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Classification: MSC: 62G08, 62G20, 62M10

Language: English

Great Britain Journals Press

LJP Copyright ID: 925662
Print ISSN: 2631-8490
Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 23 | Issue 7 | Compilation 1.0

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I. INTRODUCTION

Model selection is a challenging step in statistical modelling. Modelling any data requires characterization of the associated data generating process (DGP). The DGP is unknown and therefore we face several admissible competing models. Model selection consists on selecting a model, which mimics such unknown DGP, from a set of admissible models according to a criterion. One retains the model which makes such criterion optimal, Tibshirani et al. (2015), Ferraty and Hall (2015) and Li et al. (2017). There exist various model selection criteria in the literature when admissible models have fully parametric specification, such as the Wald test, the likelihood ratio test, the Lagrange multiplier test, the information criteria and so on, see Hamilton (1994), Greene (2003) and Hooten and Hobbs (2015). The other case, when admissible models contain simultaneously parametric and nonparametric specifications, seems underdeveloped, Hendry et al. (2008). Encompassing tests appear to be helpful for the latter situation where an encompassing model is intended to account the salient feature of the encompassed model. Therefore, encompassing test can detect redundant models among the admissible models.

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Encompassing tests are based on two points, that the encompassing model ought to be able to explain the predictions and to predict some mis-specifications of the encompassed model, Hendry et al. (2008). We know that there are various considerations and developments of encompassing tests, we refer readers to Mizon (1984), Hendry and Richard (1989), Gouriéroux and Monfort (1995) and Florens *et al.* (1996). For an overview on the concept of encompassing tests, see Bontemps and Mizon (2008) and Mizon (2008). Applications of encompassing tests can be found inside the model selection procedure of general to specific (GETS) modelling developed by Hendry and Doornik (1994), Hoover and Perez (1999). For application in real data, see Nazir (2017).

Recently, Bontemps et al. (2008) have developed encompassing tests which cover large set of methods such as parametric and nonparametric methods. Among their results, encompassing tests for kernel nonparametric regression method are established. They provide asymptotic normality of the associated encompassing statistics under the independent and identically distributed hypothesis (i.i.d). We extend their results by relaxing the independent hypothesis. We then focus on processes with some dependence structures. This extension lies on the generalization of encompassing test to dependent processes.

The paper is organized as follows. In section 2, we provide an overview of encompassing test. In section 3, we study the asymptotic behaviors of the encompassing test associated to the linear parametric modelling and the kernel nonparametric method. In last section, we conclude.

II. ENCOMPASSING TEST

This section introduces the encompassing test and then builds the corresponding encompassing hypothesis. So, given two regression models $\mathcal{M}_1$ and $\mathcal{M}_2$, we are interested in knowing if the model $\mathcal{M}_1$ can account the result of model $\mathcal{M}_2$. In other words, we want to know if $\mathcal{M}_1$ encompasses $\mathcal{M}_2$ or in a short notation $\mathcal{M}_1\mathcal{E}\mathcal{M}_2$. Testing such a hypothesis will be done using the notion of encompassing test.

Generally speaking, model $\mathcal{M}_1$ encompasses model $\mathcal{M}_2$, if the parameter $\theta_{\mathcal{M}_2}$ of the latter model can be expressed in function of the parameter $\theta_{\mathcal{M}_1}$ of the former model. In other words, let $\Delta(\theta_{\mathcal{M}_1})$ be the pseudo true value of $\theta_{\mathcal{M}_2}$ on $\mathcal{M}_1$. In general, the pseudo-true value is defined as the plim of $\hat{\theta}_{\mathcal{M}_2}$ on $\mathcal{M}_1$, Bontemps et al. (2008). For more discussion on pseudo-true value associated with the KLIC[1], we refer to Sawa (1978) and Govaerts et al. (1994) among others. The encompassing statistic is given by the difference between $\hat{\theta}_{\mathcal{M}_2}$ and $\Delta(\hat{\theta}_{\mathcal{M}_1})$ scaled by a coefficient a_n. Specification of the encompassing test will depend on the estimation of the regression method: parametric or nonparametric methods.

Let $S = (Y, X, Z)$ be a zero mean random process with valued in $\mathbb{R}\text{x}\mathbb{R}^d\text{x}\mathbb{R}^q$ where $d, q \in \mathbb{N}^*$. For $x \in \mathbb{R}^d$ and $z \in \mathbb{R}^q$, consider the two models $\mathcal{M}_1$ and $\mathcal{M}_2$ as the conditional expectations $m(x)$ and $g(z)$, respectively. These can be defined as follows:

$$\mathcal{M}_1 : \quad m(x) = E[Y|X = x] \quad and \quad \mathcal{M}_2 : \quad g(z) = E[Y|Z = z] \tag{2.1}$$

Moreover, the general unrestricted model is given by $r(x, z) = E[Y|X = x, Z = z]$.
Following the encompassing test in Bontemps *et al.* (2008), we are interested in testing the hypothesis that $\mathcal{M}_1$ encompasses $\mathcal{M}_2$, and then introducing the null hypothesis:

$$\mathcal{H} : E[Y|X = x, Z = z] = E[Y|X = x]. \tag{2.2}$$

This null states that $\mathcal{M}_1$ is the owner model, and $\mathcal{M}_2$ will be served on validating this statement and is called the rival model. We test this hypothesis $\mathcal{H}$ through the following implicit encompassing hypothesis:

$$\mathcal{H}^* : E[E[Y|X = x]/Z = z] = E[Y|Z = z]. \tag{2.3}$$

The following homoskedasticity condition will be assumed all along this work:

$$Var[Y|X = x, Z = z] = \sigma^2. \tag{2.4}$$

Moreover, a necessary condition for the encompassing test relies on the errors of both models where the intended encompassing model $\mathcal{M}_1$ should have smaller standard error than the encompassed model $\mathcal{M}_2$.

In general, $\mathcal{M}_1$ or $\mathcal{M}_1$ can be estimated using nonparametric or parametric regression method. We will consider these different situations when the processes $(S_n)_n$ are dependent. We begin by constructing the encompassing statistic associated to each of these four situations and then discuss their asymptotic behaviors.

III. ASYMPTOTIC BEHAVIOR OF THE ENCOMPASSING STATISTIC

We are interested on the asymptotic behavior of the encompassing statistic associated to the null hypothesis $\mathcal{M}_1\mathcal{E}\mathcal{M}_2$. We can encounter the following four situations: $\mathcal{M}_1$ and $\mathcal{M}_2$ are both estimated parametrically, $\mathcal{M}_1$ and $\mathcal{M}_2$ are both estimated nonparametrically, $\mathcal{M}_1$ is estimated nonparametrically and $\mathcal{M}_2$ parametrically and $\mathcal{M}_1$ is estimated parametrically and $\mathcal{M}_2$ nonparametrically. We will consider the kernel regression estimate for nonparametric methods and the linear regression for parametric methods. For both dependent processes, we will study and establish the asymptotic normality of the corresponding four encompassing tests.

[1]Kullback-Leiber Information Criterion

Consider a sample $S_i = (Y_i, X_i, Z_i)$, $i = 1, ..., n$, which can be viewed as realization of the random process $S = (Y, X, Z)$ with valued in $\mathbb{R}$x$\mathbb{R}^d$x$\mathbb{R}^q$ where $d, q \in \mathbb{N}^*$. We suppose that S_i, $i = 1, ..., n$ has a joint density f. Moreover, $\varphi(.,.)$, $\varphi(. \mid .)$ and $\varphi(.)$ will denote the joint, the conditional and the marginal densities of the process (Y, Z), respectively. That is, for $y \in \mathbb{R}$ and $z \in \mathbb{R}^q$, $\varphi(y, z)$, $\varphi(y \mid z)$ and $\varphi(z)$ correspond to the density of the following processes (Y, Z) at point (y, z), $(Y \mid Z = z)$ at point y and Z at point z, respectively. Similarly, h will denote the joint, the conditional and the marginal densities of the process (Y, X), according to the argument that it takes.

To get the asymptotic normality of the associated encompassing statistic, we need the following assumptions from Bosq (1998). The first assumption characterizes the dependence structure.

Assumption 3.1. *(S_t) is α-mixing with $\alpha(n) = O(n^{-\rho})$ where $\rho > \frac{\nu^2+4}{2\nu}$ for some positive ν.*

The next assumption collects regularity conditions on the continuity and on the differentiability of the density functions.

Assumption 3.2. *φ and $g*\varphi$ are $C_{2,d}(b)$ for some real b where $C_{2,d}(b)$ the space of twice continuously differentiable real valued functions f, defined on $\mathbb{R}^d$, such that $||\varphi||_\infty \leq b$ and $||\varphi^{(2)}||_\infty \leq b$ with $\varphi^{(2)}$ denotes any partial derivative of order 2 for φ. Next, $Sup_{t \geq k}||\varphi(Z_1, Z_t)||_\infty < \infty$ and last, $\varphi(.)E[Y_1^2|Z_1 = .]$ is continuous at z.*

The last assumption concerns finiteness of the moments of $(Y_n, Z_n)_n$.

Assumption 3.3. *$||E[|Y_1|^{4+\nu}|Z_1 = .]||_\infty < \infty$; $E[|Z_1|^{4+\nu}] < \infty$ for some positive ν; $Sup_{t\in\mathbb{N}}||E[Y_1^i Y_t^j|Z_t = ., Z_1 = .]||_\infty < \infty$ where $i \geq 0$, $j \geq 0$, $i + j = 2$.*

Throughout this section, we assume the existence of continuous version of the various joint and marginal density functions and of the three conditional means m, g and r. In addition, the square integrability will be assumed.

For more precision, $N(\mu, v)$ will denote the Gaussian distribution with mean μ and variance v. We now consider the first case that is the encompassing test when the two models $\mathcal{M}_1$ and $\mathcal{M}_2$ have parametric specification.

3.1 Parametric modelling for M1 and M2

Encompassing test for parametric modelling has been developed a lot in the literature. We discuss briefly one parametric encompassing test where models $\mathcal{M}_1$ and $\mathcal{M}_2$ have linear parametric specification. In that case, the two models $\mathcal{M}_1$ and $\mathcal{M}_2$ are given in relation (3.1) with the nesting model r:

$$\begin{aligned} &m(x) = \beta' x \ \ with \ \ \beta = (E[XX'])^{-1}E[XY] \\ &g(z) = \gamma' z \ \ with \ \ \gamma = (E[ZZ'])^{-1}E[ZY] \\ &r(x,z) = \alpha' w \ \ with \ \ \alpha = E[WW']^{-1}E[WY] \ \ and \ \ W = (X, Z). \end{aligned} \tag{3.1}$$

We can get the estimates $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\alpha}$ of the paramaters β, γ and α, respectively, using the sample $S_i = (Y_i, X_i, Z_i)$, $i = 1, ..., n$. Now, testing $\mathcal{M}_1\mathcal{E}\mathcal{M}_2$ corresponds to the test of the null hypothesis $\mathcal{H}$ where the conditional mean is just the linear projection. Therefore, the encompassing statistic of the null $\mathcal{M}_1\mathcal{E}\mathcal{M}_2$ can be written as follows.

$$\hat{\delta}_{\beta,\gamma} = \hat{\gamma} - \hat{\gamma_L}(\hat{\beta}), \tag{3.2}$$

where $\hat{\gamma_L}(\hat{\beta})$ is an estimate of the pseudo-true value $\gamma_L(\beta)$ associated with $\hat{\gamma}$ on $\mathcal{H}_1$. Remarking that the pseudo-true value is defined by $\gamma_L(\beta) = (E[ZZ'])^{-1}E[ZX']\beta$, we state in the following theorem the asymptotic behavior of the encompassing statistic in relation (3.2).

Theorem 3.1. *Assume that the relation 2.4 is satisfied. When the sample $S_i = (Y_i, X_i, Z_i)$, $i = 1, ..., n$ are i.i.d., then under $\mathcal{H}$, we get:*

$$\sqrt{n}\hat{\delta}_{\beta,\gamma} \rightarrow N(0, \sigma^2\Omega) \quad in \quad distribution \quad as \quad n \rightarrow \infty. \tag{3.3}$$

where $\Omega = Var(Z)^{-1}E[Var(Z \mid X)]Var(Z)^{-1}$.

For development on this asymptotic behavior of the encompassing statistic, we refer to Gouriéroux et al. (1983) and Mizon and Richard (1986). For recent discussion on this encompassing test for fully parametric case, Bontemps et al. (2008) is a good reference.

Development of the parametric encompassing test goes beyond independent processes in the literature. As encompassing test for dynamic stationary models and time series regressions have been discussed in Govaerts et al. (1994), Hendry and Nielsen (2006), among others.
Next, we will study the completely nonparametric case.

3.2 Nonparametric modelling for M1 and M2

We now consider the case where the two models $\mathcal{M}_1$ and $\mathcal{M}_2$ defined in (2.1) are estimated using nonparametric techniques. To test the hypothesis " $\mathcal{M}_1$ encompasses $\mathcal{M}_2$ ", we build the corresponding encompassing statistic and establish asymptotic property of such statistic.

Considering the functional estimates m_n and g_n of the unknown functions m and g in relation (2.1) respectively, we define the encompassing statistic as follows:

$$\hat{\delta}_{m,g}(z) = g_n(z) - \hat{G}(m_n)(z), \tag{3.4}$$

where $\hat{G}(m_n)$ is an estimate of the pseudo true value $G(m)$ associated with g_n on $\mathcal{H}$, which is defined by $G(m) = E[m \mid Z = z]$.

Using the sample $S_i = (Y_i, X_i, Z_i)$, $i = 1, ..., n$, the kernel regression estimates m_n of the function m, and g_n of the function g have the following expressions:

$$m_n(x) = \frac{\frac{1}{nh_{1n}^d}\sum_{i=1}^n K_1(\frac{x-X_i}{h_{1n}})Y_i}{\frac{1}{nh_{1n}^d}\sum_{i=1}^n K_1(\frac{x-X_i}{h_{1n}})} \quad g_n(z) = \frac{\frac{1}{nh_{2n}^q}\sum_{i=1}^n K_2(\frac{z-Z_i}{h_{2n}})Y_i}{\frac{1}{nh_{2n}^q}\sum_{i=1}^n K_2(\frac{z-Z_i}{h_{2n}})} \tag{3.5}$$

where h_{jn} and K_j, $j = 1, 2$ are window widths and kernel densities, respectively. The kernel densities satisfy

$$K_j(u) \geq 0 \ \ and \ \ \int K_j(u)du = 1 \ \ j = 1, 2. \tag{3.6}$$

We provide in the following, a theorem establishing the asymptotic convergence of the encompassing statistic.

Theorem 3.2. *Suppose that assumptions 3.1-3.3 hold. Moreover, suppose that relation (3.6) is satisfied. Then, under $\mathcal{H}$, we get:*

$$\sqrt{nh_{2n}^q}\hat{\delta}_{m,g}(z) \to N(0, \frac{\sigma^2 \int K_2^2(u)du}{\varphi(z)}) \quad in \quad distribution \quad as \quad n \to \infty. \tag{3.7}$$

$\varphi(z)$ is the marginal density of the Z at z and $\sigma^2 = Var[Y|X = x, Z = z]$.

Proof of theorem 3.2 The proof of this theorem will be based on the decomposition of the expression of the encompassing statistic into two parts as follows:

$$
\begin{aligned}
\sqrt{nh_{2n}^{q}}\hat{\delta}_{m,g}(z) &= \sqrt{nh_{2n}^{q}}(g_n(z) - \hat{G}(m_n)(z)) \\
&= \sqrt{nh_{2n}^{q}}(\sum_{t=1}^{n} \frac{K_2(\frac{z-Z_t}{h_{2n}})}{\sum_{t=1}^{n} K_2(\frac{z-Z_t}{h_{2n}})} Y_t - \sum_{t=1}^{n} \frac{K(\frac{z-Z_t}{h_n})}{\sum_{t=1}^{n} K(\frac{z-Z_t}{h_n})} m_n(x_t)) \\
&= \sqrt{nh_{2n}^{q}} \sum_{t=1}^{n} \frac{K_2(\frac{z-Z_t}{h_{2n}})}{\sum_{t=1}^{n} K_2(\frac{z-Z_t}{h_{2n}})} (Y_t - m(x_t)) \qquad (3.8) \\
&+ \sqrt{nh_{2n}^{q}} \sum_{t=1}^{n} \frac{K(\frac{z-Z_t}{h_n})}{\sum_{t=1}^{n} K(\frac{z-Z_t}{h_n})} (m(x_t) - m_n(x_t)) \\
&= C_1 + C_2.
\end{aligned}
$$

The first part C_1 coincides to the kernel regression of the residuals $\epsilon_t = Y_t - m(x_t)$ onto Z_t. When assumptions 3.1-3.3 hold, then under $\mathcal{H}$, we achieved the convergence in distribution of the first part to a normal distribution using Rhomari's result in Bosq (1998). The second part C_2 reflects the limit in probability of the supremum of the difference $m_n(x_t) - m(x_t)$ at $x_t \in \mathbb{R}^d$ scaled by $\sqrt{nh_n^q}$ and its convergence can be derived from the rate of covergence of the uniform convergence of the estimate $m_n(x_t)$ which has been provided by Bosq (1998).

3.3 Parametric modelling M1 vs nonparametric modelling M2

We consider the case that model $\mathcal{M}_1$ is a linear parametric model and $\mathcal{M}_2$ is estimated by kernel nonparametric technique. Therefore, the hypothesis $\mathcal{H}$ will have linear parametric specification. The encompassing statistic associated to the null $\mathcal{M}_1\mathcal{E}\mathcal{M}_2$ can be written as follows:

$$
\hat{\delta}_{\beta,g}(z) = g_n(z) - \hat{G}_L(\hat{\beta})(z), \qquad (3.9)
$$

where $\hat{G}_L(\hat{\beta})$ is an estimate of the pseudo-true value $G_L(\beta)(z)$ associated with g_n on $\mathcal{H}$, which is defined by $G_L(\beta)(z) = \beta' E[X \mid Z = z]$.

For the nonparametric specification of $\mathcal{M}_2$, we consider the estimate g_n as the kernel regression estimate of g given in (2.1). Since the rival model g is estimated using kernel method, the various assumptions on kernel density and window width will be maintained.

Even the process exhibits some dependences, we can still establish the asymptotic normality of the encompassing statistic defined in relation (3.9).

Theorem 3.3. *Assume that relation 2.4 and assumptions 3.1-3.3 are satisfied. Then, under $\mathcal{H}$ with linear specification and when the bandwidth h_{2n} satisfy kernel regularity condition, we get:*

$$\sqrt{nh_{2n}^q}\hat{\delta}_{\beta,g}(z) \to N(0, \frac{\sigma^2 \int K_2^2(u)du}{\varphi(z)}) \quad in \quad distribution \quad as \quad n \to \infty. \tag{3.10}$$

$\varphi(z)$ is the marginal density of the Z at z.

Proof of theorem 3.3 Using similar techniques as previously, we can write the encompassing statistic as follows:

$$\begin{aligned}
\sqrt{nh_{2n}^q}\hat{\delta}_{\beta,g}(z) &= \sqrt{nh_{2n}^q}(g_n(z) - \hat{G}_L(\hat{\beta})(z)) \\
&= \sqrt{nh_{2n}^q}(\sum_{t=1}^{n} \frac{K_2(\frac{z-Z_t}{h_n})}{\sum_{t=1}^n K_2(\frac{z-Z_t}{h_{2n}})} Y_t - \sum_{t=1}^{n} \frac{K(\frac{z-Z_t}{h_n})}{\sum_{t=1}^n K(\frac{z-Z_t}{h_n})} \hat{\beta}' X_t) \\
&= \sqrt{nh_{2n}^q} \sum_{t=1}^{n} \frac{K_2(\frac{z-Z_t}{h_n})}{\sum_{t=1}^n K_2(\frac{z-Z_t}{h_{2n}})} (Y_t - \beta' X_t) \qquad (3.11) \\
&+ \sqrt{nh_{2n}^q} \sum_{t=1}^{n} \frac{K(\frac{z-Z_t}{h_n})}{\sum_{t=1}^n K(\frac{z-Z_t}{h_n})} X_t'(\beta - \hat{\beta}) \\
&= D_1 + D_2.
\end{aligned}$$

When assumptions 3.1-3.3 hold, then under $\mathcal{H}$, D_1 converges in distribution to a normal law with mean zero and variance $\frac{\sigma^2}{\varphi(z)} \int K_2^2(u)du$, see Bosq (1998). Concerning D_2, using central limit theorem for linear processes in Peligrad and Utev (1997), the normality asymptotic of the linear process has been established. Therefore, this implies the normality asymptotic of $\sqrt{n}(\beta - \hat{\beta})$. The remaining expression in D_2 vanishes to zero as n tends to infinity. Thus, D_2 converges in distribution to zero. This completes the proof.

3.4 Nonparametric modelling M1 vs parametric modelling M2

We consider the owner model $\mathcal{M}_1$ to be estimated using a nonparametric method and the rival model $\mathcal{M}_2$ to be a linear parametric method. Therefore, the encompassing statistic associated to the null $\mathcal{M}_1 \mathcal{E} \mathcal{M}_2$ is given by:

$$\hat{\delta}_{m,\gamma} = \hat{\gamma} - \hat{\gamma}(m_n), \tag{3.12}$$

where $\hat{\gamma}(m_n)$ is an estimate of the pseudo-true value $\gamma(m)$ associated with $\hat{\gamma}$ on $\mathcal{H}$, which is defined by $\gamma(m) = (E[ZZ'])^{-1}E[Zm]$.

When the estimate of model $\mathcal{M}_1$ is obtained from the kernel regression and the model $\mathcal{M}_2$ is from linear parametric modelling, we summarize the asymptotic results in the following theorem.

Theorem 3.4. *Assume that relation (2.4) is satisfied. When the kernel K_1 and the bandwidth h_{1n} satisfy the usual regularity condition and when we have one of the following points:*

Assumption 3.1 holds and the kernel regression estimate m_n and the process $(Y_n, X_n)_n$ satisfy assumptions 3.2 and 3.3.
Then, under $\mathcal{H}$, we get:

$$\sqrt{n}\hat{\delta}_{m,\gamma} \to N(0, \Sigma) \quad in \quad distribution \quad as \quad n \to \infty. \tag{3.13}$$

where $\Sigma = plim_{n\to\infty} Var(\sqrt{n}\hat{\delta}_{m,\gamma})$.

Proof of theorem 3.4 We split the encompassing statistic $\sqrt{n}\hat{\delta}_{m,\gamma}$ into two parts. The first part yields $F_1 = \sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} Z_i Z_i)^{-1}(\frac{1}{n}\sum_{i=1}^{n} Z_i(Y_i - m(x_i)))$ which gives the asymptotic normality of the theorem, Peligrad and Utev (1997).

The second part is $F_2 = \sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} Z_i Z_i)^{-1}(\frac{1}{n}\sum_{i=1}^{n} Z_i(m(x_i) - m_n(x_i)))$. Again, we bound this by taking the supremum with respect to x_i. Thus, F_2 vanishes to zero from the uniform convergence of $m_n(x_i)$, Bosq (1998). This completes the proof of theorem 3.4.

We remark that we should be careful about mutual encompassing of both models which concerns the bijection of the pseudo true value function $G(.)$.

IV. CONCLUSION

We have considered encompassing test for functional parameters. As stated in Hendry et al. (2008) that the work of Bontemps *et al.* (2008) is the starting treatment of such type of encompassing test based on nonparametric methods. We have extended that work to dependent process.

When using kernel method and linear regression as estimator of conditional expectations, we have established asymptotic normality of the encompassing test for dependent processes. These results would be helpfull for analysing non-nested non-parametric and parametric models.

Development of encompassing test to nonparametric methods opens new research direction in theory as well as in practice. Application of the various results on real data would accelerate such development.

Funding statement : this work was performed as part of my employement at the Ministry of Higher Education and Scientific Research of Madagascar.

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