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Quantum Mechanics as Structuralist Chimera

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One must do no violence to nature, nor model it in conformity to any blindly formed chimaera.

Bolyai

ABSTRACT

I argue that properties and relations are in the same boat with respect to quantum mechanics. That just as properties cannot be considered as "hidden variables" so also neither can the relation of being correlated with. Nevertheless properties and relations can both be understood as incomplete expressions: they are both contextual, properly understood. The argument on this leverages a neglected proof by Adán Cabello. We show that this latter argument extends and strengthens an argument given by van Fraassen in his (2006). The argument given also consid- erably strengthens the arguments given previously by Cao (2003) and Psillos (2006). I end by sketching a way of understanding that this contextuality is similar across relativity theory and quantum theory.

I. INTRODUCTION

We have grown accustomed, to the extent one can, to the following idea: certain 'qualities' of quantum systems are strangely dispositional; before a measurement is made they are merely present *in potentia* — it is the measurement which, with a given probability, *realises the potential* to give one particular result or another. So the spin of a spin-½ system (like an electron), will give a value of spin-up or spin- down upon measurement in some chosen direction in space. However before the measurement is made this value is not, and cannot be, a pre-existent feature of the state of the particle. We have this proven in two sets of theorems, the Bell theorems, and the Kochen-Specker theorems: both constitute high barriers to taking these qualities as unconditionally present. What I argue in this paper is that this situation also applies to relations as well, in particular to the relation of *correlation* . This also is dispositional, dependent upon a measurement to release this potential to exhibit some value or other. We thus have a generalisation of the transition from pre-existence to actuality — it was applied *first to Property and now to Relation*.

To appreciate this we need to retrace some steps through history. Plato concentrated his attention on the properties of things, and took those properties to have the backing of Forms, as a way of vouchsafing the properties an objectivity and stability. So an action or person being good may have the backing of a Form of goodness which is here being instantiated. But relations were not considered in this. There was no Form of *taller than*. To the extent that one individual was taller than another it was supervenient on the heights of the two individuals, where these heights do have forms. So the relation of taller than is a property of properties. Qualities were thus the ontological focus. When Aristotle came to formulate his syllogistic logic it was similarly focussed on qualities. Relations were left out. In the *Categories* relations were considered, and much discussed, but this discussion mostly favoured a non-realistic view of relations (see Brower in Marmodoro and Yates (2016)). As the Platonic and the Aristotelian metaphysics passed from the Medieval world the focus of concerns was in understanding the Forms in a consistent way. What was this idea of Instantiation? Did it also have a Form? It certainly appears to be a relation of some sort, but if so what were the implications of this. The western tradition was focussed on the objective existence of qualities, i.e. primary qualities, and the lesser, dependent reality of relations could not easily be made to fit. The result was an assortment of positions that were distilled down, in time, to Leibniz's view that an accident (i.e., quality) could not be in two subjects at the same time.

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In the second half of the 19th Century there were movements in two different, indeed opposite, directions. On the one hand F.H. Bradley drew on his Hegelian background to reason, via an infinite regress argument, that relations were wholly unreal. Bradley's argument had a wide and deep influence, in part because it exonerated philosophers from the need to explain them. Charles Sanders Peirce had in the meantime worked hard to bring relations in from the dark by developing a logic of relations. (It should be noted that Peirce's knowledge of the medieval discussions very much outshone that of his contemporaries.) However his chosen symbolism was difficult to follow and hard to reproduce. It has been almost completely neglected in the 150 years that have intervened. Russell took up both halves of this challenge; both attempting to refute Bradley's argument, in multiple papers (in particular his (1907), for which see Russell (1959)), and establishing a logic of relations 'which must serve as a foundation for mathematics' (see Russell (1901)). (The notation for this was, frankly, not much of an improvement on Peirce.) In the end, by the 1930s, a simpler solution was found. The logic of relations would simply be folded up into first order quantificational logic, the monadic parts would represent the logic of attributes and the polyadic parts the logic of relations. Both would be treated together; Aristotle's logic would be replaced by something far more versatile, and Plato's fixation on forms for attributes would be turned from metaphysics into notation. Quine would make the most of this transformation — a paved-over paradise — with logical structure becoming a replacement for metaphysical thought.

A great deal of subtlety was lost in this process. One obvious thing was that certain seeming attributes, what we call "secondary qualities", were obviously not able to be understood in any simple way. Sweet and bitter were dependent on additional context: who was tasting them? what species was doing the tasting? what was their biochemistry? And more. But even more profoundly, attributes like velocities, from Galileo onwards, required the context of a frame of reference. 'Alice's velocity is 100 mph' looks to be contradicted by 'Alice's velocity is 200 mph' but obviously need not be if we fill in the missing context of the different Galilean frames of reference. Velocity attributions are strictly meaningless without a specification of the context, namely the frame of reference. But it is not obvious how to supply this context within the framework of first-order quantificational logic. (At first blush it looks like it would need something like a governing modal operator). There is the same need for context in the specification of positions and, since the advent of Einstein's theory of relativity, the specification of elapsed time and measured distance. Both of these require the specification of a Lorentz frame. Accelerations do not require a specification of a Lorentz frame, but they do require a specification of the gravitational field: are they positive accelerations or are they zero accelerations, i.e. free falls along geodesics of a curved space-time?

The more we look at attributes the more we find that they require the specification of a context.

We should take this as the default requirement — at least we take such as our proposal. Thus we should reject the picture that we've inherited from the Greeks — specifically from Plato and Aristotle, and which has been turned into a notational blind-spot in first order logic — of qualities as context-free attributions, of relations as reducible to monadic properties. If a quality *appears* to be context-free we should treat that as an anomalous case, requiring extra scrutiny. ('Alice is tall' is thus not context-free, it requires the context of the average height in the reference population. And so on for many of the historical examples of qualities.)

This was an interesting path to follow and it might have been followed if we had been doing metaphysics conscientiously in the post-war period. The groundwork had been well-laid in the pre war period by both philosophers and physicists. But it did not happen.

So here we want to consider two realistic views of accidental properties and relations. We can call these the *non-contextual* and the *contextual* views. The non-contextual view is a classical sort of Realism, monadic and polyadic properties and relations exist, much as they might appear to do in classical logic. The contextual view, no surprise, puts in a contextual requirement, where seeming monadic properties are really relational, they relate to, or are conditional upon, a context. We've already seen some examples of this, most notably those incorporating Galilean and Einsteinian relativity. The contextual requirement could be null, but if it *is* then this must still be entered. What we have not done thus far is make any comment on how either of these views might accommodate quantum mechanics. This is the subject of this paper. There are three philosophical or metaphysical issues that I want to address — all are connected with one another.

a. The non-individuality of particles.
b. The measurement problem for particles.
c. The status of entanglement as a relation.

There is a view that has considerable popularity at the moment. It is called Ontological Structural Realism (OSR) which promises to solve various problems in QM, namely a) and c). As the name suggests, it takes a realistic view of the mathematical structures underlying quantum mechanics, in particular the Hilbert space structure and the self-adjoint operators that act upon it. The claim is that this structure can exist without any interpretation as a state space of any particles, merely as a structure. An early objection to this idea was made by Michael Redhead, and so called Redhead's Problem. It is this:

> If structure is understood in relational terms — as it typically is — then there needs to be relata and the latter, it seems, cannot be relational themselves. In other words, the question is, how can you have structure without (non-structural) objects? (French and Ladyman 2003, 41)

As they also note, on the same page: 'If the structural realist cannot answer this question, then the whole metaphysical project threatens to come undone.' We can agree that the point is a crucial one. The problem has been voiced by others, including Dorato (1999), Cao (2003), Psillos (1999) and van Fraassen (2006) (2007).

This problem does not face OSR alone, but any structuralist view that wants to claim that we can know relational facts in the world but not the things-in-themselves that are so related, as for example in Russell's view. Frank Jackson's (1998) confronts the problem directly. 'An obvious extension of this possibility leads to the uncomfortable idea that we may know next to nothing about the intrinsic nature of our world. We know only its causal cum relational nature.' (24) This is particularly problematic if one includes in the relational characteristics (as is done here) the causal ones. Suppose one thing interacts with another, by bumping into it (say), then the causal relation, the *bumping into*, is revealed to us by a change in the object(s), the effect upon them — say a change in momentum, or some deformation of one or both. And conversely, if we can't know of such effects then we can't know of the causal relations after all'. This may be behind why Jackson rejects OSR: 'This, to my way of thinking, is too close to holding that the nature of everything is relational cum causal, which makes a mystery of what it is that stands in the causal relations.' Quite so: but this is a manifestation of Redhead's Problem in a particularly acute form. And we can see already that it will be very difficult to include causal relations into OSR — impossible if we continue to insist that we have *no* idea of the relata, or that the relata needn't exist.

This problem is interesting but it is by now a well-trodden path. It is in effect the application to relations of a well known consequence of realism about monadic Universals, namely that they can exist as Forms but be uninstantiated. Uninstantiated relations are perhaps more of a shock to our intuitions, but they are not of an entirely different kind. The question I want to focus on here is how this is supposed to help in QM.

The answer lies in the notion of '*weak discernibility*'. Quantum particles have traditionally been seen as lacking individuality in virtue of their statistics — Bose-Einstein for Bosons and Fermi-Dirac statistics for Fermions. The statistics are in turn related to their subspaces: the symmetric subspace for bosons and antisymmetric subspace for fermions. In the latter case we are supposed to be able to recover a weak form of discernibility of the fermions that may be sufficient for individuating them. The metaphysically traditional idea of *absolute* discernibility is thus no longer required for something to be considered an individual. In this more traditional case: 'two objects are *absolutely discernible* if there is a sentence in one free variable such that one object satisfies that sentence but the other doesn't' Quine 1976: 113). Absolute discernibility is tied to the idea that things are different when they have different properties — it being the differential possession of these properties that make them distinct from one another. But QM does not have this character. Measurement produces properties depending on what observables one chooses to measure and one cannot attribute these properties to the particles before a measurement is made. It thus looks as though, if we were to adhere to the Leibniz standard of the identity of indiscernibles, that particles are not individuals.

Weak discernibility does not seek to individuate entities by properties, but rather by relations between them, specifically the possession of a binary irreflexive, symmetric relation (ISR). If two fermions satisfy a particular ISR then they must be different, because they do not have the ISR to themselves but they do have it to the other particle. The examples given by Simon Saunders in his 2006 are the two Black spheres weakly discerned by the predicate of being one mile apart (see Black (1952)); two fermions in the singlet state (more on such states shortly), in which the predicate becomes '. . . has opposite component of spin to..'. This last is paraphrased as the particles being anti-correlated. But the point is generalisable:

> On the strength of this we can see, I think, the truth of the general case: so long as the state of an *N*-fermion collective is antisymmetri zed,[1] there will be some totally irreflexive and symmetric *n*-ary predicate that they satisfy. Fermions are therefore invariably weakly discernible. (Saunders 2006: 59)[2]

This does not follow: weak discernibility is a *binary* relation; we do not have any reason to believe that it can be generalised to more than two objects. The reply will come: surely we can apply it pairwise to all the pairs in the *n* set. But we couldn't do this without being able to distinguish them — effectively, by pairing them with ordinals — and in this way knowing that we've exhaustively run them through the binary ISR formula. But we can't do that. Once we see this it is easy to see that we can't do it even in the case of two objects — like Max Black's spheres — because, again we would need to form sequences of these two objects, which requires pairing with ordinals, for which they would need to be absolutely distinguishable. Max Black in his discussion and defence of his example was perfectly clear that this couldn't be done without assuming the spheres to be absolutely distinguishable in some way, contrary to hypothesis. Even if we think of pairing them up in some colloquial way, say by speaking of *this one* and the *other one*, we have no way of knowing which one is referred to by 'this one'. We can even allow that we have two names for the two balls, say 'Castor' and 'Pollux'. But it is an empty idea, for we can't say which ball has which name, so we are back again at square one. Thus there is no intermediary position between *absolutely distinguishable* and *absolutely indistinguishable*. *Weak distinguishability does not exist, unless it is applied to entities that are already absolutely distinguishable*. It is a chimera.[3]

But there is worse to come. In the literature there is widespread use of something called 'permutation invariance' where this is to be applied to particles, such as electrons. The problem is that we *can't* permute indistinguishable entities.

[1] This sentence carries the odd implication that it is somehow an *option* that the state of the fermion collective be antisymmetric. If it is not antisymmetrized we do *not* have fermions.

[2] Similar remarks can be found in later publications on this subject, for example Saunders 2018: 170.

[3] As noted the notion of weak discernibility comes from Quine 1976. But Quine does not give a worked example of this notion, so we don't get to see how it was supposed to apply to, for example, Black's spheres. As soon as we try to set up the formal notion of satisfaction the problems appear.

(And what *could* it mean to permute particles themselves, to change their order in a sequence?) So let us take four electrons. We would have to have a 1 :1 correspondence with four ordinals (i.e. a numbering of them) in order to permute them, which would, again, mean that they were absolutely distinguishable. What is really intended here by permutation invariance is that the permutations be applied not to the particles, but to the tensor product state space — made, in our example, of four Hilbert spaces which have been turned into a tensor product in all of ways they can be by permutation of the components — i.e. the component Hilbert spaces. What this signifies is that there is no privileged way of aligning one such component with any one particle. All of the particles can only be associated with any, and all, of the permutations equally. Thus the association is said to be *permutation invariant*. In fact, strictly, when we consider the permutations in the context of bosons and fermions, we are only interested in two conjugacy classes of the full set of permutations constituting the symmetric group (in our example of 4 spaces): the *symmetric* and the *antisymmetric* classes. These are only two of the five such classes, the remaining three are 'thrown away' as having no physical significance (a sometimes disputed claim). As the number of spaces that are combined together in the tensor product grow, the number of conjugacy classes that are thrown away grows quite quickly. With 5 component spaces there would be 7 conjugacy classes in total, 5 conjugacy classes thrown away. With 100 components it is 190,569,290 thrown away.

But let us return to the philosophical discussion.

Ladyman claims that the singlet state of a 2-particle quantum system can be represented in a graph-theoretic form, as an unlabelled graph of two nodes.[4] He says:

> The case of weak discernibility, without absolute or relative discernibility, is exemplified by the following unlabelled graph *G* with two nodes and one edge. This is the graph- theoretic counterpart of Black's two-spheres universe (or the complex field substructure consisting of the imaginary units i and $-i$, or the singlet state of two fermions): . . . - (Ladyman 2007: 34)

I agree with his claim that this graph can represent the two roots of −1, and I agree that it can represent the two Black spheres. I don't agree that it represents, without very great loss, the 'singlet state of two fermions' in QM. So while it could be said that we have here an asymmetric relation between two things that cannot be distinguished, it does not represent an *anti*symmetric relation between two fermions in the singlet state. The remainder of this essay is dedicated to this point.

The simplest way to make this point is to consider how little the graph represents of the singlet state. Here is a good description of the singlet state, which in itself does not give a full picture of how the entangled states sit in the space CP^3 (complex projective 3-space) with respect to the conic surface on which the disentangled states sit.

We recall that for orthogonal states the Fubini-Study distance is π, the greatest distance possible for two states. On the other hand, the maximum distance an entangled state can have from the closest disentangled state, in the case of two spin-½ particles, is π . For example, with respect to a given choice of spin axis, the spin 0 singlet state ϵ^{AB} can be expressed as an antisymmetric superposition of two disentangled states, i.e., an up-down state and a down-up state. The two disentangled states are mutually orthogonal, and the singlet state lies 'half way' between them. Brody and Hughston (2001) p. 13

The first thing we may note is that we have here a maximal value of the relevant metric, which here is the Fubini-Study metric. This is nowhere presented in the graph model. Secondly, the superposition of the two disentangled states is for *every* choice of spin axis, not just one, but this is nowhere represented in the graph model. Thirdly, the two vertices of the graph cannot by themselves represent states, as they are in fact

[4] The idea of the fundamental metaphysical status of graph theory seems to have originated in the Peircean scholar Randall Dipert, in his 1997. Dipert's main point was the ontological importance of asymmetric graphs.

required to: instead the vertices are misrepresented as particles. In other words the graph is taken to represent particles that are only weakly distinguishable, where this has nothing to do with representing the singlet state in which the vertices must represent those orthogonal states. This confusion of purpose is in evidence throughout and makes it seem as though the unsustainable notion of weak discernibility has been vindicated by conflating it with the entanglement relation of the singlet state. These are just different things; the first one false, the second part of physics.

If one looks at the geometry of the singlet state — it is well represented by Brody and Hughston (2001), sect's 8–10 — one can see that it is fearsomely intricate and that it is situated in a complex projective space of three dimensions. It is not a simple graph (in one real dimension). It should also be noted that this one example cannot convince anyone of the significance of the use of graphs, for the simple reason that it does not give us any idea of what the graph would be in the case of three or more entangled particles. Two entangled particles have special properties that do not scale. For example the Schmidt decomposition is only available in the bipartite case.

The envoi of this section is: entanglement of a pure state of any bipartite system may be fully characterised by its Schmidt decomposition. All entanglement monotones are functions of the Schmidt coefficients. Bengtsson and Życzkowski (2017) p. 454.

A theory based on a single case is not even a theory!

But there is a very general question that we may ask: can the entanglement correlations be re-garded as pre-existent, in the way hidden variables were meant to be? Are these relations objective 'elements of reality', to use Einstein's phrase: is it the case that *Relations are All*!, as the slogan would have it. Surely if monadic properties are not hidden and preexistent, then *dyadic* properties should be the same.

Perhaps the definitive argument on this point is contained in a paper by Adán Cabello (Cabello 1999), which is an adaptation of the GHZ argument concerning three spin-½ particles.[5] In Cabello's argument we are concerned with three *pairs* of particles, six particles in all, and the measurements that can be made on them on a $2^6 = 64$-dimensional Hilbert space H. However instead of looking at values of measurements on single particles and seeing what happens if they are assumed to exist prior to measurement, as in the GHZ argument, the idea is to look at correlations and ask what would happen if these were pre-existent. For this purpose two sets of operators are defined on the pairs, {1, 2}, {3, 4} and {5, 6} on H. These pairs are owned by Alice, Bob and Charlie, respectively. One set of operat-ors consists of the Bell operators, B operators, (as defined in Braunstein, Mann, and Revzen (1992)) which measures pairs of particles to see in which of four Bell states they are. These operators are defined on the three pairs and have eigenvalues ±1, ±2. For example here it is defined on the pair {1, 2}

$$B_{12} = 2\hat{\phi}^{+}_{12} + \hat{\psi}^{+}_{12} - \hat{\psi}^{-}_{12} - 2\hat{\phi}^{-}_{12}.$$

And mutatis mutandis for the other two pairs {3, 4} and {5, 6}. Here $\hat{\phi}^{+}$ is the projection operator onto one of the four Bell basis vectors:

$$|\phi^{+}\rangle_{12} = \frac{1}{\sqrt{2}}\left(|+\rangle_1\,|+\rangle_2 + |-\rangle_1\,|-\rangle_2\right).$$

B_{12} can be considered to be an answer to the question: is the {1, 2} system in an eigenstate of $|\phi^{+}\rangle$ (with eigenvalue 2), or eigenstate $|\psi^{+}\rangle$ (with eigenvalue 1), or $|\psi^{-}\rangle$ (with eigenvalue −1), or $|\phi^{-}\rangle$ (with eigenvalue −2)?6

Then we have the A operators. These are also defined on pairs of particles, with the same eigenvalues as above.

$$A_{12} = 2\hat{\alpha}_{12}^{++} + \hat{\alpha}_{12}^{+-} - \hat{\alpha}_{12}^{-+} - 2\hat{\alpha}_{12}^{--}$$

Of course mutatis mutandis for the two other pairs, Here $\alpha^{\hat{}++}$ is the projection operator $|\alpha^{++}\rangle$ (which is defined on the tensor product with the + result for both particles 1 and 2). It can be interpreted in terms of the question: 'is the system {1, 2} in the state $|++\rangle$, or $|+-\rangle$, or in state $|-+\rangle$ or in state $|--\rangle$, with eigenvalues 2, 1, −1, −2, respectively?'.

We now take products of three operators acting on the three different pairs where the latter are assumed to be space-like separated from one another. The experimenters Alice, Bob and Charlie are each able to apply their observables A or B to their pair. We consider four operators that commute with one another and thus share a basis of eigenvectors. $|\mu\rangle$ is one of those eigenvectors for the space of all six particles and it is assumed to represent the initial state of the whole.

$$\begin{aligned} (A_{12}A_{34}B_{56})\,|\mu\rangle &= |\mu\rangle \\ (A_{12}B_{34}A_{56})\,|\mu\rangle &= |\mu\rangle \\ (B_{12}A_{34}A_{56})\,|\mu\rangle &= |\mu\rangle \\ (B_{12}B_{34}B_{56})\,|\mu\rangle &= -\,|\mu\rangle\,. \end{aligned} \tag{1}$$

If measurements were made by Alice, Bob and Charlie on their pairs then the values that could result from an A measurement results in an m-number and a B measurement listed as an n-number.

In particular the results for the above triple sets can be given as

$$\begin{aligned} m_{12}m_{34}n_{56} &= 1 \\ m_{12}n_{34}m_{56} &= 1 \\ n_{12}m_{34}m_{56} &= 1 \\ n_{12}n_{34}n_{56} &= -1. \end{aligned} \tag{2}$$

Each m and n number on the left hand side appears twice, so the product of all twelve together must be positive. However the product of the 4 numbers in the right column is negative, i.e. −1.

We want to show that the assumption that the correlations of a system are present from the outset leads to a contradiction. Thus suppose that two experimenters, say Alice and Bob, make a measure- ment to get a value on the correlation of their pair. Alice measures using her B_{12} observable, and Bob his B_{34}. Suppose they both get eigenvalue 1. They can thus predict that n_{56} must equal −1. This represents a singlet state $|\psi^{-}\rangle$ (by the last line of (3) meaning n_{56} are anti-correlated particles). But this was predicted without any interaction with Charlie's system and so it must initially also have been in the eigenstate with eigenvalue −1.

But what if Alice had instead chosen her A_{12} observable and Bob still his B_{34} observable? Then, whether Alice gets an eigenvalue of 1 or −1, the eigenvalue for A_{56} is predictable since (by the second line of (2)) the product must equal 1. This would be the eigenvalue for the correlation of the z -spin components, which again, since no physical interference has taken place, must have been present in the initial state μ. We give Cabello the final words:

> Such predictions with certainty and without interaction would lead us to assign values to the six types of correlations given by A_{12}, B_{12}, A_{34}, B_{34}, A_{56}, and B_{56}. However, such an assignment cannot be consistent with the rules of quantum mechanics because the four equations [in (3)] cannot be satisfied simultaneously, since the product of their left-hand sides is a positive number (because each value appears twice), while the product of the right-hand sides is −1. Therefore, the whole information on the correlations between the particles of the three pairs cannot be encoded in the initial state as we assumed. (Cabello 1999, 2)

Cabello has shown that this argument can also be interpreted as a Kochen-Specker-style "no-go" theorem on a pentagram diagram. His argument reinforces the point that entanglement in a system of three particles or more has a contextual character, an entanglement *of* entanglement (Krenn and Zeilinger (1996)).

Thus the correlations of particles cannot, as a matter of necessity, be pre-existent, objective, relations that experiment simply reveals in a passive way. Just as measurement creates the spin eigenvalue so it also creates the correlation eigenvalue. Thus QM cannot — again, as a matter of necessity — be interpreted in the way proposed by OSR.

Why might this result have been expected? Suppose that correlations *had* been objective, pre-existent relations; we would expect then that what they are correlating also be objective, pre-existent properties of entities — for what would correlation be without things being correlated thus-and-so? But this would take us to hidden variables. And these in turn take us to distinguishable particles, distinguishable by their pre-existent, objective *accidents* — the spin being + along the z-axis, for example. This chain of implications is compelling reason to think that Cabello's argument has an inevitability to it once we accept the fact of indistinguishability. For by modus tollens, indistinguishability would imply no hidden variables, which would in turn imply no determinate, pre-existent objective correlation relations.

A metaphysics of pre-existent relations does not arise from the mathematics underlying the physics, and it is a conspicuous fact that quantum mechanics is *not* written in the language of graph theory.[7] QM is, as we have noted, written in the language of convex sets, as realised by density matrices. What we see in the convex sets is a world of composites and the many ways in which they may be reduced to components. We have been led by commonplace metaphysical assumptions to imagine hidden variables. Now we are imagining *dyadic* hidden variables: relations. But unfortunately these go the same way as their poorer monadic brethren.

Post-measurement anti-correlations are not a sign of relations existing between particles that might exist prior to measurement and irrespective of what we choose to measure. We must beware of making correlation relations the new "hidden variable", present irrespective of measurement, part of a reified metaphysics of fundamental relations with particles as individuals by virtue of being the bearers of said relations. We may say that just as an electron's spin is not there prior to a measurement being made *neither is the correlation between a pair of particles*. This must be emphasised to make it clear that the structuralist has no retreat to the idea that the relation is only between eigenstates.

This false idea comes from the mistake about the singlet state we noted earlier: once we are down to eigenstates the particles are separable rather than entangled.

However it is possible that John Worrall himself steered things in the wrong direction in the following sentence, that

> It is a mistake to think that we need to understand the nature of the quantum state at all; and *a fortiori* a mistake to think that we need to understand it in classical terms. Worrall (1989)

If the vaunted structure was *not* going to be the structure that underlies the quantum state then what should it be? The view from 1989 might have made this claim seem plausible, but given what we now know it couldn't have been more wrong. Understanding the quantum state has led to extraordinary gains — including understanding that it can't be understood classically. So if realism is not to be concerned with the quantum state then what content could it have? Only the relation between particles seems left. And this then became the basis for French and Ladyman's form of structural realism. Structural realism was given the task of shedding light on how particles might be individuals after all, or at least quasi-individuals — of, in the words of French and Ladyman, '...the need to provide an ontology that can dissolve some of the metaphysical conundrums of modern physics.'

Let us agree that this is a worthy ambition and go back to our discussion of the nature of *n*-adic properties from the opening section. What happens if we compare the idea of velocity in relativity theory with the eigenvalues of observables in quantum theory? The appearance of the latter given a measurement is the mystery we want to dissolve: the *measurement problem*.

(I don't know whether it is possible to *solve* this problem but it may be possible to throw a *small* amount of light on it.) It should go without saying that we treat properties and relations on a par.

When we look at what is *elapsed, time* or *velocity*, or *distance in space*, in relativity theory we see that there is an imposition of a Lorentzian frame of reference, as a result of which some particular value results. But what value was present before the choice of the frame of reference? The answer obviously is that there was no such attribute; there is no velocity absent the application of such a frame. These qualities are being brought from being *in potentia* into what may be seen as actuality. And it is obviously true that there is no one single natural frame of reference: rather there can be many frames with very different values in each frame, all of which may have some claim to being a natural frame for us. But 'natural for us' is an anthropocentric imposition: the space-time itself is indifferent to our choices. For it, these properties, so important to us, have no significance to it, As Minkowski said, with relation to his space-time model: 'Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.'

These words were thoughtfully and deliberately chosen and have perhaps only rarely been properly understood. Thus we have the fact that light has the same velocity in all frames and thus is absolute, or non-relative to a frame. In fact it may be more correct to say that the frames are relative to the null cone, which is the Absolute.

Now let us compare this with the situation in quantum mechanics. Here we have particles and certain characteristics which are at best latent, prior to measurement. When we make the measurement there is, in a way that is not understood, the appearance of a value for that measurement of that particular observable: an eigenvalue. We may say, as we said in the above case of velocity, that we have brought the value, which was only *in potentia*, into actuality. We may also say that the measurement constitutes the imposition of a frame of reference: applied to an observable it yields up an eigenvalue. To ask whether that value was there prior to the measurement is like asking whether the velocity

[5] For GHZ see Greenberger Horne and Zierlinger (1989). Cabello's argument was directed initially against Mermin's Ithaca interpretation of QM, for which see Mermin (1999).

[6] In equation (1) above we followed Saunders in using up and down arrows but these are the same as the plus and minus signs here.

[7] To forestall one possible response we note that graphs *did* attain a use in illustrating the impossibility proof of the Kochen-Specker theorem as well as Cabello's theorem. But these graphs essentially showed how QM could *not* be represented. We are not anti-graph or anti- any useful mathematics.

was present prior to the choice of frame in which it appeared. It is meaningless, and both meaningless in the same way. The eigenvalues are relative to the measurement. And in fact, instead of speaking of the 'measurement' we might blend the term together with the relativity case and speak of the 'measurement-frame'. Thus if we try to seek the position for the particle we may find it present in a particular space-time location. This position has no meaning with respect to Minkowski space-time, but does have a meaning with respect to some Lorentzian frame of reference and we may be understood to have chosen that frame in making that measurement. Thus these position measurements may be reconciled with relativity theory — and the same can be said of momentum measurements. Not only *can* they be reconciled with relativity theory, they *must* be. For position and measurement only makes sense in relativity theory with respect to Lorenztian frames of reference, so the measurement here must be relative to a measurement-frame. Again it is meaningless to ask about the position of the particle absent the application of a measurement-frame. This is strikingly different to the situation with respect to spinors which can be, and have been, defined directly on the light cone (the 'Absolute') itself.

There are two significant differences between relativity theory and quantum theory and that is the appearance of probabilities in the latter and the noncommutative nature of the possibilities of measurement. These are doubtless the manifestation of a single underlying fact, namely the projective geometry of Minkowski space-time, which is also what is underlying the above picture of the relation between properties and measurement-frames. This provides the contextuality that we signalled in the opening section for both properties and relations. We end on this suggestive note.

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[8] Rovelli's argument for RQM comes to mind immediately here, but, as will be obvious, I part company with it at an early stage, as also with Dorato's use of Rovelli that also diverges from it. Nevertheless I think it's first inspiration was a great step forward.

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