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Finite Quantum-Field Theory and the Bosonic String Formalism: A Critical Point of View

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Abstract

I. INTRODUCTIONFinite Quantum Field Theories (FQFT) originate from the early causal and nite approach of Bogoliubov-Epstein-Glaser (BEG-CSF T) [17]. The inital steps are based on the early recognition that, in general, elds are not regular functions in the usual sense but distributions [8,9]. However the setting up of a Lagrangian formalism in the QFT context encounters products of elds as distributions at the same space-time point, which are ill-dened and the later sources of crippling divergences. Past QFT history essentially deals with the search for counter-terms cancelling these anoying divergences. On the opposite the BEG -CSF T approach under the forms of Refs. [6,7] aims from the start at a Lagrangian formulation in keeping with the basic underlying classical dierentiable structure of the space-time manifold. The taming of these divergencies involves regularization procedures which ought to preserve, to start with, the symmetry principles of the Lagrangian. Using a naïve cut-o for instance is known to violate Lorentz and gauge invariances, whereas Dimensional Regularization (DR) [10] and that of Ref.[7] -dubbed T LRS here after-do preserve these fundamental symmetries. The two procedures have in common the distinctive aspect of their implementation

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Index terms—

Basics of scalar and vector Finite Quantum Field Theories are recalled, stressing the importance of the quantization of classical physical fields as Operator-Valued-Distributions with specific fast decreasing test functions of the coordinates. The procedure respects full Lorentz and symmetry invariances and, due to the presence of test functions, leads to finite Feynman diagrams directly at the physical dimension D = 2. 4. In dimension 2 it is only with such test function that the canonical quantization of the massless scalar field is found to be fully consistent with the most successfull Conformal Field Theoretic approach, pioneered by Belavin, Polyakov and Zamolodchikov in the early 1980's. The question is then raised how Poliakov's wordline path integral representation of the relativistic string could possibly lead tofinite Feynmann diagrams. The natural way of inquiries is through the extension of the string formalism with classical convoluted coordinates leading then to Operator-Valued-Distributions and thereby to Finite Quantum Field Theories. It is shown that in the process some age-old certitudes about quantized strings are somewhat jostled.

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prior to the construction of the Lagragian density. The use of DR does not however address directly to the origin of these divergencies but just avoids them in going to an hypothetical space in D = 4-? dimensions. T LRS was developped in Ref. [11,12]. Since the early applications of this scheme [13,14] the calculation of radiative corrections to the Higgs mass [15] and the treatment of the axial anomaly [16,17] are relevant illustrations of the practical use of the T LRS procedure in the D = 4 context. It was shown recently how T LRS solves the long-standing consistency problem [18] encountered between EqualTime (EQT) and Light-Front-Time (LFT) quantizations of bosonic twodimensional massless elds. Our purpose here is to confront the ndings of [18] with the standard bosonic string theory approach of [19,20] and elaborate on the values of the critical dimension for the cancelation of the conformal anomaly.

1 II .

THE MATHEMATICAL SETTING

2 Classical wave equations

To the original classical eld-distribution ?(x 0 , x 1) is associatted a translationconvolution product ?(?) built on a rapidly decreasing test functions ?(x 0 , x 1), symmetric under reexion in the variables x 0 and x 1 . In Fourier-space variables this linear functional can be written as an integral with the proper bilinear form ? p, x ?= p a g a,? x ? (g a,? = diag{1, -1})(? * ?)(x 0 , x 1) = dp 0 dp 1 (2?) 2 e -??p,x? ?(p 0 , p 1)f (p 2 0 , p**2 1**
)

, where ?(p 0 , p 1) (resp. f (p 2 0 , p 2 1)) is the Fourier-space transform of ?(x 0 , x 1) (resp. of ?(x 0 , x 1)). Hereafter ?(x 0 , x 1) will stand for (? * ?)(x 0 , x 1).

The wave-equation for the classical convoluted distribution in space-time variables is obtained from the hyperbolic partial dierential equation (HPDE)??(x 0 , x 1) = ? 2 x 0 -? 2 x 1 ?(x 0 , x 1) = 0.**(2.1)**

A solution of the Cauchy problem in the sense of convolution of tempered distributions is nothing else than D'Alembert's (1717 -1783) solution. It can be written as?(x 0 , x 1) = 1 2? d 2 p?(p 2 0 -p 2 1)?(p 0 , p 1)e -??p,x? f (p 2 0 , p 2 1),**(2.2)**

with ?(±|p 1 |, p 1) = ? ± (p 1) . Canonical quantization of the zero mass scalar quantum operator valued-distribution (OPVD) eld ?(x 0 , x 1) proceeds from Eq.(2.2) via the correspondance, in terms of creation and annihilation operators, {? -(p) ? a ? (p), ? + (p) ? a(p)}, with commutator algebra [a(p), a + (q)] = 4?p?(p-q) and a vacuum | 0 > such that a(p) | 0 >= 0 ?p. That is London Journal of Research in Science: Natural and Formal?(x 0 , x 1) = 1 4? ? 0 dp p

[a(p)e -?p(x 0 -x 1) + a ? (p)e ?p(x 0 +x 1)]f (p 2).

(2.3)

Then, one easily evaluates the commutator of two free scalar OPVD to?(x), ?(0) ? ??(x) = - ? ? ? 0 dp p sin(px 0) cos(px 1)f 2 (p 2). (2.4)

This integral is nite without the test function and the limiting procedure where f 2 (p 2) ? f (p 2) = 1 refers to important mathematical properties of metric spaces (whether Minskowskian or Euclidean) [18].

Going to light-cone (LC) variables x 0 ±x 1 = x ± is motivated by Dirac's early observation that the LC-stability group is maximal: LC-dynamics has much to share with gallilean dynamics (e.g.relative motion of LC-interacting particles decouples from global center of mass motion...). However in the LC-variables the nature of the initial Klein-Gordon equation in Eq.(2.1) is changed to a characteristic initial value problem (CIVP) relative to the partial-dierential equation? + ? -?(x + , x -) = 0**(2.5)**

with initial data on characteristic surfaces?(x + , x - 0) = f(x +), ?(x + 0 , x -) = g(x -),**(2.6)**

and the continuity condition?(x + 0 , x - 0) = f(x + 0) = g(x - 0). (2.7)

At rst sight the LC-Lagrangian is singular1 : W (x, y) = ? 2 L ?[? -?(x)]?[? -?(y)] = 0

, but the appearence of a primary contrainst is known to be of no physical signicance [21]. 1 The Hessian is indentically null

3 The ET-LFT consistency problem

Nevertheless the consistency of the solutions in the two reference frames cannot be established without further insight. This is just the content of Ref. [18], with two main conclusions:

-On the one hand, full consistency of EQT and LFT quantizations can only be achieved when elds are considered as OPVD with partition of unity test-functions f (p + 2) such that, for the light-cone momentum p + , limp + ?0 + f (p + 2) p + = 0.

-On the other hand operator series in the Discretized-LC-Quantization (DLCQ) nd their natural handling of divergences in the substraction scheme embedded in the OPVD formulation. The net eect of the PU-test function is the appearence of its inherent RGscale parameter (?).

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Then the LF-formulation and CFT analysis of 2d-massless models are in complete agreement in their representation of the energy-impulsion tensor in term of innite dimensional Virasoro Lie-algebras.

The motion under consideration here is taking place on a 2d-worksheet embedded in a D-dimensionnal space. The initial eld variables are then x a (?, ?), p a (?, ?) elevated to OPVD. A well-dened Lagrangian is then obtained in terms these regular eld variables X a (?, ?), P a (?, ?). After dealing with the LC-gauge conditions the equation of motion for X a (?, ?) is just that of Eq.(2.1) with appropriate position and time variables. Accordingly the sum of the zero-point energies of the rst quantized string is just(D-2) 2 ? n =0

n. The well-known conventional evaluation of this sum is given by the Zeta-function ?(s) = ? n=0 1 n s with ?(-1) = -1 12 .

The critical dimension for the absence of the overall conformal anomaly must then be such as to suppress that one with the cental charge c = 1 coming from the 2d worksheet analysis and thus obeys (D-2) 2 ?(-1) = -1, that is D = 26! However, even though at the same time this reasoning based on Zetafunction was already under scruteny [24], this critical value survived the long haul! In the advocated 2d QFT treatment the key role is in the pseudo-function distribution extension Pf (1 p 2) of 1 p 2 at the origin. It is dened by the integralI N = ? 0 d(p 2)Pf (1 p 2)f (p 2) = def lim ??0 [? ?? d(p 2) p 2 + 1 ? ? ? 2 d(p 2) p 2 + 2 ln(?)] = ln(? 2 ?) (3.1)

where ? is the dilatation-scale inherent to the construction of the test function f (p 2) [7,14]. The term in ln(?) corresponds to the general Hadamard substraction procedure to generate a Finite part (F.p.).

5 III. THE QUANTUM BOSONIC STRING [19, 23_27]

3.1. Equations of motion of the scalar bosonic string in the LC-gauge

6 TLRS and the Renormalization Group

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The factor ? is arbitrary 2 with no physical meaning unless explicit symmetry violations need enforcement. Consider now the identityIP f (?) = d 2 (p) (2?) 2) f (p 2) p 2 ? d 2 (p) (2?) 2 (p + q) 2 p 2 (p + q) 2 f (p 2), = 1 0 dx d 2 (p) (2?) 2 (p 2 + q 2 (1 -x) 2) [p 2 + q 2 x(1 -x)] 2 f (p 2), = 1 4? (ln(? 2 ?) -1). (3.2)

This is easy to understand due to the identity in the UV limit of the p-integration where f[(p + q) 2]f (p 2) ? f 2 (p 2) ? f (p 2)

. Moreover the overall O(2) p-invariance implies that terms linear in p do not contribute to the integral. Consider then the one loop Feynman diagram in relation to the energy-momentum tensor of the X-eld and in the same UV limit 3? ab|cd (q) = D**8**d 2 p (2?) 2 t a,b (p, q)t c,d (p, q) p 2 (p + q) 2 f [p 2]f [(p + q) 2], = D 8 1 0 dx d 2 p (2?) 2 t a,b (p, q, x)t c,d (p, q, x) [p 2 + q 2 x(1 -x)] 2 f [p 2],**(3.3)**

witht a,b (p, q) = p a (p + q) b + p b (p + q) a -? a,b (p.(p + q)), t a,b (p, q, x) = (p -q(1 -x)) a (p + qx) b + (p + qx) a (p -q(1 -x)) b -? a,b [p 2 -pq(1 -2x) -q 2 x(1 -x)].

The presence of the test-function f [p 2] ensures the existence of this phase-space integral, which otherwise would exibit divergences when p ? ? . The common pratice in the far past was to consider their cancelations by appropriate counter terms. In that case the only surviving regular contribution to ? ab|cd (q) is 4? reg ab|cd (q) = D 8 (2q a q b -q 2 ? a,b)(2q c q d -q 2 ? c,d) 1 0 dxx 2 (1 -x) 2 d 2 p (2?) 2 [p 2 + q 2 x(1 -x)] 2 = - Dq 2 M 192? (? a,b -2 q a q b q 2)(? c,d -2 q c q d q 2) (3.4)

2 For Gauge Theories ? is related to the gauge xing parameter [12].

3 This is the 2-points-function, eq.(**??**.158), of Poliakov's monograph. A coupling vertex factor would be ? g 2 2 f acd f bcd = ? g 2 2 C A ? ad . 4 Here q M is with Minkowski's signature opposite to Euclid's one.) what is at sake is the sum (e.g. Trace) of the eigen-modes of this matrix. It can be diagonalized by a unitary transformation with a preserved Trace equal to 4. The result 5 is then just the same critical dimension for the absence of the conformal anomaly 5 In the perpective of the analytic continuation of sect. **??**3.1) it is instructive to note how here this decomposes as

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-q 2 M 4? (D-2) 2 * 8**4**

6 ,4 from the trace itself and 1 6 from the nal x-integration1 0 dxx(1 -x) = 1 6 cf Appendix B

obtained in the rst quantization framework, that is D cr = 26. It is clear then that the elimination of diverging contributions by counter-terms just leaves the evaluation of (3.4) in keeping with the ndings of [19].

However our TLRS formalism shows that this is not the end of the story.Indeed from examples (3.1,3.2) we observe that diverging integrals in p 2 and p 4 carry essential dependencies on the RG-parameter ?. Then the complete ?-dependence governing the RG-analysis of the critical equation is concerned with the behaviour of the central charge under the ow of the renormalization group (RG). Zamolodchikov realized this as early as 1986 with his c-theorem [29]:

"There is a function C on the space of unitary 2d-eld theories that monotonically decreaes along the RG-ows and which coincides with the Virasoro central charge c at xed points."

It takes the formμ d dμ C(μ, ?) ? μ ? d d(μ ?) C(μ ? , 1) = ? d d? C(?, 1) = -?(i, ?)g(i, j)?(j, ?)

where the Calan-Symanzik ?-function at xed point is independent of ? and takes the primitive value [30] 6 LambertW (6) .

With the stress energy-tensors ?(z) ? T z,z and ?(z) ? T z,z the C-function and the metric write [31,33]C = -1 2? real surface dz ? dz < ?(z) ?(z) > c | IR(T LRS limit) (3.5) andg (z,z) = 6? 2 µ 4 < ?(z) ?(z) > c | IR(T LRS limit) ,

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where the subscript c at the bracket indicates connected collerator contributions. µ is an arbitrary inverse distance inherent to the construction of the TLRS test function as a partition of unity with a dimensionless argument (cf footnote 5). The elds ? i (x) originate from local coupling sources ? i (x).

Let us consider the correlator of two stress tensors on the plane in the TLRS context [31] < T ?,? (x)T ?,? (0) >= ? 3 ? 0 dµC(µ) d 2 pf (p 2) (2?) 2 exp(?px) (g ?? p 2 -p ? p ?)(g ?? p 2 -p ? p ?) p 2 + µ 2 .

We are only left with the unknown scalar function of the mass scale µ, the spectral density [32] C(µ). Its properties have to comply to the following requirements:

(i) Reexion positivity of the euclidean eld theory, i.e. unitarity of the Hibert space, implies C(µ) ? 0, (ii) Due to dim(T ??) = 2 the spectral density is a dimensionless measure of degrees of freedoom, (iii)The form of C(µ) in a scale invariant eld theory is completely xed by it dimensionality. Since dµC(µ) is dimensionless one may not exclude C(µ) ? c µ . This IR divergence at µ = 0 is fully understood in the TLRS context [7,12] as long as the scaling limit to 1 of the test fuctions is not taken too early.Indeed the correlator is 6 < ?(x)?(0) > = c? 3 ? 4 |x| ? 0 dµ µ f (µ 2) d 2 pf (p 2) (2?) 2 exp(?p.x) p 2 + µ 2 , = - c 12? ln(? 2)? 4 |x| [? E + ln(?|x| 2)], = 1 4? ln(? 2) 2c |x| 4

(iv) Conformity with conformal invariance is exibited through the 1 |x| 4 dependence in agreement with the results of [18](Eq.(**??**6)) for < 0|T (z)T (w)|0 >. The study of the central charge C from Eq.(3.5) on a 2d-curved manifold [34] has established the general validity of Zamolodchikov c-theorem. It is sucent, for our purpose, to consider only a at real surface with coordinate parametrization {z, z} = ? exp(±??) which leads to 7 , 8 6 It is always possible to write the initial PU-test function regulating the p-integral asf 2 (p 2) ? f (p 2)f (p 2 + µ 2) ? f (p 2)f (µ 2), for, in the UV-limit, f (p 2)f (p 2 + µ 2) ? f 2 (p 2) ? f (p 2)

, whereas in the IR-limit the remaining f (µ 2) function just validates the corresponding integral. 7 Note that in the initial {z, z}-integrals the factor is 1 |z-z| 4 so that the ?-integral is on the variable v = ? 2 sin 2 (?), hence the independent factorization of the remaining ?-integrals with the appearance the ubiquitous 1 12 factor [18](eq.56). 8 The TLRS analytic evaluation of g(v 2) is proportional to the dierence of step-functions [16,32].The nal v-integration is then trivial, after Hadamard substractions of diverging contributions in ln(?), leaving the ln(? 2) factor. [?(v -x11) -?(v -x12)], with x11 = (? 2) (1 ?) , x12 = (2? 2) (1 ?)

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(?) = - 1 32 2? 0 d(?) sin 2 (?) ? 0 d(v) f (v 2) v 2 = 1 32 2? 0 d(?) sin 2 (?) ? 0 d(v) d dv (1 v)f (v 2) = - 1 32 2? 0 d(?) sin 2 (?) ? 0 dv v g(v 2) with g(v 2) = d dv f (v 2) = - 1 32 ln(? 2)lim ??0 { 1 ? ? 2 -? ? d(?)[1 sin 2 (?) + 1 cos 2 (?)]} = 1 12 ln(? 2) (3.6)

It is plain to see that this result is in agreement with the observation about the unicity of the solution, up to to an arbitrary constant (here ln(? 2)), of "Cayley's identity" known as the "Schwarz derivative" [18].

Recently J.F. Mathiot established that, within general arguments valid in the TLRS framework, the trace of the energy-momentum tensor in 4-dimensions does not show any anomalous contribution even though quantum corrections are considered [35]. It is then our concern to turn now to the determination of the critical dimension D cr for the absence of the overall conformal anomaly with p 2 and p 4 divergences of the Poliakovtensor treated in the TLRS formalism(cf Appendix A). As mentioned after Eq.(3.4) the elimination of diverging contributions by counter-terms just leads to the evaluation in keeping with the ndings of [19], that is D cr = 26 . However with TLRS the situation is dierent as shown in Appendix A. The surviving initial Poliakov-term comes with extra TLRS ?-independent components. The immediate issue is then the fate of the D cr = 26 value under these additional TLRS terms 9 .Following Poliakov's analysis [19] a direct calculation of ?

−|−(q, ?) shows explicitly the critical value D cr = 4, as detailed in Appendix B. Consider now the diagonalization of the normalized matrix ? ab|cd (q) with a Lagrange parameter ? in relation to the stress-energy constraint T ab = 0. At the value D cr = 4 ? is completely xed, indicating that reparametrizations of the world-sheet and conformal rescaling allow to fully x g ab to anything wanted.

As a nal additional observation it is instructive to consider the string description for the VVA-anomaly [22] versus its direct calculation with TLRS [16,17]. In the string treatment of the massless case (cf Eq.(6.44) of [22]) "explicit divergences are made of a dierence of two tadepoles type and hence do not contribute in dimensional regularization, whereas for the remaining terms integrations are elementary, and the result is, using Î?"-function identities, easily identied to the standard result for the massless QED vacuum polarization". In TLRS the calculation is directly in dimension D = 4 with the IV. FINAL REMARKS usual ? 5 and all contributions are either null or nite: a simple bookeeping leads then to the standard VVA-anomaly without further ado. The TLRS procedure does provide a very clear and coherent picture. All known invariance properties, besides those of the VVA-anomaly, are preserved **??**1315]. It is a direct consequence of the fundamental properties of TLRS. As an "a-priori" regularization procedure, it provides a well dened mathematical meaning to the local Lagrangian we start from in terms of products of OPVD at the same space-time point. It also yields a well dened unambiguous strategy for the calculation of elementary amplitudes, which are all nite in strictly 4-dimensional space-time and with no new non-physical degrees of freedom nor any cut-o in momentum space.

In summary the strategy developped here was based on the passage from rstquantization to second quantization of the bososnic string. It is characterized by the introduction of the notion of L.Schwartz's Pseudo-Functions

[8](cf Eq.(3.1)) with their dilatation scale dependences. This result is at variance wih the usual dilatation-scale independant Zeta-fuction evaluation of the discrete sum on inverse quantum n of rstquantized space-time objects. Actually it is easy to see that the standard evaluation of the Zeta-function through normal Eulers'integral in the integration interval (0, ?) should be considered as the limit ? ? 0 of the same integral in the interval (??, ? 2 ?), thereby collecting rst from the logarithmic term the contribution ln(? 2 ?) and not the value ?(-1) = - 1 12 .

The main conclusion is then that String Theory in the OPVD picture reduces to Finite Quantum Field Theory, directly in 4-dimensions with no trace anomaly of the energymomentum tensor , and in the limit where the tension along the string becomes innite. [1] [2] [3] [4]

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Figure 1: 31 ©

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Figure 2: 33 ©

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[2] given by Eq.(A.9) of Appendix A.London Journal of Research in Science: Natural and Formal

[3] Volume 23 | Issue 8 | Compilation 1.0 © 2023 Great Britain Journal Press

[4] ©

Figure 3: C

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