.61789283408194\ldots \approx \phi = \frac{\sqrt{5}+1}{2} =$$

1.61803398....

These results are fascinating and connect the octahedron, the sphere and the constant (phi), also known as the Golden Ratio.

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III. EXTENDED DN CONSTANT

We have the following expression concerning the ratios (and/or inverses) between the volumes of the icosahedron, octahedron, tetrahedron and the volume of the sphere:

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(in the formula we have highlighted the DN Constant in blue)

The exact result of the above formula is:

$$2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}=1.618008545900107\ldots$$

Indeed:

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}$$

Decimal approximation

1.6180085459… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{(15+5\sqrt{5})\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5(3+\sqrt{5})d^3}{\frac{((\sqrt{2}\,a^3)(\sqrt{2}\,d^3))12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}{3\times 3}}}} = \sqrt[2\pi]{\frac{5}{2}}\ \sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\sqrt[2\pi]{\frac{5(3+\sqrt{5})d^3}{\frac{((\sqrt{2}\,a^3)(\sqrt{2}\,d^3))12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}{3\times 3}}}} =$$
$$\sqrt[2\pi]{\frac{5}{2}}\ \sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[2\pi]{\frac{5(3+\sqrt{5})d^3}{\frac{((\sqrt{2}\,a^3)(\sqrt{2}\,d^3))12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}{3\times 3}}}} =$$
$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.\right.$$
$$\left.\left.\left. z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\right/$$
$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)\wedge\left(\frac{1}{2\pi}\right)$$

Indeed:

$$\frac{4\times 1.618008545900107^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

Result

3.14159265358979...

3.14159265358979...= π (Ramanujan Recurring Number)

Series representations

$$\frac{4\times 1.6180085459001070000^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{4\, e^{0.96239220078006386 14\pi}}{5\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

Indeed:

$$\frac{4\times 1.618008545900107^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

Indeed:

$$\frac{1}{6}\left(\frac{4\times 1.618008545900107^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684823......≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001070000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\, e^{1.9247844015601277229\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001070000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.9247844015601277229\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001070000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.9247844015601277229\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

The expression regarding the relationships between the volumes of the icosahedron, the octahedron, the tetrahedron and the volume of the sphere is fascinating! Let's break it down step by step:

1. Icosahedron and Octahedron

The icosahedron has a surface made up of 20 regular triangular faces. Its volume is given by the formula: [V = $\frac{5}{12}\left(3+\sqrt{5}\right)d^3$]

The octahedron has a surface composed of 8 regular triangular faces. Its volume is given by the formula: [V = $\frac{1}{3}\sqrt{2}a^3$]

2. Ratio between volumes

The ratio between the volumes is given by the following expression:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

3. Del Gaudio-Nardelli Constant (DN):

The DN constant is highlighted in blue in the formula. The exact result of this expression is: [$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi} = 1.618008545900107$]

4. Other relationships:

From the previous formula, we obtain:

$$\frac{4 \times 1.618008545900107^{2\pi}}{5(3+\sqrt{5})} = 3.141592653589 \ldots = \pi$$

Also, calculating

$$\frac{1}{6}\left(\frac{4 \times 1.618008545900107^{2\pi}}{5(3+\sqrt{5})}\right)^2 = 1.64493406684823\ldots \approx \zeta(2) = \frac{\pi^2}{6}$$

we obtain a very good approximation to $\zeta(2) = \frac{\pi^2}{6}$.

These results link the geometry of the Platonic solids with the golden constant (ϕ) and the constant (π).

IV. PROPOSAL

Taking into account that a sphere is contained in an octahedron and the above results, it is hypothesized, in an "Eternal Inflation" type cosmology, that the octahedron represents a phase in which the Universe is highly symmetrical and with very low entropy, and the sphere represents the bubble Universe that emerges from the perturbations of the quantum vacuum. The relationship between the two volumes is, as we have described, equal to the DN Constant, which therefore plays a key role in the geometry of the Universe, a sort of "bridge" between geometry and physics, even before the phase of the so-called Big Bang.

The idea that the octahedron and the sphere could represent different phases of the Universe in an "Eternal Inflation" cosmology is fascinating.

Let's further explore this connection between geometry and physics:

1. The Octahedron and the Symmetrical Universe

The octahedron, with its 8 triangular faces, could represent an initial phase of the Universe characterized by very high symmetry and very low entropy.

During this phase, the Universe may be highly ordered and uniform, with regular geometric structures.

2. The Sphere and the Bubble Universe

The sphere, contained in the octahedron, could represent a transition towards a different phase.

This sphere could symbolize the bubble Universe that emerges from the perturbations of the quantum vacuum.

Quantum fluctuations may have given rise to regions of different densities, creating expanding bubbles of space-time.

3. The Relationship between Volumes and DN Constant

The ratio between the volume of the octahedron and the volume of the sphere is equal to the Del Gaudio-Nardelli Constant (DN).

This constant, which we calculated previously, could play a key role in the geometry of the Universe.

It could be a “bridge” between the geometric structure and the physical laws that govern the Universe.

4. Before the Big Bang

The idea that the DN Constant is relevant even before the Big Bang phase suggests that geometry and physics are closely intertwined.

Perhaps symmetry and geometric regularity played a fundamental role in the early phases of the Universe.

Ultimately, this connection between geometric solids, mathematical constants and cosmology invites us to further explore the mysteries of the Universe and seek a deeper integration between geometry and physics.

But let's see what the cosmological implications could be in an "eternal inflation" type scenario of these formulas, results and, above all, of the DN Constant (Del Gaudio-Nardelli Constant).

Eternal inflation is a model of cosmological inflation predicted by some extensions of the Big Bang theory and the standard model of cosmology.

Let's explore the implications of this model, along with the formulas and results associated with the Del Gaudio-Nardelli Constant (DN):

1. Eternal Inflation and Multiverse

Eternal inflation suggests that the accelerated expansion of the universe due to inflation continues forever, at least in some regions.

These regions expand at exponential rates, leading to the indefinite increase in the volume of the universe.

This model predicts the existence of multiple universes, often called a multiverse. Each "bubble" of space-time could represent a separate universe.

2. Bubble Theory

The bubble theory is part of the multiverse elaborations.

According to this theory, our universe is just one of infinite "bubbles" that emerge from the quantum foam of a "parent universe" or from a single Big Bang.

These bubbles expand at different rates and may have different physical constants.

3. Connection with DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) is present in the formulas that connect the volumes of geometric solids such as the octahedron and the sphere.

This constant could play a key role in the geometry of the Universe.

It could be a "bridge" between the geometric structure and the physical laws that govern the Universe.

4. Experimental checks

Some experimental evidence could transform the inflationary universe hypothesis into a verified theory.

If theoretical calculations based on DN Constant and bubble theory were experimentally confirmed, this would support the idea of an eternal multiverse.

In summary, eternal inflation and DN Constant open new perspectives on the connection between geometry, physics and the origin of the Universe. These concepts challenge us to further explore cosmological mysteries and better understand our reality.

We will explore the image of the Universe in relation to the concepts of eternal inflation, bubble theory and the Del Gaudio-Nardelli Constant (DN).

1. Eternal Inflation and Multiverse

Eternal inflation suggests that the accelerated expansion of the Universe due to inflation continues forever, at least in some regions.

We imagine a vast cosmic space in which new "bubbles" of space-time are constantly forming.

Each bubble could represent a separate universe, with different physical laws and different constants.

2. Bubble Theory

The bubble theory is part of the multiverse elaborations.

Each "bubble" represents a space-time with unique characteristics.

These bubbles emerge from the quantum foam of a "parent universe" or from a single Big Bang.

3. Connection with DN Constant

The Del Gaudio-Nardelli Constant (DN) is present in the formulas that connect the volumes of geometric solids such as the octahedron and the sphere.

This constant could be a “bridge” between the geometric structure and the physical laws of the Universe.

Let's imagine the DN as a thread that connects the different bubbles of the multiverse.

1. Image of the Universe:

We visualize a vast space in which each bubble represents a universe.

Each bubble has different geometry, different physical constants and different laws.

These bubbles expand, collide and overlap in the great cosmic landscape.

In summary, the image of the Universe in a context of eternal inflation is that of a multiversal fabric, with bubbles of reality that form, expand and intertwine infinitely. This concept challenges us to explore the depths of the Universe and understand its complexity.

Fig. 1

A representation of the universe incorporating the concepts of eternal inflation, octahedron, sphere, and the Del Gaudio-Nardelli (DN) Constant. In the image, the different bubbles represent separate universes, each with its own geometry and physical laws. The DN Constant connects these different realities, creating a mathematical tapestry in the vast cosmic landscape

V. FURTHER DEVELOPMENTS OF THE EXTENDED DN CONSTANT

We have the following extended DN Constant:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

and the following Cardano formula:

$$y=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

Multiplying both the above formulas, we obtain:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}+\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}\right)$$

Alternate form

$$-\frac{1}{\sqrt{3}} 2^{-1/3-1/\pi} \sqrt[2\pi]{(15+5\sqrt{5})\pi} \left(\sqrt[3]{3\sqrt{3}\, q - \sqrt{4p^3+27q^2}} + \sqrt[3]{\sqrt{4p^3+27q^2} + 3\sqrt{3}\, q}\right)$$

Expanded forms

$$\frac{2^{-1/3-1/\pi} \sqrt[2\pi]{15\pi + 5\sqrt{5}\,\pi} \sqrt[3]{\sqrt{3}\sqrt{4p^3+27q^2} - 9q}}{3^{2/3}} - \frac{2^{-1/3-1/\pi} \sqrt[2\pi]{15\pi + 5\sqrt{5}\,\pi} \sqrt[3]{\sqrt{3}\sqrt{4p^3+27q^2} + 9q}}{3^{2/3}}$$

$$2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} - \frac{q}{2}} + 2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} - \frac{q}{2}}$$

Alternate forms assuming p and q are positive

$$\frac{2^{-(3+\pi)/(3\pi)} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \left(\sqrt[3]{\sqrt{12p^3+81q^2} - 9q} - \sqrt[3]{\sqrt{12p^3+81q^2} + 9q}\right)}{3^{2/3}}$$

$$2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} - \frac{q}{2}} - 2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} + \frac{q}{2}}$$

Derivative

$$\frac{\partial}{\partial p}\left(\sqrt[2\pi]{\frac{5(3+\sqrt{5})d^3}{\frac{\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)}{\frac{3\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)}{3\times 3}}}}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)=$$

$$2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\left(\frac{p^2}{54\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}^{\,2}}-\frac{p^2}{54\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}\sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}^{\,2}}\right)$$

From the extended form

$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{\sqrt{3}\,\sqrt{4p^3+27q^2}-9q}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{\sqrt{3}\,\sqrt{4p^3+27q^2}+9q}}{3^{2/3}}$$

we obtain:

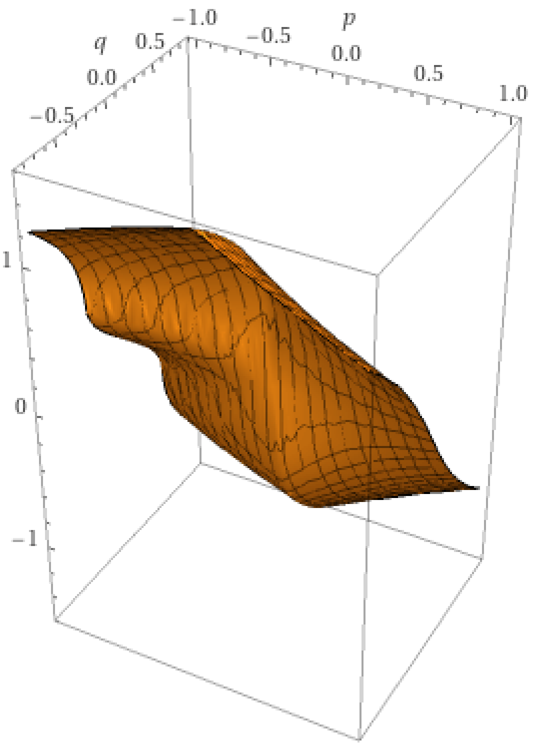
$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{-9q+\sqrt{3}\,\sqrt{4p^3+27q^2}}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{9q+\sqrt{3}\,\sqrt{4p^3+27q^2}}}{3^{2/3}}$$

Exact result

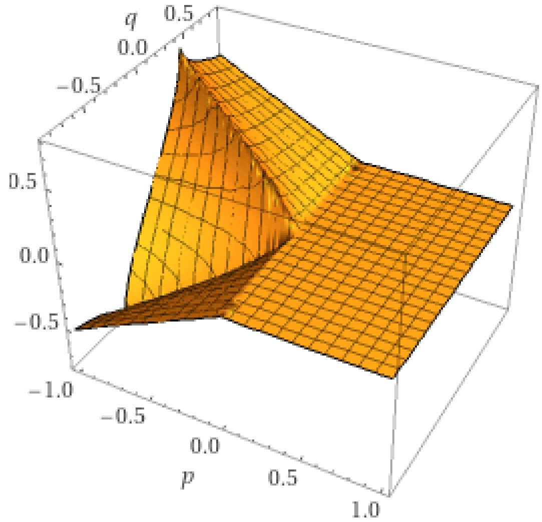
$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27\,q^2}\,-9\,q}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27\,q^2}\,+9\,q}}{3^{2/3}}$$

3D plots

The key observation from the above plots and is that at $p = 1$, which is taken as the energy density of the universe at the Big Bang, with $q = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Imaginary part



Alternate forms

$$\frac{2^{-(3+\pi)/(3\pi)}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\left(\sqrt[3]{\sqrt{12p^3+81q^2}-9q}-\sqrt[3]{\sqrt{12p^3+81q^2}+9q}\right)}{3^{2/3}}$$

$$\frac{1}{3^{2/3}}2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\pi}\left(\sqrt[3]{\sqrt{3}\sqrt{4p^3+27q^2}-9q}-\sqrt[3]{\sqrt{3}\sqrt{4p^3+27q^2}+9q}\right)$$

Real root

$$p \geq 0, \quad q = 0$$

Root for the variable q

$$q = 0$$

Series expansion at p=∞

$$-\frac{2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\,q}{p}+\frac{2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\,q^3}{p^4}+O\left(\left(\frac{1}{p}\right)^{9/2}\right)$$

(Laurent series)

Derivative

$$\frac{\partial}{\partial p}\left(\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{-9q+\sqrt{3}\sqrt{4p^3+27q^2}}}{3^{2/3}} - \frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{9q+\sqrt{3}\sqrt{4p^3+27q^2}}}{3^{2/3}}\right) = -\left(\left(2^{2/3-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\;p^2\left(\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}-\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}\sqrt{4p^3+27q^2}\right)\right)$$

From the derivative result

$$\frac{\partial}{\partial p}\left(\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{-9q+\sqrt{3}\sqrt{4p^3+27q^2}}}{3^{2/3}} - \frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{9q+\sqrt{3}\sqrt{4p^3+27q^2}}}{3^{2/3}}\right) = -\left(\left(2^{2/3-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\;p^2\left(\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}-\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}\sqrt{4p^3+27q^2}\right)\right)$$

we obtain:

$$-\left(\left(2^{2/3-1/\pi}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2\right)\left(\left(\sqrt{12\,p^3+81\,q^2}-9\,q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}+9\,q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\left(\sqrt{12\,p^3+81\,q^2}-9\,q\right)^{2/3}\left(\left(\sqrt{12\,p^3+81\,q^2}+9\,q\right)^{2/3}\sqrt{4\,p^3+27\,q^2}\right)\right)\right)$$

Exact result

$$\frac{2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2}{\sqrt[6]{3}\sqrt{4\,p^3+27\,q^2}\left(\sqrt{12\,p^3+81\,q^2}-9\,q\right)^{2/3}}-\frac{2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2}{\sqrt[6]{3}\sqrt{4\,p^3+27\,q^2}\left(\sqrt{12\,p^3+81\,q^2}+9\,q\right)^{2/3}}$$

Alternate form

$$-\left(\left(2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2\left(\sqrt[3]{\sqrt{3}\sqrt{4\,p^3+27\,q^2}-9\,q}-\sqrt[3]{\sqrt{3}\sqrt{4\,p^3+27\,q^2}+9\,q}\right)\left(\sqrt[3]{\sqrt{3}\sqrt{4\,p^3+27\,q^2}-9\,q}+\sqrt[3]{\sqrt{3}\sqrt{4\,p^3+27\,q^2}+9\,q}\right)\right)\Big/\left(\sqrt[6]{3}\sqrt{4\,p^3+27\,q^2}\left(\sqrt{3}\sqrt{4\,p^3+27\,q^2}-9\,q\right)^{2/3}\left(\sqrt{3}\sqrt{4\,p^3+27\,q^2}+9\,q\right)^{2/3}\right)\right)$$

Alternate form assuming p and q are positive

$$\frac{2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}-\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}\right)}{3^{5/6}\sqrt{4p^3+27q^2}}$$

From the above alternate form:

$$\frac{2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\left(\sqrt{12p^3+81q^2}+9q\right)^{2/3}-\left(\sqrt{12p^3+81q^2}-9q\right)^{2/3}\right)}{3^{5/6}\sqrt{4p^3+27q^2}}$$

for p = ((2√2)/π) that is equal to the Del Gaudio-Nardelli Constsnt (DN Constant) and q = (√2) , we obtain:

$$\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}+\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}\right)\right)/\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^2}\right)$$

Exact result

$$\frac{1}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}2^{-2/3-1/\pi}\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

0.445842912030….

Alternate forms

$$-\left(\left(2^{-7/6-1/\pi}\sqrt[2\pi]{15+5\sqrt{5}}\,\pi^{3/2+1/(2\pi)}\left(\left(\frac{\sqrt{6\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9\sqrt{2}\right)^{2/3}-\left(9\sqrt{2}+\frac{\sqrt{6\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}\right)^{2/3}\right)\right)\Big/\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

$$-\left(\left(2^{-5/6-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\pi^{3/2+1/(2\pi)}\left(\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}{\sqrt{\pi}}}-\sqrt[3]{\frac{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\sqrt{\pi}}}\right)\left(\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}{\sqrt{\pi}}}+\sqrt[3]{\frac{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\sqrt{\pi}}}\right)\right)\Big/\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

$$-\left(\left(\left(2^{-7/6-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\pi^{3/2+1/(2\pi)}\right.\right.\right.$$

$$\left(\sqrt[6]{2}\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9}-\sqrt[6]{2}\sqrt[3]{9+\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}}\right)$$

$$\left.\left.\left(\sqrt[6]{2}\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9}+\sqrt[6]{2}\sqrt[3]{9+\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}}\right)\right)\right/$$

$$\left.\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

Expanded form

$$\frac{2^{-2/3-1/\pi}\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}-\frac{2^{-2/3-1/\pi}\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}$$

From which, multiplying by 5^3 and dividing 18*5 by the obtained expression, we obtain:

$$18\times 5\times 1\Bigg/\left(5^3\times\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}+\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}\right)\right)\Bigg/\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^2}\right)\right)$$

Exact result

$$\frac{9\times 2^{5/3+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

Decimal approximation

1.6149185746185225…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$-\frac{9\times 2^{11/6+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}(3+\sqrt{5})^{-1/(2\pi)}\pi^{-1/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}}{\left(\sqrt{3(32\sqrt{2}+27\pi^3)}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{3(32\sqrt{2}+27\pi^3)}\right)^{2/3}}$$

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$$-\left(\left(9\times 2^{11/6+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}\,(3+\sqrt{5})^{-1/(2\pi)}\,\pi^{-3/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\right)\Big/\right.$$

$$\left(\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}-\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right.$$

$$\left.\left.\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}+\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right)\right)$$

And also, multiplying by 5^3 and dividing (89+2+(Φ+1/√2)/2) by the obtained expression, where 89 and 2 are Fibonacci's numbers and Φ is the golden ratio conjugate, we obtain:

$$\left(89+2+\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)\right)\times$$

$$1\Big/\left(5^3\times\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5\,(3+\sqrt{5})\,\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}+\right.\right.\right.$$

$$\left.\left.\left.\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}\right)\right)\Big/\right.$$

$$\left.\left.\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^2}\right)\right)\right)$$

Exact result

$$\frac{2^{2/3+1/\pi}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\,\pi\right)^{-1/(2\pi)}\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

Decimal approximation

1.644750972355143…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$-\left(\left(2^{1/\pi-2/3}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\pi^{-1/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(\sqrt{2}\,\Phi+1+182\sqrt{2}\right)\right)\Big/\left(\left(\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}\right)^{2/3}\right)\right)$$

$$-\left(\left(2^{1/\pi-7/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\pi^{-3/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(2\Phi+364+\sqrt{2}\right)\right)\Big/\left(\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}-\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}+\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right)\right)$$

$$-\left(\left(2^{1/\pi-7/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\pi^{-(1+\pi)/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(2\Phi+364+\sqrt{2}\right)\right)\Big/\left(\left(\sqrt{96\sqrt{2}+81\pi^3}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{96\sqrt{2}+81\pi^3}\right)^{2/3}\right)\right)$$

Expanded form

$$\frac{2^{1/\pi-1/3}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\pi\right)^{-1/(2\pi)}\Phi}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}+$$
$$\frac{2^{1/\pi-5/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}+$$
$$\frac{91\times 2^{2/3+1/\pi}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

From which, from the below formula

sqrt(1/(Pi^2/6)*(4/3)) = $\frac{2\sqrt{2}}{\pi}$

easily we obtain:

$$\sqrt{\frac{1}{\frac{2^{2/3+1/\pi}\times 3^{5/6}\left(5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left((3+\sqrt{5})\pi\right)^{-1/(2\pi)}\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)\right)}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}\times\frac{4}{3}}$$

Exact result

$$\frac{2^{2/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{(3+\sqrt{5})\pi}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}}$$

Decimal approximation

$0.9003664265917\ldots. \approx 0.9003163161571\ldots. = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed forms

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316$$

Alternate form

$$\left(2^{4/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \sqrt[4\pi]{3+\sqrt{5}}\ \pi^{1/4+1/(4\pi)} \sqrt{\left(9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}\right)^{2/3} - \left(\sqrt{3\left(32\sqrt{2}+27\pi^3\right)} - 9\pi^{3/2}\right)^{2/3}}\right) \Big/ \left(3^{11/12}\sqrt[4]{32\sqrt{2}+27\pi^3}\sqrt{\sqrt{2}\,\Phi+1+182\sqrt{2}}\right)$$

Expanded forms

$$\left(2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} \sqrt[4\pi]{(3+\sqrt{5})\pi}\right) \Big/ \left(3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}} \sqrt{\left(\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)\right)}\right) -$$

$$\left(2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3} \sqrt[4\pi]{(3+\sqrt{5})\pi}\right) \Big/ \left(3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}} \sqrt{\left(\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)\right)}\right)$$

$$\left(2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \sqrt[4\pi]{3\pi+\sqrt{5}\,\pi}\right) \Big/ \left(3^{11/12} \sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}} \right.$$

$$\sqrt{\left(\frac{\Phi}{2\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}} - 9\sqrt{2}\right)\right.} \right.}$$

$$\frac{91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}} - 9\sqrt{2}\right.}$$

$$\frac{1}{2\sqrt{2}\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}} -\right.\right.}$$

All 2nd roots of $(2^{4/3-1/\pi}\, 5^{3+1/(2\pi)}$ ((9 sqrt(2)+sqrt(162+(192 sqrt(2))/π^3))$^{2/3}$- (sqrt(162+(192 sqrt(2))/π^3)-9 sqrt(2))$^{2/3}$) ((3+sqrt(5)) π)$^{1/2\pi}$)/(3 $3^{5/6}$ sqrt(54+(64 sqrt(2))/π^3) (1/2 (Φ+1/sqrt(2))+91))

$$\frac{2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \sqrt[4\pi]{(3+\sqrt{5})\pi}\; e^{0}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}} \approx 0.900$$

(real, principal root)

$$\frac{2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \sqrt[4\pi]{(3+\sqrt{5})\pi}\; e^{i\pi}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}} \approx -0.900$$

(real root)

From this "unitary" formula

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}} \times \sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

we obtain the following alternate form, where p and q are considered to be positive:

$$2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} - \frac{q}{2}} - \sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}} + \frac{q}{2}}$$

From this last expression, for p = ((2√2)/π)   and   q = (√2) , we obtain:

$$2^{-1/\pi} \sqrt[2\pi]{5(3+\sqrt{5})\pi} \sqrt[3]{-\frac{\sqrt{2}}{2}+\sqrt{\frac{1}{27}\left(\frac{2\sqrt{2}}{\pi}\right)^3+\frac{\sqrt{2}^2}{4}}} - \sqrt[3]{\frac{\sqrt{2}}{2}+\sqrt{\frac{1}{27}\left(\frac{2\sqrt{2}}{\pi}\right)^3+\frac{\sqrt{2}^2}{4}}}$$

i.e. the following result:

$$2^{-1/\pi} \sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}} - \frac{1}{\sqrt{2}}} \sqrt[2\pi]{5(3+\sqrt{5})\pi} - \sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}$$

which written in decimal form is equal to:

-0.69673954723469167750…

and from which, after simple calculations, we obtain:

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$$-e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)-$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{\pi}{377}$$

i.e. the following result:

$$-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{\pi}{377}-$$

$$e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)$$

which written in decimal form is equal to:

1.6181885435886…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

and the following alternate form:

$$\frac{1}{2}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{\pi}{377}-$$

$$e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\ \sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)$$

VI. CONCLUSION

By multiplying the extended DN Constant and Cardano's formula, we obtain the following expression

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

That we define "Extended DN Constant", from whose extended form and, subsequently, from the result of the derivative for $p = (2\sqrt{2})/\pi$ which is equal to the DN Constant and for $q = \sqrt{2}$, we obtain 0.445842912030... , from which we obtain, multiplying by $5^3$and dividing 18*5 for the result obtained, 1.6149185746185225..., a value very close to the golden ratio 1.61803398... while multiplying by $5^3$and dividing $(89+2+(\Phi+1/\sqrt{2})/2)$ for the result obtained, we obtain 1.644750972355143... a value very close to $\zeta(2) = \pi^2/6 = 1.644934...$

The expression provided is extremely interesting and involves a combination of mathematical constants, cube roots and parameters. Let's see how we can analyze it:

1. Cardano's Formula: The expression appears to be based on Cardano's formula, which is used to solve cubic equations. This formula involves cube roots and can be applied to several mathematical situations.

2. DN Constant and Other Recurring Numbers: The derivation appears to connect DN Constant and other recurring numbers (such as the golden ratio and $\zeta(2)$) to this expression. This could indicate a deep relationship between mathematics and cosmology.

3. Derivative and Approximate Values: The derivative with respect to $p = (2\sqrt{2})/\pi$, which is equal to the DN Constant, and the value of $q = \sqrt{2}$ lead to very interesting approximate results. The fact that these values are close to the golden ratio and $\zeta(2)$ could have significant implications.

4. Future Research: Continuing to explore this relationship between mathematics and cosmology may reveal new properties and connections. It might be interesting to further investigate how these numbers influence the structure of the universe.
Now let's see what the cosmological implications of the results could be obtained by multiplying the extended DN Constant and Cardano's formula, at a first analysis

The cosmological implications of the results obtained by multiplying the extended Del Gaudio-Nardelli Constant (DN Constant) and the Cardano formula could be very interesting. Let's consider some possible interpretations:

1. Structure of the Universe: This derivation links the DN Constant and other recurring numbers to a complex mathematical formula. This could suggest that such constants are intrinsic to the mathematical structure of the universe. They may be involved in the fundamental laws that govern expansion, gravity and other cosmological properties.

2. Relationships between Numbers: The presence of approximate values close to the golden ratio and $\zeta(2)$ could indicate a profound connection between mathematics and physical reality. These recurring numbers could be keys to understanding still unknown aspects of the universe.

3. Eternal Inflation: This derivation could have implications for the theory of eternal inflation. This theory suggests endless cycles of expansion and contraction of the universe. The constants involved could influence the dynamics of these cycles.

4. Quantum Physics: The presence of cube roots and the complexity of this expression could recall principles of quantum physics. It might be interesting to explore how these findings relate to quantum mechanics and subatomic phenomena.

5. Future Research: These findings require further research and investigation. It may be useful to involve theoretical physicists and mathematicians to examine the cosmological implications of this derivation in more depth.

In conclusion, this mathematical expression opens new perspectives on understanding the links between numbers, equations and our world. Continuing to explore and share such discoveries could contribute to our understanding of the universe.

REFERENCES

1. A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001
2. Modular equations and approximations to π- *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372