CrossRef DOI of original article:

Scan to know paper details and author's profile A Multiple Contracts Version of the SACRE

Received: 1 January 1970 Accepted: 1 January 1970 Published: 1 January 1970

Abstract

Index terms—

1 I. INTRODUCTION

In 1996, the "Caixa Econômica Federal" (CEF), which is the main institution for housing financing in Brazil, introduced a debt amortization scheme named "Sistema de Amortizações Reais Crescentes" -SACRE (system of increasing amortizations in real terms).

In its original version, this very peculiar amortization system is not financially consistent. Namely, even if all contractual payments are dutifully made, a residual debt remains, which must be paid in full by the borrower, usually one month after the end of the term of the contract.

Given that de Faro and Lachtermacher (2022) proposed a financially consistent variant of the SACRE, the purpose of this paper is to formulate a multiple contracts version of this system. Similar to cases of the adoption of either the constant payments scheme or the constant amortization scheme of debt financing, which were considered in De-Losso et al **??**2013) and in de Faro (2022), it will be shown that the financial institution granting the loan, depending on its cost of capital, may derive substantial income tax reductions in terms of present values.

2 II. THE CASE OF A SINGLE CONTRACT

Denoting by F the loan amount, and by i the periodic rate of compound interest , suppose that, in the case where a single contract is considered, it is stipulated by the financing institution granting the loan that the debt must be repaid in n periodic payments, in accordance with the SACRE scheme.

Since the SACRE scheme is a combination of the constant payments scheme with the constant amortization scheme, the number n of payments is divided into â??" subperiods, each with m payments. The numbers n, â??" and m are integer numbers with n = â??" ? m , and with m constant payments in each of the first sub-periods. , for (2)? ? = 1 + ? ( ) × ? ? -1 -? ? ? = 1 , 2 ,..., ?

Therefore, using the presumed recurrence method to determine the debtor's balance, we have:

(

where .? = 1 + ? ( ) ? -1 [ ] / ?

This relationship, in view of the value of P 1 presented in Equation **??**, can be rewritten as:

(At this point, as suggested in de Faro and Lachtermacher (2022), rather than being constant, the last m payments should decrease linearly in accordance with an arithmetic progression of ratio equal to , with , which is a procedure assured to be ? × ?â??" -1 ( ) / ? ? â??" -1 ( ) ? + 1 = ? â??" -1 ( ) ? × 1 + ? × ? ( ) / ?

financially consistent whenever the interest rate, i, is less than 10% per month, and which is far above the current rates charged in the Brazilian house-financing system. Currently, the monthly rate is reflected at 1.5%.

In summary, the sequence of the first payments will be as follows: ? -? With regard to the sequence of the parcels of amortization, it should be noted that, as shown in de Faro and Lachtermacher (2012, p. 243), and similar to the case of the constant payments scheme, the parcels of amortization, in each set of constant payments, follow a geometric sequence of ratio equal to 1+ i . Accordingly, we have:

3 III. THE MULTIPLE CONTRACTS ALTERNATIVE

Rather than engaging a single contract, the financial institution has the option of requiring the borrower to adhere to n subcontracts; one for each of the n payments that would be associated with the case of a single contract, with the principal of the k -th subcontract being the present value, at the same interest rate i , of the k -th payment of the single contract.

Namely, the principal of the k -th subcontract, denoted by , is:? ? (**15**)

In this case, the parcel of amortization associated with the k -th payment, which will be denoted by , ? ? will be:(16)

Ergo, the parcel of amortization associated with the k -th subcontract is exactly equal to the value of the corresponding principal.

Conversely, from an accounting point of view, it follows that the parcel of interest associated with the k -th subcontract, which will be denoted by , is:? ? (**17**)

From a strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the subcontracts, the total interest payments is the same in both cases. However, in terms of present values, and depending on the financial institution opportunity cost, it is possible that the financial institution will be better off if it adopts the multiple contracts option.

4 A simple numerical example

Before presenting a numerical illustration, it is appropriate to give due credit to the one who has introduced the idea of associating a specific contract with each of the payments of the main contract.

As far as we know, the concept was originally proposed by Sandrini (2007), in his Master's thesis for the Federal University of Paraná. However, an actual contract for each of the payments was not effectively proposed. The goal was to imply, specifically for the case of the constant payments scheme of debt amortization, the occurrence of what is named, in legal terms, anatocism -to wit, the charge of interest upon interest.

Later, De-Losso et al. (**??**013) presented a formalization of the concept of multiple contracts. Focusing on the case of the constant payments scheme. Later de Faro (2022) extended the analysis to consider the Constant Amortization System. Now, as a numerical illustration, consider a loan of 12,000 units of capital, for the case of ? = 12 periodic payments, with , and , with the periodic rate of interest, i, being equal to 1% per ? = 3 â??” = 4 period.

Table 1 presents the sequence of the 12 payments, which is the same both in the case of a single contract, as well as in the 12 individual contracts.

Also, in Strictly from an accounting point of view, there is no gain if a single contract is substituted by multiple contracts since the sums of the corresponding parcels of interest are the same. Hence, Yet, depending on the opportunity cost of the financial institution, which will be denoted as , the ? financial institution may derive substantial financial gains in terms of income tax deductions.

In other words, it is possible that:(18)

where the interest rate is supposed to be relative to the same period of the interest rate i . ? Moreover, as the sequence of differences has only one change of sign, thus characterizing what is ? ? termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this particular case null, it follows that ? = ? 1 ? ( ) -? 2 ? ( ) > 0 ð ??”?? ? > 0 .

Figure **??** outlines the evolution of Additionally, we also have the evolution of , ?, ð ??”?? 0 ? ? ? 5% . when the interest rate, i , is equal to 0.5%, 1%, 1.5%, 2% and 3%.

5 | |

A Multiple Contracts Version of the SACRE

Figure 1

For instance, if per period, and if per period, we will have ? = 1% ? = 2% units of capital. Namely, the financing institution ? = ? 1 2% ( ) -? 2 2% ( ) =

6 IV. GENERAL ANALYSIS

In the previous section, focusing attention on the case of a contract with only 12 payments, it was verified that the sequence, , of differences of the interest payments yielded just one change of sign, ? ? thereby assuring us of the uniqueness of the corresponding internal rate of return, which was known to be zero.

However, when the number of payments is increased, it is possible to have instances wherein more than one change of sign can occur.

This possibility is illustrated in Figure **??**, which refers to the case where a loan of 1,200,000 units of capital has a term of 15 years (180 months), with â??” = 15, monthly payments, and with the monthly interest rate, i, going from 0.5% up to 3%.

7 22

A Multiple Contracts Version of the SACRE

Figure 2

Wherefore, for the cases where the monthly interest rate i assumes the values of 1%, 2% and 3%, we have three changes of sign in the sequences of differences with only one change of sign in the other ? ? three cases.

However, considering a classical result first stated by Norstrom (1972), which is based on the sequence of the accumulated values of the sequence , we can still guarantee the uniqueness of the ? ? corresponding internal rate of return, and which we already know is null. Moreover, we are also assured that the difference of present values is positive whenever the opportunity cost is greater ? ? than zero.

Taking into consideration that in Brazil the monthly interest rates charged in house-financing contracts do not exceed 2% per month, in real terms, Tables **??**345 The results presented in Tables 2 to 5 are self-evident. They illustrate a compelling support for the substitution of a single contract by multiple contracts.

For instance, if the interest rate i charged by the financial institution granting the loan is 0.5% per month, the percentual value of can be as high as 47% when its opportunity cost is 30% annually, the ? contract has a 5-year term, and with a percentage fiscal gain over 248%, if the contract is of 30 years, and =30%per year. ? ? Furthermore, even though the fiscal gain decreases when the interest rate, i , being charged is increased, the percentage gain is no less than 35% in every case.

Accordingly, one can conclude that the financial institution is well advised whenever it substitutes a single contract by multiple contracts, one for each of the payments of the single contract, whenever using our version of SACRE scheme.

A Multiple Contracts Version of the SACRE

8 V. A COMPARISON WITH TWO ALTERNATIVE SYSTEMS OF AMORTIZATION

Given that the financial institution granting the loan may have the option of choosing an alternative system of amortization, this section addresses two such possibilities, since both alternatives have also been considered in the Brazilian House-Financial program.

The first one is the system of constant payments. In this case, as shown in De-Losso et al. (**??**013) and also in de Faro (2022), the present value of the sequence of interest payments, if multiple contracts are adopted, is equal to:

where and . ? = ? × ? / 1 -1 + ? ( )-? [ ] ? = ? + ? + ? × ? ( )

Tables 6 to 9 illustrate the percentage increase of the fiscal gain , ? ’ = ?1 ? ? ( ) / ? 3 ? ? ( ) -1 [ ] × 100

wherein the financial institution adopts the multiple contracts version of the SACRE instead of the constant payments scheme. As indicated in the overwhelming majority of the cases, the financial institution should not choose the multiple contracts version of the SACRE. That is, if possible, the best option is to adopt the multiple contracts version of the constant payments scheme.

On the other hand, in the case of the system of constant amortization, the present value of the sequence of interest payments, where multiple contracts are adopted as shown in de Faro (2022), is equal to:

Tables 10 to 13 portray the percentage increase of the fiscal gain , when ? = ? Similarly, it is clear that in the overwhelming majority of cases, the financial institution should opt for the multiple contracts version of the constant amortization scheme.

9 VI. CONCLUSION

In similarity to the cases where either the constant payments system or the constant amortization system is adopted, a financial institution which implements our version of the SACRE, will be well advised if a multiple contract scheme, rather than a single contract, is implemented.

10 | |

However, if the financial institution has the option of rather than adopting the SACRE, choosing either the constant payment system or the constant amortization one, in the vast majority of cases, SACRE is not the best option. [1] [2]

111

Figure 1: â??” - 1 1 ? 1 =

6


Figure 2: 6)

Figure 3: 18A

Figure 4: A

Figure 5:

Figure 6:

$$P_1 = p_1 = A_1 + J_1 = \frac{S_0}{n} + i \times S_0 = S_0 \times \left(\frac{1}{n} + i \right) = \frac{S_0}{n} \times (1 + n \times i)$$

Figure 7:

1

? ? ? ? ? = ? ? -? ?

Figure 8: Table 1 ,

1

k				
1	1,120.00	120.00	11.09	108.91
2	1,120.00	110.00	22.07	87.93
3	1,120.00	99.90	32.94	66.96
4	1,086.35	89.70	42.39	47.31
5	1,086.35	79.73	52.73	27.01
6	1,086.35	69.67	62.96	6.71
7	1,051.16	59.50	70.72	-11.22
8	1,051.16	49.58	80.43	-30.85
9	1,051.16	39.57	90.04	-50.48
10	1,011.16	29.45	95.77	-66.32
11	1,001.34	19.63	103.82	-84.18
12	991.52	9.82	111.60	-101.78
	12,776.55	776.55	776.55	0.00

Figure 9: Table 1 :

3

i =1%p.m.				? (%) ?		
n (years)	5%	10%	15%	20%	25%	30%
5	6.9818	14.0369	21.1427	28.2780	35.4240	42.5637
10	12.6403	25.8461	39.4650	53.3526	67.3789	81.4325
15	17.1122	35.2698	54.0650	73.1370	92.1981	111.0376
20	20.6713	42.6924	65.3157	87.9659	110.2641	131.9929
25	23.5050	48.4791	73.8287	98.8507	123.1814	146.6734
30	25.7789	52.9968	80.2719	106.8814	132.5477	157.2110

Figure 10: Table 3 :

4

i =1.5%p.m.				? (%) ?		
n (years)	5%	10%	15%	20%	25%	30%
5	6.4400	12.9049	19.3754	25.8347	32.2678	38.6619
10	11.0264	22.3577	33.8719	45.4608	57.0333	68.5156
15	14.2645	29.0323	44.0129	58.9738	73.7455	88.2143
20	16.6191	33.8043	51.0807	68.1246	84.7455	100.8487
25	18.3616	37.2551	56.0467	74.3817	92.1025	109.1625
30	19.6818	39.8015	59.6129	78.7830	97.2111	114.8975

Figure 11: Table 4 :

5

i =2%p.m.			? ?	(%)		
n (years)	5%	10%	15%	20%	25%	30%
5	5.9649	11.9192	17.8476	23.7367	29.5750	35.3530
10	9.7461	19.6393	29.5842	39.5008	49.3250	59.0074
15	12.1807	24.5843	37.0028	49.2816	61.3156	73.0401
20	13.8378	27.8861	41.8294	55.4706	68.7046	81.4871
25	15.0065	30.1631	45.0706	59.5276	73.4576	86.8487
30	15.8621	31.7896	47.3310	62.3081	76.6821	90.4695

Figure 12: Table 5 :

6

i =0.5%p.m.			? ?	(%)		
n(years)	5%	10%	15%	20%	25%	30%
5	5.7794	5.2285	4,6985	4.1891	3.7003	3.2316
10	9.4904	7.6829	5.9746	4.3761	2.8930	1.5266
15	12.4358	8.8406	5.5845	2.7002	0.1883	-1.9736
20	14.6772	9.0254	4.2075	0.2399	-2.9578	-5.5094
25	16.3350	8.5602	2.4188	-2.2365	-5.7050	-8.2918
30	17.4969	7.7185	0.6241	-4.3241	-7.7642	-10.2021

Figure 13: Table 6 :

7

Values of for ? ’ ? = 1 . 0% ? . ? .

Figure 14: Table 7 :

10

i =0.5%p.m.			?	? (%)		
n(years)	5%	10%	15%	20%	25%	30%
5	1.2310	1.0836	0.9439	0.8114	0.6861	0.5675
10	1.1235	0,8599	0.6193	0.4016	0.2061	0.0315
15	1.0147	0.6658	0.3650	0.1106	- 0.1015	-0.2766
20	0.8942	0.5125	0.1968	-0.0552	-0.2521	-0.4043
25	0.7927	0.3987	0.0889	-0.1445	-0.3168	-0.4438
30	0.7084	0.3151	0.0216	-0.1884	-0.3371	-0.4436

Figure 15: Table 10 :

11

i =1.0%p.m.			? ?	(%)		
n(years)	5%	10%	15%	20%	25%	30%
5	2.5282	2.2448	1.9781	1.7273	1.4915	1.2700
10	2.3441	1.8686	1.4429	1.0645	0.7298	0.4351
15	2.1678	1.5681	1.0653	0.6496	0.3087	0.0302
20	1.9540	1.3198	0.8130	0.4170	0.1106	-0.1265
25	1.7694	1.1305	0.6452	0.2849	0.0184	-0.1807
30	1.6121	0.9843	0.5301	0.2070	-0.0246	-0.1942

Figure 16: Table 11 :

12

i =1.5%p.m.			?	(%) ?		
n(years)	5%	10%	15%	20%	25%	30%
5	3.8908	3.4788	3.0936	2.7336	2.3972	2.0829
10	3.6425	2.9811	2.3967	1.8829	1.4330	1.0398
15	3,4074	2.5917	1.9182	1.3674	0.9186	0.5529
20	3.0991	2.2443	1.5721	1.0511	0.6482	0.3350
25	2.8242	1.9654	1.3226	0.8473	0.4941	0.2278
30	2.5846	1.7397	1.1366	0.7078	0.3981	0.1689

Figure 17: Table 12 :

13

i =2.0%p.m.			?	? (%)		
n(years)	5%	10%	15%	20%	25%	30%
5	5.3187	4.7826	4.2842	3.8207	3.3898	2.9890
10	5.0078	4.1727	3.4415	2.8038	2.2487	1.7659
15	4.7111	3.6897	2.8547	2.1760	1.6247	1.1756
20	4.3022	3.2309	2.3969	1.7531	1.2553	0.8670
25	3.9287	2.8489	2.0482	1.4577	1.0179	0.6847
30	3.5992	2.5325	1.7778	1.2421	0.8542	0.5657

Figure 18: Table 13 :

[Norstrom ()] ‘A Sufficient Condition for a Unique Non Negative Internal Rate of Return’. C Norstrom . *Journal of Financial and Quantitative Analysis* 1972. 3 p. .

[De Faro and Lachtermacher ()] C De Faro , G Lachtermacher . *Introdução à Matemática Financeira, FGV/Saraiva*, 2022.

[De-Losso et al. ()] R De-Losso , B C Giovannetti , A S Rangel . *Sistema de Amortização por Múltiplos Contratos: a Falácia do Sistema Francês*, 2013. 4 p. .

[De Faro and Lachtermacher ()] ‘O SACRE no Regime de Juros Compostos’. C De Faro , G Lachtermacher . *Estudos e Negócios Academics*, 2022. 4 p. .

[De Faro ()] ‘On the Internal Rate of Return Criterion’. C De Faro . *The Engineering Economist* 1974. 19 p. . (Nº)

[Sandrini ()] J C Sandrini . *Sistemas de Amortização de Empréstimos e a Capitalização de Juros: Análise dos Impactos Financeiros e Patrimoniais*, 2007. Federal University of Paraná (Master Thesis)

[De Faro (2022)] ‘The Constant Amortization Scheme With Multiple Contracts’. C De Faro . *Revista Brasileira de Economia* april-june, 2022. 76 p. .