An Alternative for Time Series Models

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Two methods for construction of new stochastic processes with discrete time are presented. One of the methods employs as the defining tool 'triangular (more specifically 'pseudoaffine') transformations' which are extended from the Euclidean Rnto infinite dimension space. They transform any well-known discrete time stochastic process into the constructed one. The other, more flexible, method is the "method of parameter dependence", extended to infinite dimension. Properties of the obtained stochastic processes (by either method) indicate the possibility to apply them for financial analysis, as an alternative for the classical time series models. The advantage of the presented models over the existing ones first of all relies on expected better accuracy. This follows from the fact that the typically held assumption on Markovianity in the existing models can easily be relaxed. The defined processes may incorporate a quite long including, among others, the k-Markovian cases for k ≥ 2. Regardless of the non-Markovianity of the models they still are tractable in an analytical or numerical way.

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ABSTRACT

Two methods for construction of new stochastic processes with discrete time are presented. One of the methods employs as the defining tool is'triangular (more specifically 'pseudoaffine') transformations' which are extended from the Euclidean $R^n$to infinite dimension space. They transform any well-known discrete time stochastic process into the constructed one. The other, more flexible, method is the "method of parameter dependence", extended to infinite dimension. Properties of the obtained stochastic processes (by either method) indicate the possibility to apply them for financial analysis, as an alternative for the classical time series models. The advantage of the presented models over the existing ones first of all relies on expected better accuracy. This follows from the fact that the typically held assumption on Markovianity in the existing models can easily be relaxed. The defined processes may incorporate a quite long list including, among others, the k-Markovian cases for k ≥ 2. Regardless of the non-Markovianity of the models they still are tractable in an analytical or numerical way.

*The stochastic processes defined in this paper provide more flexible and more general tools than the existing time series models for modeling financial problems. Among others, they make it possible to incorporate the influence of environmental (explanatory) random variables on the underlying stochastic models' behavior. These additional features turn out to be describable by the method of parameter dependenc*e. Some suggestions for an associated preliminary statistical analysis are included.

Keywords: stochastic dependence, stochastic processes, alternative for time series financial models, parameter dependence method of construction, k-Markovianity.

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I. INTRODUCTION

In this work a pattern for construction of new stochastic models is proposed. The models modification and generalization of the classical time series frameworks for financial analysis (Tsay, 2005). As such they are considered a possible alternative to these known ones. They can be obtained by two different methods.

One of the methods employs triangular transformations (Filus & Filus& Arnold, 2010), as the defining tool and may therefore be more useful in a further statistical analysis and possible simulation studies. This method is described in Section 2 and 3. The other, described in Section 4, relies on application of the 'parameter dependence method' (Filus & Filus, 2012), (Filus & Filus, 2013), which is more flexible than the first method in the sense that it produces more models. The models obtained by either of the two methods are stochastic processes whose terms have financial meanings, especially the meaning of log returns for a single asset.

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All the stochastic processes obtained by the triangular transformations method may also be obtained by the parameter dependence (not conversely), but the possibility of a nice statistical and simulation analysis as provided by the transformations is sometimes lost. This was the reason both methods were introduced. Any of the two is very general.

The patterns employed allow us to define wide classes of conditional probability distributions_of any term X_t, given realizations $x_1, \ldots, x_{t-1}$ of all past terms $X_1, \ldots, X_{t-1}$ of the defined stochastic processes.

Notice, that such conditional distributions are very seldom explicitly given in efficient forms in the literature. The classical exception lies within the pattern of the multivariate normal case. The obtained conditional distributions are then used for further construction of joint probability distributions of all the random vectors $(X_1, \ldots, X_t)$, $t = 2, 3, \ldots$ if an initial distribution of X_1 is given.

Perhaps the most amazing fact that follows is the easy possibility of defining non-Markovian (as well as the Markovian) stochastic processes incorporating long pasts, and still analytically tractable. Additionally, the method of parameter dependence allows us to include into the model, typically occurring in practice, 'state random variables' that describe a "stochastic environment" in which the processes evolve over time.

The generality of these new models (from a financial perspective) inclined us rather to concentrate on the formulation of fundamental ideas as beginning to possibly new theories. Therefore, in order to avoid unnecessary dissipation, the number of examples was purposefully limited. Statistical analysis problems of the new stochastic models are only mentioned. Also references are limited, somewhat, especially because the results presented are possibly at first in a financial setting. However, somewhat similar, from a pure mathematical point of view, but generally different results were published in (Filus & Filus, 2008).

II. DEFINING TRANSFORMATIONS

Consider a sequence of log returns R_t of a single asset, $t = 0, 1, \ldots, T$; (Tsay, 2005), as given by the following sequence $T = 1, 2, \ldots$ of transformations:

$$R_0 = 0$$

$$R_1 = V_1(R_0)X_1 + B_1(R_0)$$

$$R_2 = V_2(R_0, R_1)X_2 + B_2(R_0, R_1)$$

$$R_T = V_T(R_0, R_1, \ldots, R_{T-1})X_T + B_T(R_0, R_1, \ldots, R_{T-1}),$$

$T = 1, 2, \ldots$, (1) where the random variables $X_1, \ldots, X_T$ are assumed to be independent and identically distributed. This is then a general white noise pattern which is a source of randomness for the considered log returns $R_1, \ldots, R_T, \ldots$. V_0 represents a nonnegative constant initial value, while the functions $V_1, \ldots, V_T$ are arbitrary positive and piecewise continuous with respect to each argument. If the variance of each random variable X_t is 1, then $V_1, \ldots, V_T$ will have the "conditional volatilities" interpretation conditioned on realizations of past returns $R_0, R_1, \ldots$ prior to a given R_t. Also conditioned on the same realizations of the past returns are the conditional expectations

$$E[R_t | R_0, R_1, \ldots, R_{t-1}] = B_t(R_0, R_1, \ldots, R_{t-1})$$

where $B_1, \ldots, B_T$ are arbitrary, piecewise continuous with respect to each argument, functions of realizations of past returns.

Example. The functions $V_t(\ )$ and $B_t(\ )$ ($t = 1, \ldots, T$) may be arbitrary continuous. However, in practical applications one could choose, for example, the following simple functions: $V_t(r_o, r_1, \ldots, r_{t-1}) = 1 + a_o r_o^2 + a_1 r_1^2 + \ldots + a_{t-1} r_{t-1}^2$, $B_t(r_o, r_1, \ldots, r_{t-1}) = b_o r_o + b_1 r_1 + \ldots + b_{t-1} r_{t-1}$,

where the coefficients $a_o, a_1, \ldots, a_{t-1}$ are real nonnegative and $b_o, b_1, \ldots, b_{t-1}$ are arbitrary real. These coefficients are to be statistically estimated.

Also, if appropriate, one can choose as model:

$$V_t(r_o, r_1, \ldots, r_{t-1}) = \exp[\, a_o r_o^2 + a_1 r_1^2 + \ldots + a_{t-1} r_{t-1}^2] \text{ and}$$

$$B_t(r_o, r_1, \ldots, r_{t-1}) = \exp[b_o r_o + b_1 r_1 + \ldots + b_{t-1} r_{t-1}]$$

with arbitrary real coefficients $a_o, a_1, \ldots, a_{t-1}$ and $b_o, b_1, \ldots, b_{t-1}$. Other examples of such functions can easily be given.

Returning to the main subject, notice that the sequence of the random vector transformations $(X_1, \ldots, X_T)$ □ $(R_1, \ldots, R_T)$ ($T = 1, 2, \ldots$) defined by (1), is the pseudoaffine version of sequence of triangular transformations R^T □ R^T, (Filus & Filus & Arnold, 2010). Here it is proposed to apply them as a general financial model_for values of log returns. This model can be seen as a slightly different version of time series, and is proposed to be named *"triangular model"*. Realize that all the transformations (1) are easily invertible, and their inverses are given as follows:

$$R_o = X_o,$$

$$X_1 = [\, R_1 - B_1(R_o)\,] / V_1(R_o)$$

$$X_2 = [\, R_2 - B_2(R_o, R_1)\,] / V_2(R_o, R_1)$$

$$X_T = [R_T - B_T(R_o, R_1, \ldots, R_{T-1})\,] / V_T(R_o, R_1, \ldots, R_{T-1})$$

$T = 1, 2, \ldots$. (1*) For realizations $x_1, \ldots, x_T$ and $r_o, r_1, \ldots, r_T$ of the underlying random variables, denoted by the corresponding capital letters, the jacobians, $J_T(r_1, \ldots, r_T) = \partial(x_1, \ldots, x_T) / \partial(r_1, \ldots, r_T)$, have the simple form of the inverse of the volatilities' products

$$J_T(r_o, r_1, \ldots, r_{T-1}) = [V_1(r_o)\, V_2(r_o, r_1) \ldots V_T(r_o, r_1, \ldots, r_{T-1})\,]^{-1}, \text{ (2) for each } T = 1, 2, \ldots .$$

One can see that if the sequence of probability densities (pdf) of the random vectors $(X_1, \ldots, X_T)$ is known (which is mostly the case), then from (1*) and (2) one immediately can derive the corresponding sequence of joint pdfs of the random vectors of the returns $(R_1, \ldots, R_T)$, $T = 1, 2, \ldots$.

In such a way, one defines a wide class of stochastic processes $\{R_T\}_{T = 1, 2, \ldots}$. (The Kolmogorov consistency theorem easily applies to this case.)

Consider these processes as *"modified time series"* processes for log returns $R_1, R_2, \ldots$. Clearly, the model given by (1) is heteroscedastic as the underlying conditional volatilities, $V_t(r_o, r_1, \ldots, r_{t-1})$, $t = 1, 2, \ldots$ (conditioned on elementary events $R_o = V_o = r_o, R_1 = r_1, \ldots, R_{t-1} = r_{t-1}$), are, in general, distinct.

It follows from (1) that the introduced model is, in general, not Markovian but still analytically tractable.

Actually, when using model (1) one can incorporate in each conditional pdf $g_T(r_T \mid r_1, \dots, r_{T-1})$ (at present time T) all the past information on the returns, and underlying calculations are still performable.

However, this computational advantage is overshadowed by limitations of a statistical nature. As T grows, the number of parameters to be estimated also grows without bounds, so some restrictions on the past must be provided. For that one can apply the notion of *k-Markovianity* that limits the past to the last k observations (k = 1, 2, ...). The case k = 1 means the ordinary Markovianity.

The general k-Markovian version of model (1) can be defined as the following sequence of transformations:

$$R_0 = X_0$$

$$R_1 = V_1(R_0)X_1 + B_1(R_0)$$

$$R_2 = V_2(R_0, R_1)X_2 + B_2(R_0, R_1)$$

$$R_j = V_j(R_0, R_1, \dots, R_{j-1})X_j + B_j(R_0, R_1, \dots, R_{j-1}) \text{ if } j-1 \le k$$

$$R_t = V_t(R_{t-k}, \dots, R_{t-1})X_t + B_t(R_{t-k}, \dots, R_{t-1}) \text{ if } t-1 \ge k$$

$$R_T = V_T(R_{T-k}, \dots, R_{T-1})X_T + B_T(R_{T-k}, \dots, R_{T-1}) \quad (3) \quad k = 1, 2, \dots, T = 1, 2, \dots, k < T.$$

The k-Markovian conditional pdfs of $R_t \mid R_0, R_1, \dots, R_{t-1}$ as derived from (3) are given by: $g_t(r_t \mid r_1, \dots, r_{t-1})$ if $t-1 \le k$ and $g_t(r_t \mid r_{t-k}, \dots, r_{t-1})$ if $t-1 \ge k$.

Thus, in this setting, the (conditional) distribution of the present asset log return R_T only depends on the last k moments (months, years) in the past. The earlier times are considered irrelevant and are neglected.

Nevertheless, even in the case k = 2 (bi-Markovian) the amount of information incorporated in the stochastic model is significantly bigger than in the Markovian case, so one may expect more accurate predictions.

3. *Examples*

The following examples are based on (1) and (3).

Example 1. Assume that, for each T, the random variables $X_1, \dots, X_T$ are independent, each having the standard normal N(0, 1) pdf.

Using standard calculations based on the knowledge of (1*) and (2) one first obtains the (unconditional) normal pdf $g_1(r_1) = N[B_1(R_0), V_1(R_0)]$ for R_1 and then for each t = 2, 3, ... ,T one obtains the conditional pdf: $g_t(r_t \mid r_1, \dots, r_{t-1})$

$$= [V_t(r_0, r_1, \dots, r_{t-1})\sqrt{2\pi}\,]^{-1}\exp[-(1/2)\{(\mathbf{r_t} - B_T(r_0, r_1, \dots, r_{t-1}))\,/\,V_t(r_0, r_1, \dots, r_{t-1})\}^2]. \quad (4)$$

Realize that the latter conditional pdf is normal with respect to the single variable $\mathbf{r_t}$. The joint probability density $g_T(r_1, \dots, r_T)$ for each random vector $(R_1, \dots, R_T)$, T = 2, 3, ... , is given by the common formula: $g_T(r_1, \dots, r_T) = g_1(r_1)\,\Pi_{t=1}^{T} g_t(r_t \mid r_1, \dots, r_{t-1})$, **(5)** where $g_t(r_t \mid r_1, \dots, r_{t-1})$ is given by (4).

The so obtained T-dimensional pdf is the FF-normal (former name "pseudonormal"), (Kotz & Balakrishnan & Johnson, 2000).

Example 2.

Consider the following "pseudolinear" part of the pseudoaffine transformation (1) which one obtains by setting in (1) all the "pseudotranslation" coefficients $B_t(R_o, R_1, \dots, R_{t-1})$ to zero. One then has the pseudolinear transformations:

$$R_o = X_o$$

$$R_1 = V_1(R_o)X_1$$

$$R_2 = V_2(R_o, R_1)X_2$$

$$R_T = V_T(R_o, R_1, \dots, R_{T-1})X_T \quad \mathbf{(6)} \quad T = 1, 2, \dots .$$

Investigate how the transformations (6) act on set of independent *Pareto* distributed random variables X_t ($t = 1, 2, \dots, T$; $T = 1, 2, \dots$) so, in this case, the expected values of X_t' s are positive.
Recall that the Pareto density is given by

$f_t(x_t) = 1 / \beta (1 + x_t / \beta\gamma)^{1+\gamma}$, (7) where β and γ are positive real parameters.
Using (6), for every $t = 1, \dots, T$, express $\mathbf{x_t}$ as

$$x_t = r_t / V_{t-1}(r_o, r_1, \dots, r_{t-1}) \text{ (assuming } V_{t-1}(r_o, r_1, \dots, r_{t-1}) \neq 0).$$

Also realize, that the jacobian of inverse to (6) equals to the inverse product: $J_T(r_o, r_1, \dots, r_{T-1}) = [V_1(r_o) V_2(r_o, r_1) \dots V_T(r_o, r_1, \dots, r_{T-1})]^{-1}$.

As the next step, one obtains (for each $t = 1, 2, \dots, T$) the conditional pdfs $g_t(r_t | r_o, r_1, \dots, r_{t-1})$ of each rv $\mathbf{R_t}$, given the past realizations $r_o, r_1, \dots, r_{t-1}$ of the rvs $\mathbf{R_o}, R_1, \dots, R_{t-1}$, as follows:

$$g_t(r_t | r_o, r_1, \dots, r_{t-1}) = f(x_t) \, | \partial x_t / \partial r_t |$$

$$= f(r_t / V_{t-1}(r_1, r_2, \dots, r_{t-1})) \, | V_{t-1}(r_1, r_2, \dots, r_{t-1}) |^{-1}$$

$$= 1 / \{ \beta \, | V_{t-1}(r_1, r_2, \dots, r_{t-1}) | \, [1 + r_t / \beta \, | V_{t-1}(r_1, r_2, \dots, r_{t-1}) | \, \gamma]^{1+\gamma} \}. \quad (8)$$

So, the effect of each t^{-th} line in transformation (6) on the rv X_t is to change its Pareto density (7) for the (conditional) Pareto density (8) of R_t.

The two Pareto densities (7) and (8) only differ by the scale parameters, namely**:** β in (7) was transformed into the product $\beta \, | V_{t-1}(r_1, r_2, \dots, r_{t-1}) |$ in (8).

Given the conditional densities (8) one obtains the joint density of each random vector $(R_o, R_1, \dots, R_T)$, $T = 1, 2, \dots$ using formula (5). In such a way the "Pareto stochastic process" $\{R_T\}_{T=1, 2, \dots}$ is well defined.

Example 3.

In the same way as for the independently Pareto distributed random variables $X_1, \dots, X_T$, ($T = 1, 2, \dots$), one can apply transformation (6) to any sequence of independent identically, and exponentially distributed random variables that will be denoted by the same symbols X_t's.

If, for any $t = 1, 2, \ldots$, $g_t(x_t)$ is the exponential density of X_t given by the expression $(1/\theta) \exp[-x_t / \theta]$ then it can easily be verified that the corresponding conditional density of $R_t | R_o, R_1, \ldots, R_{t-1}$ will be given as follows: $h_t(r_t | r_o, r_1, \ldots, r_{t-1}) = (1 / \theta \mid V_{t-1}(r_o, r_1, \ldots, r_{t-1}) \mid) \exp[- r_t / \theta \mid V_{t-1}(r_o, r_1, \ldots, r_{t-1}) \mid]$. It is then clear that as in Example 2, the parameters θ is multiplied by the "coefficient" $\mid V_{t-1}(r_o, r_1, \ldots, r_{t-1}) \mid$.

The same actually will happen with the parameter σ in Example 1, if one would assume all the random variables X_t in (1) are normal $N(0, \sigma)$. Also in this case, the parameter σ will be turned to the conditional volatility of R_t: $\sigma \mid V_{t-1}(r_o, r_1, \ldots, r_{t-1}) \mid$.

This regularity for the parameter transformations will be applied in the next section.

III. PARAMETER DEPENDENCE MODELS

4.1 In all three examples in the previous section transformation (1) or (6) were used in order to obtain the conditional densities, say, $\varphi_t(r_t | r_o, r_1, \ldots, r_{t-1})$ describing the stochastic dependence of the return R_t on the past.

Realize that in this derivation the underlying operations only result in changing the value of a parameter of the given density of X_t, into other value that depends on the past return values $r_o, r_1, \ldots, r_{t-1}$.

This observation opens the way for the method of conditioning (on values $r_o, r_1, \ldots, r_{t-1}$), which is significantly more efficient than the method of triangular transformations (1) or (6). This method, called the "parameter dependence", is presented in (Filus & Filus, 2012) and (Filus & Filus, 2013).

In the considered framework one can describe this method as follows.

Suppose there is given a sequence of independent random variables (now, instead of X_t, denoted by $R_{f\,t}$ $t = 1, 2, \ldots$) all having the same arbitrary probability density $f_t(r_t ; \alpha)$, $\alpha \in A$. In this situation any past in this artificial "no memory process" has no influence on the current density $f_t(r_t ; \alpha)$ of $R_{f\,t}$.

The density depends on a constant (original) scalar or vector parameter α. Instead of applying transformation (1) or (6) to the random vectors $(R_{f1}, \ldots, R_{fT})$ one can "directly transform" each density $f_t(r_t ; \alpha)$ into a conditional density $\varphi_t(r_t | r_o, r_1, \ldots, r_{t-1})$ of $R_t | r_o, r_1, \ldots, r_{t-1}$ just by setting the parameter α of $f_t(r_t ; \alpha)$ to "become" a function of the values $r_o, r_1, \ldots, r_{t-1}$.

In such a way one defines the sequence of conditional pdfs by the formula: $\varphi_t(r_t | r_o, r_1, \ldots, r_{t-1}) = f_t(r_t ; \alpha_t(r_o, r_1, \ldots, r_{t-1}))$, $t = 1, 2, \ldots$ (9) which, for an arbitrary function $\alpha_t(r_o, r_1, \ldots, r_{t-1})$, defines a legitimate density with respect to r_t if all the values $\alpha_t(r_o, r_1, \ldots, r_{t-1})$ still belong to the set A of the parameters α of $f_t(r_t ; \alpha)$. Each sequence of the so obtained conditional densities $\{\varphi_t(r_t | r_o, r_1, \ldots, r_{t-1})\}_{t = 1, 2, \ldots}$ defines a corresponding stochastic process $\{R_t\}_{t = 1, 2, \ldots}$.

The parameter dependence method allows for relatively free choice for the functions $\alpha_t(r_o, r_1, \ldots, r_{t-1})$ and therefore the class of the so obtained stochastic processes is much wider than that obtained by the triangular transformation from the same sequence of independent random variables X_t or R_{ft}. On the other hand, the factor that, in applications, often may limit the range of choices of the functions $\alpha_t(r_o, r_1, \ldots, r_{t-1})$ is reality.

Every "educative guess" for such a function must be statistically verified. So, first of all, the chosen function itself usually has its own parameters (parametric approach) that must be estimated by any statistical method such as, for example, the maximum likelihood method. Then the properly arranged parametric hypothesis should be verified. Finally the choice of the best fitting to data function $\alpha_t(r_o, r_1, \ldots, r_{t-1})$ should be based on statistical methods as to be the best one from several candidates (the choices made in the beginning). This then should be declared as the final model.

4.2 It is common (Tsay, 2005), that the general stochastic model for log returns of a given single asset from a portfolio is a joint probability distribution

$$P(R_1 < r_1, \ldots, R_T < r_T \mid Y_1, \ldots, Y_k) = G_T(r_1, \ldots, r_T; Y_1, \ldots, Y_k)$$

$$= G_1(r_1; Y_1, \ldots, Y_k)\, \Pi_{t=2}^{T} G_t(r_t \mid r_1, \ldots, r_{t-1}; Y_1, \ldots, Y_k) \quad (10)$$

where $G_1(r_1; Y_1, \ldots, Y_k)$ is the cdf of the random variable R_1 and, for t = 2, 3, ... , T, $G_t(r_t \mid r_1, \ldots, r_{t-1}; Y_1, \ldots, Y_k)$ is the conditional distribution function of R_t, given realizations $r_1, \ldots, r_{t-1}$ of the random variables $R_1, \ldots, R_{t-1}$.

However, the above joint and conditional distributions also depend on the state random variables $Y_1, \ldots, Y_k$ that summarize the "environment" in which asset return is determined, see (Tsay, 2005), page 13.

One can apply the parameter dependence method to define the conditional distribution functions $P(R_1 < r_1, \ldots, R_T < r_T \mid y_1, \ldots, y_k)$, where $y_1, \ldots, y_k$ are (measured) realizations of the states $Y_1, \ldots, Y_k$.

For that it is enough to set parameter $\alpha_t(r_o, r_1, \ldots, r_{t-1})$ **(**which already determines the conditional distribution $G_t(r_t \mid r_1, \ldots, r_{t-1})$ **)** to be additionally dependent on the values $y_1, \ldots, y_k$. Thus, for a given t, the conditional distribution of $R_t \mid r_1, \ldots, r_{t-1}; y_1, \ldots, y_k$ will be determined by a parameter(s) $\alpha_t(\,)$ of R_t's distribution as follows:

$G_t(r_t \mid r_1, \ldots, r_{t-1}; y_1, \ldots, y_k) = G_t(r_t; \alpha_t(r_1, \ldots, r_{t-1}; y_1, \ldots, y_k))$. (11) If the values (realizations) $y_1, \ldots, y_k$ are measured then the joint distribution (10) is already determined. If not, one needs to have joint probability density $f(y_1, \ldots, y_k)$ of the random vector $(Y_1, \ldots, Y_k)$. It seems that often one may assume stochastic independence of the components $Y_1, \ldots, Y_k$ of this vector.

Finally, as typically, it may be needed to multiply the resulting G_T's distribution (10) conditioned on $y_1, \ldots, y_k$ by the density $f(y_1, \ldots, y_k)$.

As an example of the parameter function $\alpha_t(r_1, \ldots, r_{t-1}; y_1, \ldots, y_k)$ one may consider the following:

$$\alpha_t(r_1, \ldots, r_{t-1}; y_1, \ldots, y_k) = \alpha\,(1 + a_1 r_1^2 + \ldots + a_{t-1} r_{t-1}^2)\exp[\, b_1 y_1 + \ldots + b_k y_k\,],$$

where α is the constant original parameter of the density $f_t(r_t; \alpha)$ of the random variable R_{ft} (Recall, $\{R_{ft}\}_{t=1,2,\ldots}$ is the original stochastic process with the independent terms). Furthermore, $a_1, \ldots, a_{t-1}$ and $b_1, \ldots, b_k$ are real coefficients. Obviously, when all the coefficients $b_1, \ldots, b_k$ are small enough then the impact of the states $y_1, \ldots, y_k$ on the parameter **(**so on the conditional distribution**)** is insignificant.

According to my knowledge, the above application of the parameter dependence method to incorporate the random states $Y_1, \ldots, Y_k$ impact on the returns' distributions is not yet present in literature.

Final Remark

The core achievement when employing either of the two methods, is opening the way for easy constructions of the conditional probability distributions of $X_t \mid X_1, \ldots, X_{t-1}$, given in compact analytical forms ready for the calculations. Underlying calculations can be analytical or, if necessary, relatively simple numerical. Having the conditional distribution functions (11) in analytical forms allows for extending many classical *regression models,* usually being in the form of conditional expectation, say, $E[\, R_t \mid r_1, \ldots, r_{t-1}; y_1, \ldots, y_k\,]$, by replacing them with the full probability distribution

(11). Notice that the latter regression is the expected value of (11) so it is only part of the wider model considered here. In what is called *"enforced regression"* (Filus & Filus, 2014), the numerical characteristics like conditional expectations or covariance coefficients can be replaced by richer functional characteristics such as the conditional distributions or joint probability distributions respectively. This idea is, apparently, different from that (nonparametric) considered by (Koenker & Bassett, 1978), and followers. For a wider discussion of this subject, see (Filus & Filus, 2014).

REFERENCES

1. J. K. Filus & L. Z. Filus, Parameter Dependence as 'Weak Transformation'; Method of Construction of Multivariate Probability Densities, Biometrie und Medizinische Informatik Greifswalder Seminarberichte, Heft 23 Statistical and Biometrical Challenges, Theory and Applications, 133 – 147, 2014.
2. J. K. Filus & L. Z. Filus, A Method for Multivariate Probability Distributions Construction via Parameter Dependence, Communications in Statistics: Theory and Methods, Vol. 42, Num. 4, 15, 716-721, 2013.
3. J. K. Filus & L. Z. Filus, Multivariate "Pseudodistributions" as Natural Extension of the Multivariate Normal Density Pattern – Theory, American Institute of Physics Conference Proceedings 1479, Numerical Analysis and Applied Mathematics, Vol. 1479, 1417-1420, 2012.
4. J. K. Filus & L. Z. Filus, B. C. Arnold, Families of multivariate distributions involving "triangular" transformations, Communications in Statistics - Theory and Methods, Volume 39, Issue 1, 107-116, 2010.
5. J. K. Filus & L. Z. Filus, Construction of New Continuous Stochastic Processes, Pak. J. Statistics, Vol. 24(3), 227-251, 2008
6. R. Koenker & G. W. Bassett, Regression Quantiles, Econometrica, 46, pp 33-50, 1978.
7. S. Kotz & N. Balakrishnan & N. L. Johnson, Continuous Multivariate Distributions, Volume 1. Second Edition. J. Wiley & Sons, Inc, New York, pp 217-218, 2000.
8. R. S. Tsay, Analysis of Financial Time Series, Second Edition.
9. J. Wiley & Sons, Inc, 2005.